# INTER LAMINAR THERMAL STRESS ANALYSIS IN SYMMETRIC COMPOSITE LAMINATED METALLIC

**Juan José Delfín Vázquez***

**Jorge Carrera Bolaños****

**Mario Acosta Flores*****

**Eusebio Jiménez López******

**Abstract.**

*A model is presented that allows the evaluation of global and interlaminar thermal stresses in symmetrical metallic laminated composites. Besides being fully compatible with experiments, it is relatively simple. It is analogous to the model used by Acosta [1] to determine interlaminar and global stresses in composites under tension. It is linear, the superposition principle is valid. To validate the model electrical strain gauge technology and a special heat chamber were used. Experimental tests were performed in two composites metal laminates; one of two layers of aluminum and a stainless steel intermediate layer (AL-AI-AL), and another with two stainless steel layers and an intermediate layer of aluminum (AI-AL-AI). The experimental data were feed up in the model; and interlaminar and global thermal stresses and thermal expansion coefficients (CET) were determined for each layer. Experimental results show consistency and reliability in the application of the model.*

*Key words*: Thermal stress, interlaminar stress, thermal expansion coefficient, thermal strain, composite material, experimental methods

* Centro de Tecnología Avanzada del Instituto Tecnológico Superior de Cajeme

** División de Estudios de Posgrado de la Facultad de Ingeniería, UNAM

*** División de Estudios de Posgrado de la Facultad de Ingeniería, UNAM

**** CINNTRA de la UTS-ULSA Noroeste-IIMM

## Introduction

The determination of interlaminar thermal stresses in composite materials is a complex but needed task. This problem is usually solved numerically, but in such away that is not compatible with experimental techniques [2, 3, 4, 5]. These stresses have also been analyzed in free edges [6, 7]. There are several models currently used to determinate these stresses, for example the model of displacement Zig-Zag function [8], the Global-Local Theory [9] or two-dimensional model based on Mixed Variacional Theorem [10]. But they are generally complicated.

In contrast, the model here proposed is a relatively simple linear model, able to evaluate global (average) and local (interlaminar) thermal stresses in laminated composites subject to temperature changes. The experimental data required are obtained by applying controlled changes in temperature and measuring strain at the borders. Besides the experimental data, the model requires the elastic and thermal properties of the components of the material. It is validated using an inverse method, collecting data through electrical strain gauge [1].

## 1. Analytic model

The premises are:

a) The layers constituting the compound are perfectly joined, the adhesive thickness is zero (it doesn´t affect mechanically) and there are Dirichlet conditions of continuity between the layers.

b) Layers and laminates are thin and the thickness remains constant.

c) The laminate is symmetric

d) The physics of the problem can be adequately approximated by a linear model.

f) The distribution of interlaminar thermal stresses caused by thermal temperature changes is considered uniform throughout the thickness.

Under these premises, when there are temperature changes in the laminated composite, the average stress state for each point is a state of plane stress (Figure 1)

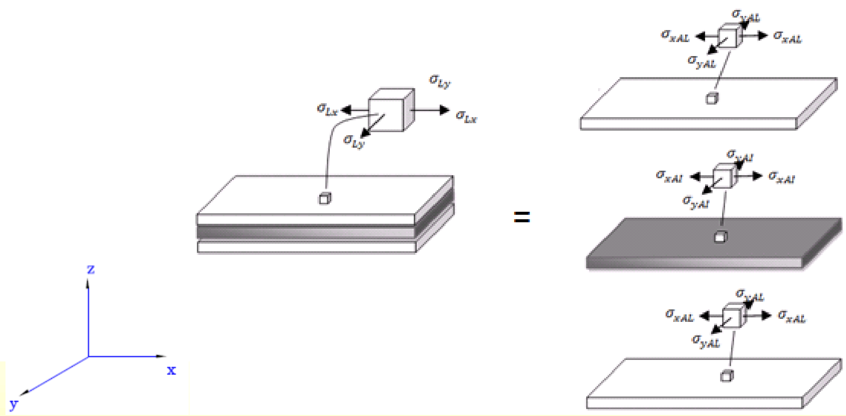



Figure 1.Planes Stress problem in laminated composite subjected to thermal loads

### 1.1 Analytical model for thermal stresses

**2.1. The global problem of thermal stresses**. It is assumed that the strain state is represented by a homogeneous strain [11]. Then the strains in the main directions$\varepsilon_1^0$and$\varepsilon_2^0$are constantat all points, [1],[12]:

$$\varepsilon_1(z) = \varepsilon_1^0 \;\; ; \;\; \varepsilon_2(z) = \varepsilon_2^0. \qquad (1)$$

Where the exponent denote that the strains in function of the direction z are constants.

Applying the superposition principle, a general model that relates the global stresses to interlaminar in each layer:

$$\sigma_{G1} = \eta_I\sigma_{1I} + \eta_{II}\sigma_{1II} + \eta_{III}\sigma_{1III} + \cdots + \eta_i\sigma_{1i} \qquad (2)$$

$$\sigma_{G2} = \eta_I\sigma_{2I} + \eta_{II}\sigma_{2II} + \eta_{III}\sigma_{2III} + \cdots + \eta_i\sigma_{2i}$$

where $\sigma_{G1}$, $\sigma_{G2}$ are average global stresses in the directions $\varepsilon_1^0$ and $\varepsilon_2^0$; $\sigma_{1i}$, $\sigma_{2i}$ are thermal iterlaminar stresses, and $\eta_i$ are volumetric fraction of the material.

The volumetric fraction ($\eta$) is determined by:

$$\eta = \frac{h_i}{h} \qquad (3)$$

$$1 = \eta_I + \eta_{II} + \eta_{III} + \cdots . + \eta_i$$

were $h$ and $h_i$ are the total thickness of the laminate and the thickness of the constituent layers, respectively.

In anisotropic laminate the principal axes coincide with the axis of symmetry x, y, so thatθ= 0 (angles are measured in radians). According to[12]:

$$\varepsilon_x = \varepsilon_1 m^2 + \varepsilon_2 n^2 + \varepsilon_6 mn \text{and} \varepsilon_y \;\; = \varepsilon_1 n^2 + \varepsilon_2 m^2 - \varepsilon_6 mn$$

$m = \cos\theta\,,\; n = sen\theta$, are the direction cosines, and, therefore:

$$\varepsilon_x = \varepsilon_1 \text{and} \varepsilon_y = \varepsilon_2$$

So that:

$$\varepsilon_1(z) = \varepsilon_1^0 = \varepsilon_x^0 \text{and} \varepsilon_2(z) = \varepsilon_2^0 = \varepsilon_y^0$$

The needed constitutive equations for mechanical and thermal stresses, for each of the layers are:

$$\sigma_{xn} = \frac{E_n}{1-\nu_n{}^2}\left(\varepsilon_x^0 + \nu_n \varepsilon_y^0\right) - \frac{E_n \alpha_n \Delta T}{1-\nu_n} \quad (4)$$

$$\sigma_{yn} = \frac{E_n}{1-\nu_n{}^2}\left(\varepsilon_y^0 + \nu_n \varepsilon_x^0\right) - \frac{E_n \alpha_n \Delta T}{1-\nu_n}$$

for each n-layer. $\sigma_{xn}$ and $\sigma_{yn}$, are interlaminar thermal stresses in the x, y directions; $E_n$ is the elastic modulus: $\nu_n$ is Poisson's ratio; $\alpha_n$ the thermal expansion coefficient ofthe *n* layer;$\varepsilon_x^0$, $\varepsilon_y^0$ are homogeneous longitudinal and transverse strain, ΔT is the temperature change.

**2.2. The model as a system of polynomial equations**

The constitutive equations (4) are incorporated into the general model (2). Assuming a composite of two materials $C_1$ and $C_2$, the following resulting system of equations allows the calculation of the global and interlaminar thermal stresses of the constituent layers:

$$\sigma_{Gxc1} = 0 = \eta_{ALc1}\sigma_{xALc1} + \eta_{AI1}\sigma_{xAIc1} \quad (5)$$

$$\sigma_{Gxc2} = 0 = \eta_{ALc2}\sigma_{xALc2} + \eta_{AI2}\sigma_{xAIc2}$$

$$\sigma_{Gyc1} = 0 = \eta_{ALc1}\sigma_{yALc1} + \eta_{AI1}\sigma_{yAIc1}$$

$$\sigma_{Gyc2} = 0 = \eta_{ALc2}\sigma_{yALc2} + \eta_{AIc2}\sigma_{yAIc2}$$

$$\sigma_{xALc1} = \frac{E_{AL}}{1-\nu_{AL}{}^2}\left(\varepsilon_{xc1}^0 + \nu_{AL}\varepsilon_{yc1}^0\right) - \frac{E_{AL}\alpha_{AL}\Delta T_{c1}}{1-\nu_{AL}} \quad (6)$$

$$\sigma_{xAIc1} = \frac{E_{AI}}{1-\nu_{AI}{}^2}\left(\varepsilon_{xc1}^0 + \nu_{AI}\varepsilon_{yc1}^0\right) - \frac{E_{AI}\alpha_{AI}\Delta T_{c1}}{1-\nu_{AI}}$$

$$\sigma_{xALc2} = \frac{E_{AL}}{1-\nu_{AL}{}^2}\left(\varepsilon_{xc2}^0 + \nu_{AL}\varepsilon_{yc2}^0\right) - \frac{E_{AL}\alpha_{AL}\Delta T_{c2}}{1-\nu_{AL}}$$

$$\sigma_{xAIc2} = \frac{E_{AI}}{1-\nu_{AI}{}^2}\left(\varepsilon_{xc2}^0 + \nu_{AI}\varepsilon_{yc2}^0\right) - \frac{E_{AI}\alpha_{AI}\Delta T_{c2}}{1-\nu_{AI}}$$

$$\sigma_{yALcc1} = \frac{E_{AL}}{1-\nu_{AL}{}^2}\left(\varepsilon_{yc1}^0 + \nu_{AL}\varepsilon_{xc1}^0\right) - \frac{E_{AL}\alpha_{AL}\Delta T_{c1}}{1-\nu_{AL}}$$

$$\sigma_{yAIc1} = \frac{E_{AI}}{1-\nu_{AI}{}^2}\left(\varepsilon_{yc1}^0 + \nu_{AI}\varepsilon_{xc1}^0\right) - \frac{E_{AI}\alpha_{AI}\Delta T_{c1}}{1-\nu_{AI}}$$

$$\sigma_{yALc2} = \frac{E_{AL}}{1-\nu_{AL}{}^2}\left(\varepsilon^0_{yc2} + \nu_{AL}\varepsilon^0_{xc2}\right) - \frac{E_{AL}\alpha_{AL}\Delta T_{c2}}{1-\nu_{AL}}$$

$$\sigma_{yAIc2} = \frac{E_{AI}}{1-\nu_{AI}{}^2}\left(\varepsilon^0_{yc2} + \nu_{AI}\varepsilon^0_{xc2}\right) - \frac{E_{AI}\alpha_{AI}\Delta T_{c2}}{1-\nu_{AI}}$$

where:

$\sigma_{Gxc1}$, $\sigma_{Gyc1}$, y $\sigma_{Gxc2}$, $\sigma_{Gyc2}$, are the average global thermal stresses in x, y, for $C_1$ and $C_2$,.

$\sigma_{xALc1}, \sigma_{yALc1}, \sigma_{xAIc1}, \sigma_{yAIc1}$, are interlaminar thermal stresses in x direction of each the aluminum caps. $C_1$.

$\sigma_{xALc2}, \sigma_{yALc2}, \sigma_{xAIc2}, \sigma_{yAIc2}$,are the corresponding stresses for Stanley steel, $C_2$.

$\varepsilon^0_{xc1}$, $\varepsilon^0_{yc1}$, $\varepsilon^0_{xc2}$, and$\varepsilon^0_{yc2}$, are longitudinal and transverse strain for both composites.

$E_{AL}, \nu_{AL}, \alpha_{AL}$, y $E_{AI}, \nu_{AI}, \alpha_{AI}$, are the elastic and thermal constant of aluminum and Stanley steel caps, $C_1$ and $C_2$.

$\Delta T_{c1}$,$\Delta T_{c2}$are temperature changes in the composites.

## 3. Experimental validation

The validation is based on an inverse method [1].

If the thermal strain, the elastic constants and temperature changes are known, this method determines the thermal expansion coefficients (CTE) of each of the constituent layers.

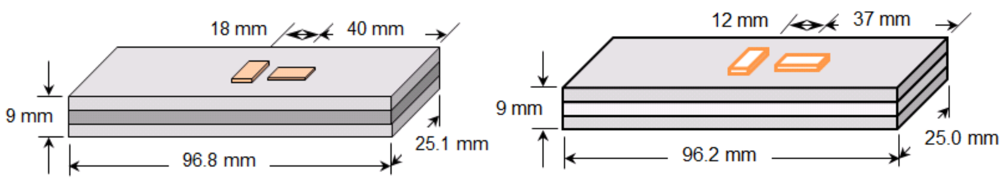
Experimentally, three-layer specimens were used, made of aluminum (AL) and stainless steel (AI) in two configurations: AL-AI-AL and AI-AL-AI. Adequate confirmation was obtained.

### 3.1 Instrumentation specimens and experimental testing

To evaluate the model a series of tests in composites materials were made.

A set of three specimens was made. One specimen was made solely AL2024-T3 aluminum, with thermal and elastic properties known. It was used as reference material. The other two were composites, AL-AI-AL (C1), and AI-AL-AI (C2), both with unknown properties.

Each specimen was instrumented with two strain gages, except the2024-T3 where just one was fixed.EA-06-120LZ-120 /E strain gages were used. Figure 2 shows the location of the strain gags in the composites.




a) In AL-AI-AL composite b) In AI-AL-AI composite

Figure 2.Dimensions and location of the strain gages in the composites

The specimens were installed in a thermal chamber where the tests were made.

### 3.2. Experiments

Experimental tests were carried out to obtain the thermal expansion coefficients and elastic constants were as follows:

**3.2.1. Tests on simple specimens.** Were carried out according to [12], [13], and [14], experimental test, results are shown in Table 1.

| Thermalproperties | Aluminum | Stanley Steel |
|---|---|---|
| Thermal Expansion Coefficient $(\alpha_s)^{\mu\varepsilon}/_{°C}$ | 24.2 | 17.6 |
| ElasticityModules E Gap | 70.1 | 202.4 |
| Poissón Ratíov | 0.337 | 0.275 |

Table 1.Materials data

### 3.2.2. Thermaltesting in thecomposites

The test was made raising the temperature 5°Cevery 20 minutes. The temperature range was 23.4to63.2°C.The results are shown in Table 2. The time constants remained deformations recorded in the table.

| SETTING | TEMP | COMPOSITE AL-AI-AL | | COMPOSITE AI-AL-AI | | ALUMINUM 2024-T3 | TIME |
|---|---|---|---|---|---|---|---|
| °C | °C | $\varepsilon^0_{xc1}$ $\mu\varepsilon$ | $\varepsilon^0_{yc1}$ $\mu\varepsilon$ | $\varepsilon^0_{xc2}$ $\mu\varepsilon$ | $\varepsilon^0_{yc2}$ $\mu\varepsilon$ | $\varepsilon_x$ $\mu\varepsilon$ | Min |
| | 23.4 | 0 | 0 | 0 | 0 | 0 | |
| 25 | 26.2 | 10 | 12 | 7 | 7 | 12 | 11 |
| 30 | 34.3 | 80 | 81 | 54 | 55 | 113 | 12 |
| 35 | 36.3 | 102 | 101 | 64 | 62 | 132 | 10 |
| 40 | 44.2 | 165 | 164 | 101 | 99 | 214 | 12 |
| 45 | 49 | 213 | 211 | 131 | 126 | 256 | 11 |
| 50 | 53 | 252 | 250 | 145 | 141 | 296 | 11 |
| 55 | 56.2 | 286 | 283 | 160 | 156 | | 10 |
| 60 | 63.2 | 359 | 354 | 189 | 184 | | 13 |
| 65 | 66.4 | 395 | 392 | 215 | 211 | | 12 |

Table 2. Experimental data obtained in the composites test

## 4. Correcting thermal strain

The corrections of the thermal strain of aluminum and stainless steel were made according to standard procedures [15]. The temperature and the values of the strains produced by the change in resistance of the strain gages were expressed in terms of increases. The reference temperature was 23.4°C and the corresponding strains $\varepsilon_{xc1}^0 = \varepsilon_{yc1}^0 = \varepsilon_{xc2}^0 = \varepsilon_{yc2}^0 = 0$, the data related to the two composites and the2024-T3 aluminum, used as reference, are shown in Table 3, Figure 3 shows the relationships of the temperature change ΔT and strains in the composites and the reference material.

| | COMPOSITE | | | | MATERIAL |
|---|---|---|---|---|---|
| TEMP. | AL-AI-AL | | AI-AL-AI | | AL 2024-T3 |
| ΔT | $\varepsilon_{xc1}^0$ | $\varepsilon_{yc1}^0$ | $\varepsilon_{xc2}^0$ | $\varepsilon_{yc2}^0$ | $\varepsilon_x$ |
| °C | $\mu\varepsilon$ | $\mu\varepsilon$ | $\mu\varepsilon$ | $\mu\varepsilon$ | $\mu\varepsilon$ |
| 2.8 | 11 | 12 | 7 | 7 | 12 |
| 10.9 | 80 | 81 | 54 | 53 | 113 |
| 12.9 | 100 | 100 | 65 | 64 | 132 |
| 20.8 | 165 | 164 | 110 | 109 | 214 |
| 25.6 | 213 | 211 | 131 | 129 | 256 |
| 29.6 | 253 | 251 | 145 | 142 | 296 |
| 32.8 | 290 | 287 | 160 | 157 | |
| 39.8 | 359 | 356 | 189 | 186 | |
| 43.0 | 395 | 392 | 203 | 198 | |

Table 3. Temperature increase and strain

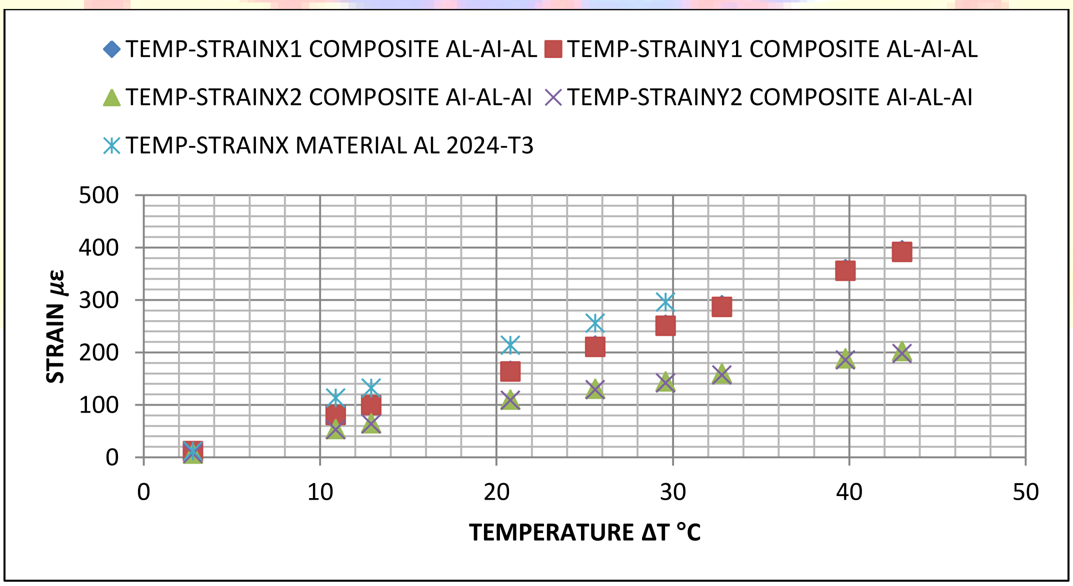



Figure3.Relation ΔT-experimental strain composites and specimen AL 2024-T3

With the thermal strain, temperature change (Table 3), and reference CET ($\alpha_R = 23.4\ ^{\mu\varepsilon}/_{\circ C}$), the average CET ($\alpha_{SPxc1}$,$\alpha_{SPxc2}$ ,$\alpha_{SPyc1}$, and $\alpha_{SPyc2}$) are obtained, and then, by multiplying the average CET per the temperature change, the strain corrected ($\varepsilon_{Cxc1}$, $\varepsilon_{Cxc1}$, $\varepsilon_{Cxc1}$, and $\varepsilon_{Cxc1}$) are calculated, see Table 4. The process to correct the thermal strain will be shown in future work.

Since $\Delta T_{c1} = \Delta T_{c2} = \Delta T$

| | AL | Average CET and corrected strains in *x* | | | | | | Average CET and corrected strains in y | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2024 -T3 | Measured Strain in *x* | | Average CET in *x* | | Corrected Strain in *x* | | Measured Strain in y | | Average CET in *y* | | Corrected Strain in *y* | |
| ΔT | $\varepsilon_R$ | $\varepsilon_{xc1}^0$ | $\varepsilon_{xc2}^0$ | $\alpha_{SPxc1}$ | $\alpha_{SPxc2}$ | $\varepsilon_{Cxc1}$ | $\varepsilon_{Cxc2}$ | $\varepsilon_{yc1}^0$ | $\varepsilon_{yc2}^0$ | $\alpha_{SPyc1}$ | $\alpha_{SPyc2}$ | $\varepsilon_{Cyc1}$ | $\varepsilon_{Cyc2}$ |
| °C | $\mu\varepsilon$ | $\mu\varepsilon$ | $\mu\varepsilon$ | $^{\mu\varepsilon}/_{\circ C}$ | $^{\mu\varepsilon}/_{\circ C}$ | $\mu\varepsilon$ | $\mu\varepsilon$ | $\mu\varepsilon$ | $\mu\varepsilon$ | $^{\mu\varepsilon}/_{\circ C}$ | $^{\mu\varepsilon}/_{\circ C}$ | $\mu\varepsilon$ | $\mu\varepsilon$ |
| 2.8 | 12 | 11 | 7 | 23.04 | 21.614 | 60.2 | 52.7 | 12 | 7 | 23.400 | 21.614 | 60.3 | 52.6 |
| 10.9 | 113 | 80 | 54 | 20.37 | 17.99 | 234.4 | 205.3 | 81 | 53 | 20.464 | 17.895 | 234.4 | 204.6 |
| 12.9 | 132 | 100 | 65 | 20.92 | 18.21 | 277.5 | 243.0 | 100 | 64 | 20.919 | 18.129 | 277.8 | 242.2 |
| 20.8 | 214 | 165 | 110 | 21.04 | 18.40 | 447.4 | 391.8 | 164 | 109 | 20.996 | 18.352 | 448.3 | 390.4 |
| 25.6 | 256 | 213 | 131 | 21.72 | 18.52 | 550.6 | 482.2 | 211 | 129 | 21.642 | 18.439 | 551.7 | 480.5 |
| 29.6 | 296 | 253 | 145 | 21.95 | 18.30 | 636.6 | 557.6 | 251 | 142 | 21.890 | 18.197 | 637.9 | 555.6 |
| SUM | | | | 129.05 | 113.02 | | | | | 129.3302 | 112.627 | | |
| AVERAGE | | | | 21.508 | 18.84 | | | | | 21,550 | 18.771 | | |

**5. Calculation of the CTE of the layers and calculation of interlaminar thermal stresses in composites AL-AI-AL and AI-AL-AI**

Equations (5) and (6), elastic constant, corrected strain, temperature change, and volumetric fractions, are now used to obtain:The interlaminares stresses$\sigma_{xALc1}, \sigma_{yALc1}, \sigma_{xAIc1}, \sigma_{yAIc1}$, and the thermal expansion coefficient of the aluminum $\alpha_{AL}$ and stainless steel $\alpha_{AI}$ layers are also calculated and used to validate the model.

The procedure to determine the CET and interlaminar thermal stresses is:

1. Use equations 5.1) and 5.2), and the equations 6.1), 6.2), 6.3) and 6.4) in the *x* direction, to obtained the CET of $\alpha_{AL}$, and $\alpha_{AI}$ of the layers, and the interlaminar thermal stresses $\sigma_{xALc1}$, $\sigma_{xAIc1}$ in the C1composite, and $\sigma_{xALc2}, \sigma_{xAIc2}$, in C2

composite, see Table 5. The relation temperature change-interlaminar thermal stresses are shown in Figures 4 and 5.

2. With expressions 5.3) and 5.4) and expressions 6.5), 6.6), 6.7) and 6.8), in the *y* direction calculate the CET of aluminum $\alpha_{AL}$ and stainless steel$\alpha_{AI}$ layers, the interlaminar thermal stresses $\sigma_{yALc1}$, $\sigma_{yAIc1}$, of the C1 composite, and $\sigma_{yALc2}, \sigma_{yAIc2}$ of C2 composite. The interlaminar thermal stress and CET both composites are similar to those of the *x* direction.

| ΔT | $\varepsilon^0_{xc1}$ | $\varepsilon^0_{yc1}$ | $\alpha_{AL}$ | $\alpha_{AI}$ | $\sigma_{xAIc1}$ | $\sigma_{xAIc2}$ | $\sigma_{xALc1}$ | $\sigma_{xALc2}$ |
|---|---|---|---|---|---|---|---|---|
| °C | $\mu\varepsilon$ | $\mu\varepsilon$ | $\mu\varepsilon/{\circ C}$ | $\mu\varepsilon/{\circ C}$ | MPa | MPa | MPa | MPa |
| 2.8 | 60.2 | 60.3 | 22.803 | 18.342 | -1.47 | -0.74 | 0.74 | 0.74 |
| 10,9 | 234.4 | 234.4 | 22.832 | 18.319 | -5.79 | -2.92 | 2.90 | 2.92 |
| 12.9 | 277.5 | 278 | 22.804 | 18.369 | -6.74 | -3.39 | 3.37 | 3.39 |
| 20.8 | 447.4 | 448.3 | 22.809 | 18.359 | -10.89 | -5.48 | 5.45 | 5.48 |
| 25.6 | 550.6 | 551.7 | 22.807 | 18.358 | -13.41 | -6.74 | 6.70 | 6.74 |
| 29.6 | 636.6 | 637.9 | 22.808 | 18.356 | -15.51 | -7.81 | 7.76 | 7.81 |
| SUM | | | 136.863 | 110.103 | | | | |
| AVERAGE | | | 22.811 | 18.351 | | | | |

Table 5. Interlaminar thermal stresses and CET in *x* direction, in C1 and C 2 composites

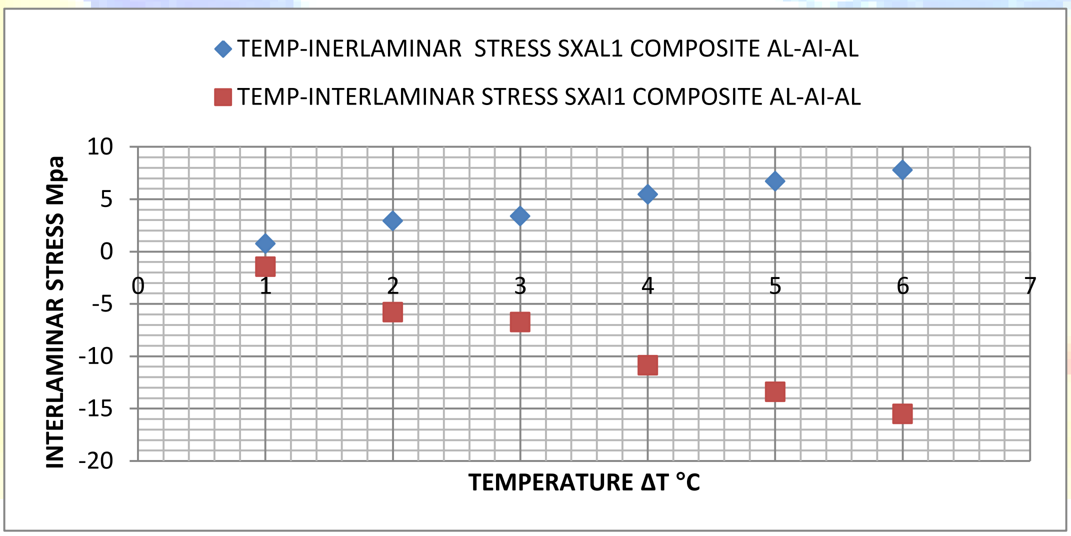



Figure 4. Relation between temperature-interlaminar thermal stresses in x, composite c1

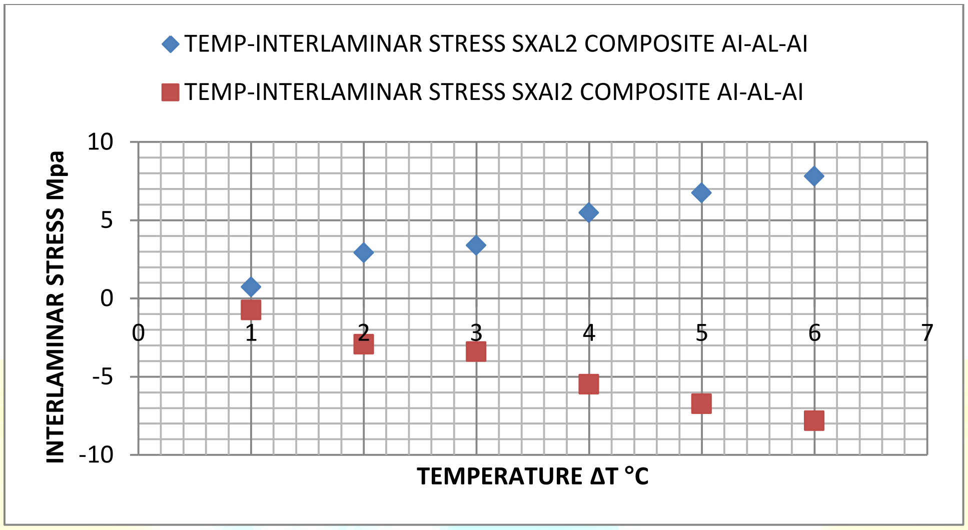



Figure 5.Relation between temperature-interlaminar thermal stresses in y, composite C2

Table 7shows a comparison between CET obtained by the inverse method, by experimental methods (Table 1), and the database MatWeb [16].

| | Method | | Database | Difference | | Percent % | |
|---|---|---|---|---|---|---|---|
| CET | MI | ME | MW | MI-ME | MI-MW | MI-ME | MI-MW |
| $\alpha_{AL}$ $({}^{\mu\varepsilon}/_{\circ C})$ | 22.811 | 24.2 | 23.4 | -1.389 | -0.589 | -5.740 | -2.517 |
| $\alpha_{AI}$ $({}^{\mu\varepsilon}/_{\circ C})$ | 18.351 | 17.6 | 17.3 | 0.751 | 1.051 | 4.267 | 6.075 |

Table 7. Comparison of CET inverse method (MI), respect to experimental method CET (ME), and MatWeb data base (MW)

*The results show that the model is consistent*,with maximal CET deviations less than 10%.

Global thermal stresses in each of the composites are equal to zero, according to the boundary conditions.

As can be seen in Figures 3, 4, and 5, the relation between temperature and strain *is linear*. The same happens between the interlaminar stresses and temperature.

## 6. Conclusions

A new system to calculate both global and interlaminar stresses in symmetric composite materials is presented. It is not only fully compatible with experimental data, in contrast to other used methods, but interacts with the experimentation at each step. *It is relatively simple and well adapted to be programmed*.

Resuming:

6.1. The model *is consistent*. *It evaluates in reliable form* the interlaminar and global thermal stresses, in symmetric laminated composites.

6.2. The experimentally obtained results *showed consistency* because the relationbetween of the states of strain and the interlaminar thermal stresses *can be considered linear; implying that the experimental test was performed in a reliable form.*

6.3. In the $C_1$ (AL-AI-AL) composite, the maximum interlaminar thermal stress in the aluminum layer $\sigma_{xALc1} = 7.76\ MPa$, represent about 7% of elastic limit$S_{LEAL}$, while the maximum in the stainless steel layer $\sigma_{xAIc1} = -15.51\ MPa$, represent 8.08% of $S_{LEAI}$. The stresses generated are small.

6.4. In the $C_2$ (A-AL-AI) composite, the maximum interlaminar thermal stresses in the aluminum layer $\sigma_{xALc1} = 7.81\ MPa$, and the stainless steel layer $\sigma_{xAIc1} = -15.51\ MPa$, represent about 9.76% and 4.06% of the respective elastic limits.

6.5. In both composites the interlaminar thermal stresses in the aluminum layers are compression, while in the stainless steel are tension.

6.6. The thermal expansion coefficient of aluminum obtained by the inverse method, $\alpha_{AL} = 22.81\ {}^{\mu\varepsilon}/_{\circ C}$, is 5.74% less than the experimental CET, and 2.57% lower than MatWeb, whereas inverse method gives for the stainless steel CET,$\alpha_{AI} = 18.351\ {}^{\mu\varepsilon}/_{\circ C}$, which is 4.627% greater than experimental CET, and 6.075% higher than MatWeb CET; however, it does not exceed 10%. *Meaning that the model is consistent and reliably*.

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## Indexing and Archiving