HEAT GENERATION AND UNIFORM SUCTION/BLOWING OF MICROPOLAR FLUID FLOW ON POROUS PLAT WITH TEMPERATURE DEPENDENT VISCOSITY AND THERMAL CONDUCTIVITY

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Abstract

The heat generation of micropolar fluid flow on porous plat with temperature dependent viscosity and thermal conductivity is investigated. The micropolar model due to Eringen is used to describe the working fluid. The partial differential equations governing the motion, angular momentum and energy are reduced to ordinary differential equations using similarity transformations and then solved numerically using Runge- Kutta shooting technique. The effect of the porosity of the medium and the characteristics of the fluid on both the flow and heat transfer is obtained. The results are presented graphically for velocity distribution, temperature distribution and micropolar distributions for various values of non-dimensional parameters. It is found that the effects of the parameters representing variable property of viscosity and thermal conductivity are significant.

Key words: stagnation point flow, stretching sheet , heat generation, viscosity, thermal conductivity.

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1. Introduction

The two dimensional flow of a fluid near a stagnation point is a classical problem in fluid mechanics. It was first examined by Hiemenz [4], who demonstrated that the Navier-Stokes equations governing the flow can be reduced to an ordinary third order differential equation using similarity transformation. Later, the problem of stagnation point flow was extended in numerous ways to include various physical effects. The axisymmetric three-dimensional stagnation point flow was studied by Homman [5]. The results of these studies are of great technical importance; for example, in the prediction of skin-friction, as well as heat/mass transfer near stagnation regions of bodies of high speed flows, and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling, and thermal oil recovery. In either the two or three dimensional case, Navier-Stokes equations governing the flow are reduced to an ordinary differential equation of the third order using a similar transformation. The effect of suction on the Hiemenz flow problem has been considered in the literature. Schlichting and Bussman [9] were the first to give the numerical results. More detailed solutions were later presented by Preston [8]. An approximate solution to the problem of uniform suction is given by Ariel [2]. In hydromagnetics, the probem of Hiemenz flow was chosen by Na [7] to illustrate the solution of a third order boundary value problem. Attia [3] studied the investigation of non-newtonian micropolar fluid flow with uniform suction/blowing and heat generation. Stagnation point flow of a non-newtonian micropolar fluid with zero vertical velocity at the surface or heat generation was studied by Amin et al[1].

The viscosity and thermal conductivity assumed to vary with temperature. The wall and stream temperatures are assumed to be constants. Mathematical formulation of the problem under consideration in presented and similarity transformations are applied to reduce the system of partial differential equations and their boundary conditions describing this problem, into a boundary value problem of ordinary differential equations. The effects of different parameters such as Prandtl number, viscosity parameter, thermal conductivity parameter and microrotation parameter on flow and heat transfer has been studied numerically. The variation of the velocity, microrortation and temperature distribution has been illustrated.

2. Mathematical Formulation Of The Problem

Consider the two dimensional stagnation point flow of an incompressible non-Newtonian micropolar fluid impinging perpendicular on a permeable wall and flowing away along the x axis. The plane potential flow that arrives from the y-axis and impinges on a flat wall placed at y=0, divides into two streams on the wall, and leaves in both directions. The viscous flow must adhere to the wall, whereas the potential flow slides along it. (u,v) are the velocity components at any point(x,y) for the viscous flow, whereas (U,V) are the velocity components for the potential flow. A uniform suction or blowing is applied at the plate with a transpiration velocity at the boundary of the plate given by $-v_0$, where $v_0 > 0$ for suction. The velocity distribution in the frictionless flow in the neighborhood of the stagnation point is given by

$$U(x) = ax\ , \quad V(y) = -ay$$

where the constant $a(>0)$ is proportional to the free stream velocity far away from the surface. The two-dimensional equations governing the flow in the boundary layer of a steady, laminar and incompressible micropolar fluid are

The equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.1)$$

The equation of momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu + h}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{h}{\rho}\frac{\partial N}{\partial y} \qquad (2.2)$$

The angular momentum equation

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - 2N\frac{k}{\rho j} - \frac{\partial u}{\partial y}\frac{k}{\rho j} \qquad (2.3)$$

and the energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{A}{\rho c_p}\ T - T_w \qquad (2.4)$$

where N is the micro rotation whose direction of rotation is in the x-y plane,

μ is the viscosity of the fluid, ρ is the density and j, γ and h are the micro- inertia per unit mass, spin gradient viscosity and vortex viscosity respectively , which are assumed to be constant , c_p is the specific heat capacity at constant pressure of the fluid, k is the thermal conductivity of the fluid and A is the heat generation/absorption coefficient.

The boundary conditions for the problem are

$$u(x,0)=0,\ \ v(x,0)=-v_0,\ \ \ N(x,0)=-n\frac{\partial u}{\partial y}\ ,\ T=T_w$$

(2.5)

$$y\rightarrow\infty : u(x,y)\rightarrow U(x)=ax,\ \ v(x,y)\rightarrow 0,\ \ N(x,y)\rightarrow 0,\ T\rightarrow T_\infty$$

Where n is a constant and $0\le n\le 1$. The case n=1/2 indicates the vanishing of the anti symmetric part of the stress tensor and denotes weak concentration of microelements which will be considered here.

Considering the similarity transformations

$$\eta=\sqrt{\frac{a}{\nu}}y\quad ,\quad u=axf',\ \ v=-\sqrt{a\nu}\,f\ ,\ \ N=ax\sqrt{\frac{a}{\nu}}g\ ,\ \ g(\eta)=-\frac{1}{2}f'' \qquad (2.6)$$

The fluid viscosity is assumed to be inverse linear function of temperature (Lai and Kulacki [6]) as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\alpha(T-T_\infty)\right],\frac{1}{\mu}=a(T-T_r)\ ,\ a=\frac{\alpha}{\mu_\infty}\ \text{and}\ T_r=T_\infty-\frac{1}{\alpha} \qquad (2.7)$$

where a and T_r are constants and their values depends on the reference state and the thermal property of the fluid. In general a>0 for liquids and a<0 for gases. T_r is transformed reference temperature related to viscosity parameter. α is constant based on thermal property and μ_∞ is the viscosity at $T=T_\infty$ Similarly, consider the variation of thermal conductivity as,

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left[1+\xi(T-T_\infty)\right],\frac{1}{\lambda}=b(T-T_k)\ ,\ b=\frac{\xi}{\lambda_\infty}\ \text{and}\ \ T_k=T_\infty-\frac{1}{\xi} \qquad (2.8)$$

where b and T_k are constants and their values depends on the reference state and thermal property of the fluid ξ is constant based on thermal property and λ_∞ is the thermal conductivity at $T=T_\infty$.

Using equation (6) , it can be easily verified that the continuity equation is satisfied automatically and using equation (6)- (8) in the equations (2) –(4) become,

$$f''' = \frac{V_r - \theta}{V_r + K(V_r - \theta)} f'^2 - \frac{Kg'(V_r - \theta)}{V_r + K(V_r - \theta)} - \frac{V_r \theta' f''}{(V_r - \theta)\ V_r + K(V_r - \theta)} - \frac{ff''(V_r - \theta)}{V_r + K(V_r - \theta)}$$

(2.9)

$$\left(1 + \frac{1}{2}K\right) g'' = K\ \ 2g + f''\ \ + f'g - fg'$$

(2.10)

$$\theta'' = \frac{EP_r\theta(T_k - \theta)}{T_k} - \frac{\theta'(T_k - \theta)}{T_k} - \frac{\theta'^2}{(T_k - \theta)} \qquad (2.11)$$

where $E = \frac{Q}{a\rho c_p}$ is the heat generation/absorption parameter.

Prime denotes the differentiation with respect to η and the corresponding boundary conditions are

$$f\ \ 0\ = A,\ \ f'\ \ 0\ = 0,\ \ g\ \ 0\ = -\frac{1}{2} f''\ ,\ \ \theta\ \ \eta\ = 1 \qquad (2.12)$$

$$f'\ \ \infty\ = 1\ ,\ \ g\ \ \infty\ = 0\ ,\ \ \theta\ \ \infty\ = 0$$

Where $K = \frac{h}{\mu} > 0$ is the material parameter and primes denote differentiation with respect to η.

3. Results and Discussion

The equations (2.9)-(2.11) together with the boundary conditions (2.12) are solved for various values of the parameters involved in the equations using algorithms based on the shooting method.

Initially solution was taken for constant values of taking Pr=0.70, G=0.51,K=2.00, Vr =-10.00, T_k=-10.00 with the viscosity parameter Vr ranging from -10 to -1 at the certain values of T_k=-10.00 Similarly the solutions have been found with varying the thermal conductivity parameter T_k ranging from -10 to -1 at the certain values of Vr=-10 keeping other values remaining same. We have considered in some detail the influence of physical parameters on velocity distribution, micrototation distribution and temperature distribution which is shown in figures 1-6. Figures 1, 2 and 3 show the velocity, microrotation and temperature profiles with the variation of viscosity parameter Vr. This indicates that both the velocity and temperature distribution decreases as Vr increases where as microrotation distribution increases. Figures 4 and 6 display the temperature profiles for the various values of Tk. From the figures it is seen that the temperatures distribution decreases for the increasing values of Tk. In case of suction, the effect of material parameter on velocity is more pronounced for higher values. From figure 5 it is seen that temperature decreases as Pr increases and the figure bring out the effect of Pr on the thermal boundary layer thickness.

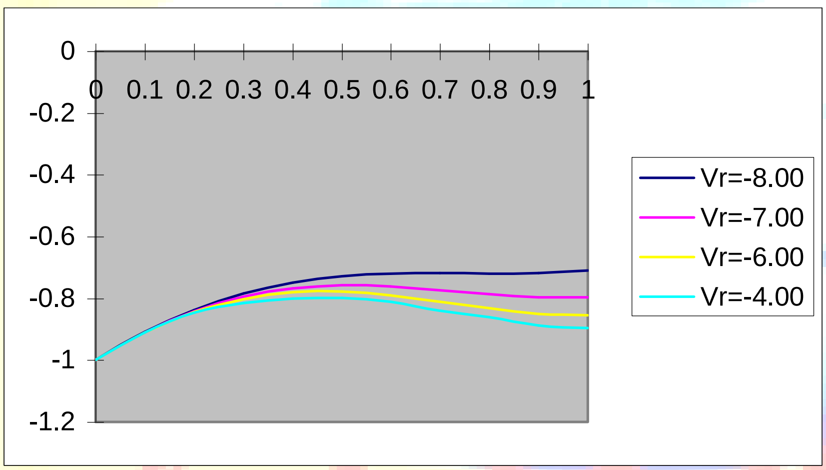

Fig 1: Variation of Velocity profiles for Pr=0.70, G=0.51,K=2.00, Tk=-10.00 and different values of Vr.

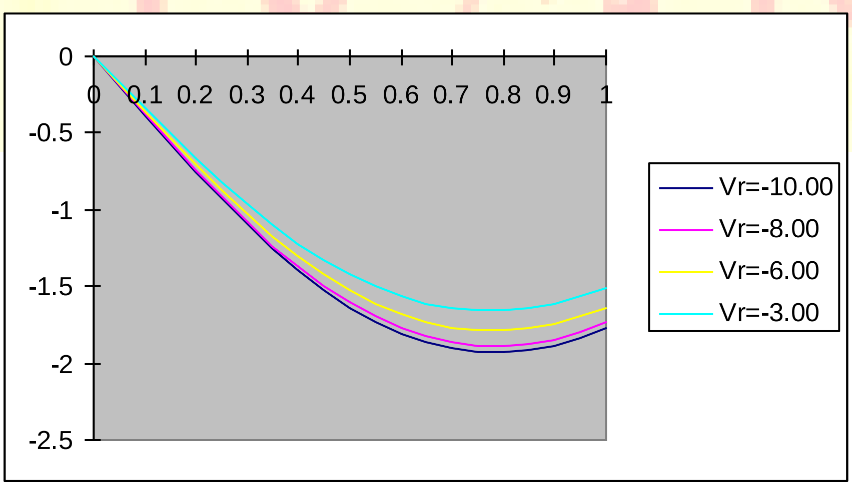

Fig 2: Variation of microrotation profiles for Pr=0.70, G=0.51,K=2.00, Tk=-10.00 and different values of Vr.

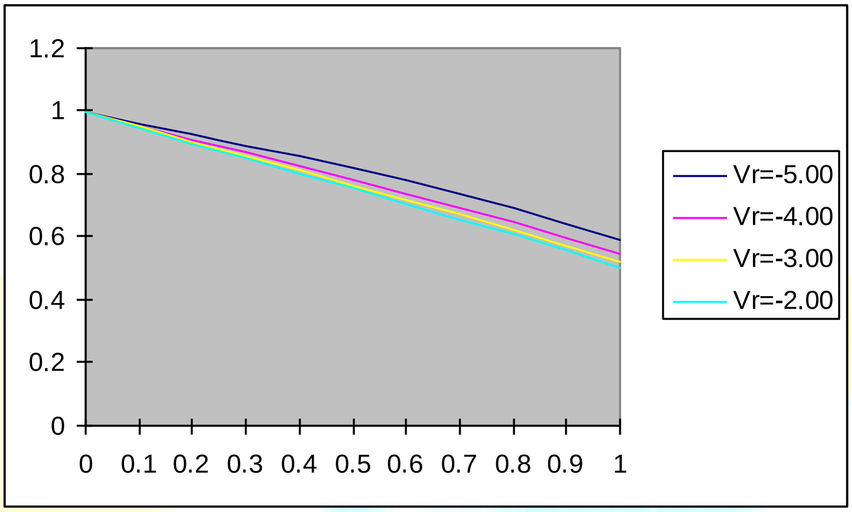

Fig 3 : Variation of temperature profiles for Pr=0.70 , G=0.51,K=2.00, Tk =-10.00 and for various values of Vr.

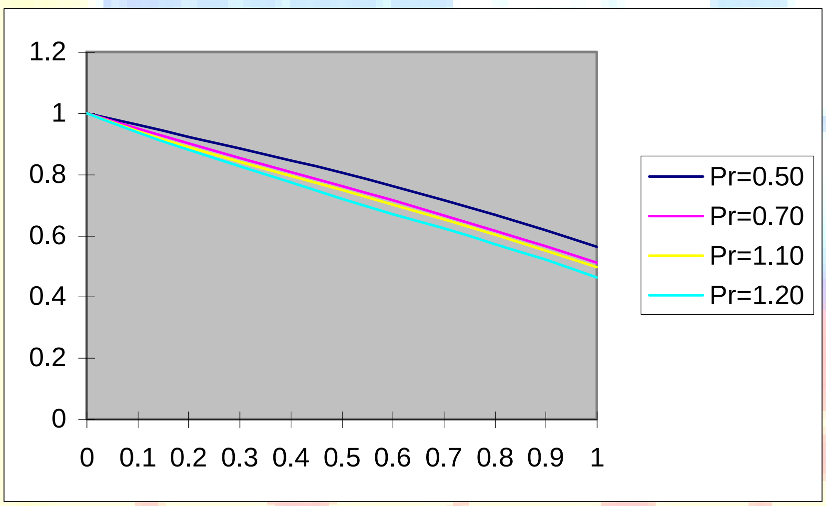

Fig 4 : Variation of temperature profiles for G=0.51,K=2.00, Vr=-10.00,Tk =-10.00 and for various values of Pr.

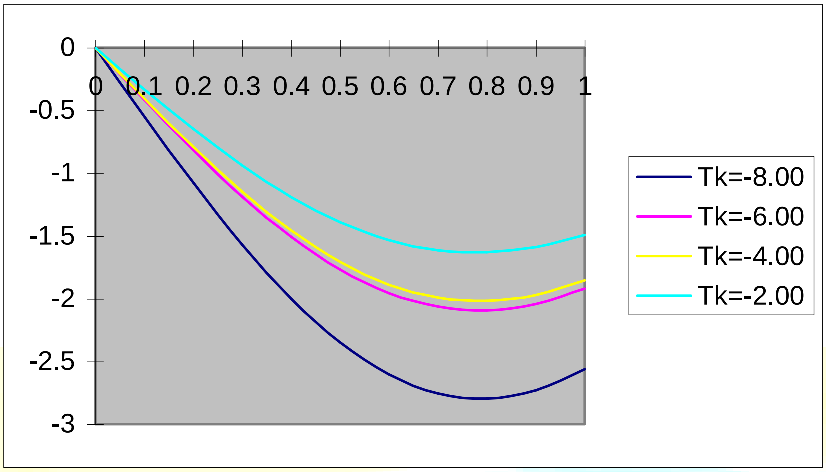

Fig 5: Variation of microrotation profiles for Pr=0.70 , G=0.51 ,K=2.00, Vr =-10.00 and for various values of parameter Tk.

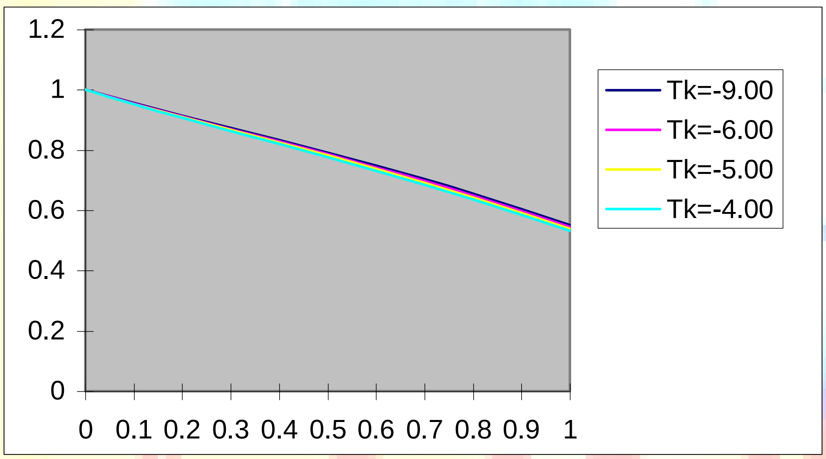

Fig 6: Variation of temperature profiles for Pr=0.70 , G=0.51 ,K=2.00, Vr =-10.00 and for various values of parameter Tk.

4. Conclusion

In this study the heat generation of micropolar fluid flow on a porous plat with temperature dependent viscosity and thermal conductivity has been investigated.the resulting partial differential equations which describe the problem are transformed into ordinary differential equations by using similarity transformations. Numerical evaluations are performed and graphical results are obtained. The results presented demonstrate clearly that the viscosity and thermal conductivity parameters have a substantial effects on velocity, micropolar and temperature distribution. The effects of Prandtl number is quite significant.

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