

INTERVAL VALUED INTUITIONISTIC Q-FUZZY GRAPHS

R.Nagarajan*

K.Balamurugan**

Abstract:

In this paper, we introduce the notion of interval α -valued intuitionistic Q-fuzzy graphs and describe various methods of their construction. We also present the concept of interval-valued intuitionistic Q-fuzzy regular graphs.

Keywords: Q-fuzzy set, Q-fuzzy graph, intuitionistic Q-fuzzy set, Interval-valued intuitionistic Q-fuzzy graph, Intuitionistic Q-fuzzy relation, Cartesian Product.

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* Associate Professor, Department of Mathematics, J J College of Engg& Tech, Trichirappalli-09

** Assistant Professor, M.A.M School of Engineering, Trichirappalli-105

1. Introduction:

In 1736, Euler first introduced the notion of graph theory. In the history of mathematics, the solution given by Euler of the well known Konigsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operation Research, optimization and computer Science. In 1975, Rosenfeld [8] introduced the concept of fuzzy graphs. The fuzzy relation between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Mordeson and Peng [4] introduced some operator on fuzzy graphs,. Shannon and Atanassov [7] introduced the concept of intuitionistic fuzzy relation and intuitionistic fuzzy graphs, and investigated some of their properties. Recently Akram and Dudek [1] have studied some properties of interval –valued fuzzy graphs. In 1975, Zadeh [10] introduced the notion of interval –valued fuzzy sets as an extension of fuzzy sets [11] in which the values of the membership degree are intervals of numbers instead of the numbers. The notion of interval –valued intuitionistic fuzzy sets was introduced by Atanassov and Gargó [3] in 1989 as a generalization of both interval –valued fuzzy sets and intuitionistic fuzzy sets. Grrstenkorn and Manko [6] re-named the intuitionistic fuzzy sets as bi fuzzy sets in 1995. A.Solairaju and R.Nagarajan [9] have introduced and defined a new algebraic structure called Q-fuzzy subgroups.

In this paper, we introduce the notion of interval –valued intuitionistic Q-fuzzy graphs and describe various methods of their construction. We also present the concept of interval-valued Q-intuitionistic regular graphs.

2. Preliminaries

We first recall some basic concept which are used to present the paper.

2.1 Definition: A Q-fuzzy graph $G = [\mu, \gamma]$ is a non-empty set V together with a pair of functions $\mu: V \times Q \rightarrow [0,1]$ and $\gamma: V \times V \times Q \rightarrow [0,1]$ such that $\gamma(\{x, y\})_q \leq \min\{\mu(x, q), \mu(y, q)\}$ for all $x, y \in V$ and $q \in Q$. Q-fuzzy graph is a graph consists of pairs of vertex and edge that have degree of membership containing closed interval of real number $[0,1]$ on each edge and vertex.

2.2 Definition: An interval number D is an interval $[a^-, a^+]$ with $0 \leq a^- \leq a^+ \leq 1$. The interval $[a, a]$ is identified with the number $a \in [0,1]$. $D[0,1]$ denotes the set of all interval numbers.

For interval numbers $D_1 = [a_1^-, b_1^+]$ and $D_2 = [a_2^-, b_2^+]$, we define

- (1) $r \min(D_1, D_2) = r \min ([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\min\{a_1^-, a_2^-\}, \min\{b_1^+, b_2^+\}]$
- (2) $r \max(D_1, D_2) = r \max ([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\max\{a_1^-, a_2^-\}, \max\{b_1^+, b_2^+\}]$
- (3) $D_1 + D_2 = [a_1^- + a_2^- - a_1^- \cdot a_2^-, b_1^+ + b_2^+ - b_1^+ \cdot b_2^+]$

2.3 Definition: The interval-valued Q-fuzzy set A in V is define by

$A = \{(x, [\mu_A^-(x, q), \mu_A^+(x, q)]) / x \in V\}$, where $\mu_A^-(x, q)$ & $\mu_A^+(x, q)$ are Q-fuzzy subsets of V such that $\mu_A^-(x, q) \leq \mu_A^+(x, q)$ for all $x \in V$ and $q \in Q$.

2.4 Definition: By an interval -valued Q-fuzzy graph $G^* = (V, E)$ we mean a pair

$G=(A,B)$, where $A=[\mu_A^-, \mu_A^+]$ is an interval-valued Q-fuzzy set on V and $B=[\mu_B^-, \mu_B^+]$ is an interval-valued Q-fuzzy relation on E.

2.5 Definition : For a non- empty set G, we call a mapping $A=(\mu_A, \gamma_A):G \times Q \rightarrow D[0,1] \times D[0,1]$

an interval –valued intuitionistic Q-fuzzy set if $\mu_A^+(x, q) + \gamma_A^+(x, q) \leq 1$ and

$\mu_A^-(x, q) + \gamma_A^-(x, q) \leq 1$ for all $x \in G$, where the mapping

$$\mu_A(x, q) = [\mu_A^-(x, q), \mu_A^+(x, q)] : G \times Q \rightarrow D [0,1] \text{ and}$$

$$\gamma_A(x, q) = [\gamma_A^-(x, q), \gamma_A^+(x, q)] : G \times Q \rightarrow D [0,1]$$

are the degree of membership functions respectively. We use $\tilde{0}$ to denote the interval-valued fuzzy empty set and $\tilde{1}$ to denote the interval-valued fuzzy whole set in a set G, and we define

$$\tilde{0}(x, q) = [0,0] \text{ and } \tilde{1}(x, q) = [1,1], \text{ for all } X \in G \text{ and } q \in G$$

2.6 Definition: If $G^* = (V, E)$ is a graph, then by an interval –valued intuitionistic Q-fuzzy relation B on a set E, we mean an interval –valued intuitionistic Q-fuzzy set such that

$$\mu_B(xy, q) \leq r \min (\mu_A(x, q), \mu_A(y, q))$$

$$\gamma_B(xy, q) \geq r \max (\gamma_A(x, q), \gamma_A(y, q))$$

for all $xy \in E$ and $q \in Q$.

3 .Interval –valued intuitionistic Q-fuzzy graphs

3.1 Definition: An Interval –valued intuitionistic Q-fuzzy graph with underlying set V is defined to be a pair $G = (A,B)$ where

- (i) The functions $\mu_A: V \times Q \rightarrow D[0,1]$ and $\gamma_A: V \times Q \rightarrow D[0,1]$ denote the degree of membership and non-membership of the element $X \in V$, respectively such that $\tilde{0} \leq \mu_A(x, q) + \gamma_A(x, q) \leq \tilde{1}$, for all $x \in V$ and $q \in Q$.

(ii) The functions $\mu_B : E \subseteq V \times V \times Q \rightarrow D[0,1]$ & $\gamma_B : E \subseteq V \times V \times Q \rightarrow D[0,1]$

and defined by $\mu_B(\{x, y\})_q \leq r \min(\mu_A(x, q), \mu_A(y, q))$ and

$$\gamma_B(\{x, y\})_q \geq r \max(\gamma_A(x, q), \gamma_A(y, q))$$

such that $\tilde{0} \leq \mu_A(x, q) + \gamma_A(x, q) \leq \tilde{1}$ for all $\{x, y\} \in E$ and $q \in G$.

We call A an interval –valued intuitionistic Q-fuzzy vertex set of V, B an interval –valued intuitionistic Q-fuzzy edge set of G, respectively. Note that B is a symmetric interval –valued intuitionistic Q-fuzzy relation on A. we use the notation xy for an element of E. Thus

$G=(A,B)$ is an interval –valued intuitionistic Q-fuzzy graph of $G(V,E)$ if

$$\mu_B(xy, q) \leq r \min(\mu_A(x, q), \mu_A(y, q))$$

$$\gamma_B(xy, q) \geq r \max(\gamma_A(x, q), \gamma_A(y, q))$$

for all $xy \in E$ and $q \in Q$.

Example 3.2: Consider a graph $G^* = (V, E)$ such that $V=\{x, y, z\}, E=[xy, yz, zx]$. Let 'A' be on IVIQFS of V and let B be on IVIQFS of $E \subseteq V \times V$ defined by

$$A = \left\langle \left(\frac{x}{[0.2,0.3]}, \frac{y}{[.0.1,0.4]}, \frac{z}{[0.3,0.5]} \right), \left(\frac{x}{[0.3,0.4]}, \frac{y}{[.0.2,0.6]}, \frac{z}{[0.1,0.7]} \right) \right\rangle$$

$$B = \left\langle \left(\frac{xy}{[0.03,0.2]}, \frac{yz}{[.0.04,0.05]}, \frac{zx}{[0.01,0.07]} \right), \left(\frac{xy}{[0.1,0.4]}, \frac{yz}{[.0.06,0.3]}, \frac{zx}{[0.07,0.3]} \right) \right\rangle$$

By routine computations, it is easy to see that $G=(A,B)$ is a IVIQFG of G.

3.3 Definition: Let $A=(\mu_A, \gamma_A)$ and $A'=(\mu_A', \gamma_A')$ be a IVIQF subsets of V_1 and V_2 and $B=(\mu_B, \gamma_B)$ and $B'=(\mu_B', \gamma_B')$ be IVIQF of subset of E_1 and E_2 respectively. The Cartesian product of two IVIQFGS. G_1 and G_2 of the graphs G_1^* and G_2^* is denoted by $G_1 \times G_2 = (A \times A', B \times B')$ and is defined as follows,

$$(i) \quad (\mu_A \times \mu_{A'}) (x_1, x_2)_q = r \min(\mu_A(x_1, q), \mu_{A'}(x_2, q))$$

$$(\gamma_A \times \gamma_{A'}) (x_1, x_2)_q = r \max(\gamma_A(x_1, q), \gamma_{A'}(x_2, q)) \text{ for all } (x_1, x_2) \in V, q \in G.$$

$$(ii) \quad (\mu_B \times \mu_{B'}) ((x_1, x_2)(y_1, y_2))_q = r \min(\mu_A(x, q), \mu_{B'}(x_2 y_2, q))$$

$$(\gamma_B \times \gamma_{B'}) ((x_1, x_2)(y_1, y_2))_q = r \max(\gamma_A(x, q), \gamma_{B'}(x_2 y_2, q))$$

for all $x \in V_1$, for all $x_2 y_2 \in E_2$.

$$(iii) \quad (\mu_B \times \mu_{B'}) ((x_1, z)(y_1, z))_q = r \min(\mu_B(x_1 y_1, q), \mu_{A'}(z, q))$$

$$(\gamma_B \times \gamma_{B'}) ((x_1, z)(y_1, z))_q = r \max(\gamma_B(x_1 y_1, q), \gamma_{A'}(z, q))$$

for all $z \in V_2$, for all $x_1 y_1 \in E_1$.

Proposition 3.4: Let G_1 and G_2 be the two interval –valued intuitionistic Q-fuzzy graphs. Then Cartesian product $G_1 \times G_2$ is an interval –valued intuitionistic Q-fuzzy graphs.

Proof: Let $x \in V_1, x_2y_2 \in E_2$ then

$$\begin{aligned} (\mu_B \times \mu_B') ((x, x_2)(x, y_2))_q &= r \min \{(\mu_A(x, q), \mu_B'(x_2y_2, q))\} \\ &\leq r \min \{(\mu_A(x, q), r \min (\mu_A'(x_2, q), \mu_A'(y_2, q))\} \\ &= r \min \{r \min (\mu_A(x, q), \mu_A'(x_2, q)), r \min(\mu_A(x, q), (\mu_A'(y_2, q))\} \\ &= r \min \{(\mu_A \times \mu_A') (x, x_2)_q, (\mu_A \times \mu_A') (x, y_2)_q\} \end{aligned}$$

$$\begin{aligned} (\gamma_B \times \gamma_B') ((x, x_2)(x, y_2))_q &= r \max \{(\gamma_A(x, q), \gamma_B'(x_2y_2, q))\} \\ &\geq r \max \{(\gamma_A(x, q), r \max (\gamma_A'(x_2, q), \gamma_A'(y_2, q))\} \\ &= r \max \{r \max (\gamma_A(x, q), \gamma_A'(x_2, q)), r \max (\gamma_A(x, q), \gamma_A'(y_2, q))\} \\ &= r \max \{(\gamma_A \times \gamma_A') (x, x_2)_q, (\gamma_A \times \gamma_A') (x, y_2)_q\} \end{aligned}$$

Let $z \in V_2, x_1y_1 \in E_1$ then

$$\begin{aligned} (\mu_B \times \mu_B') ((x_1, z)(y_1, z))_q &= r \min (\mu_B(x_1y_1, q), \mu_A'(z, q)) \\ &\leq r \min \{r \min (\mu_A(x_1, q), \mu_A(y_1, q)), \mu_A'(z, q)\} \\ &= r \min \{r \min (\mu_A(x_1, q), \mu_A'(z, q)), r \min (\mu_A(y_1, q), \mu_A'(z, q))\} \\ &= r \min \{(\mu_A \times \mu_A') (x_1, z)_q, (\mu_A \times \mu_A') (y_1, z)_q\} \end{aligned}$$

$$\begin{aligned} (\gamma_B \times \gamma_B') ((x_1, z)(y_1, z))_q &= r \max (\gamma_B(x_1y_1, q), \gamma_A'(z, q)) \\ &\geq r \max \{r \max (\gamma_A(x_1, q), \gamma_A(y_1, q)), \gamma_A'(z, q)\} \\ &= r \max \{r \max (\gamma_A(x_1, q), \gamma_A'(z, q)), r \max (\gamma_A(y_1, q), \gamma_A'(z, q))\} \\ &= r \max \{(\gamma_A \times \gamma_A') (x_1, z)_q, (\gamma_A \times \gamma_A') (y_1, z)_q\} \end{aligned}$$

3.5 Definition: Let $A = (\mu_A, \gamma_A)$ and $A' = (\mu_A', \gamma_A')$ be an IVIQF subsets of V_1 and V_2 and $B = (\mu_B, \gamma_B)$ and $B' = (\mu_B', \gamma_B')$ be IVIQF of subset of E_1 and E_2 respectively. The Composition of two IVIQFGs G_1 and G_2 is denoted by $G_1[G_2] = (A \circ A', B \circ B')$ and is defined as follows,

- (i) $(\mu_A \circ \mu_A') (x_1, x_2)_q = r \min (\mu_A(x_1, q), \mu_A'(x_2, q))$
 $(\gamma_A \circ \gamma_A') (x_1, x_2)_q = r \max (\gamma_A(x_1, q), \gamma_A'(x_2, q))$ for all $(x_1, x_2) \in V$.
- (ii) $(\mu_B \circ \mu_B') ((x, x_2)(x, y_2))_q = r \min (\mu_A(x, q), \mu_B'(x_2y_2, q))$
 $(\gamma_B \circ \gamma_B') ((x, x_2)(x, y_2))_q = r \max (\gamma_A(x, q), \gamma_B'(x_2y_2, q))$
- (iii) $(\mu_B \circ \mu_B') ((x_1, z)(y_1, z))_q = r \min (\mu_B(x_1y_1, q), \mu_A'(z, q))$

$$\begin{aligned}
 & (\gamma_B \circ \gamma_B')((x_1, z)(y_1, z))_q = r \max(\gamma_B(x_1 y_1, q), \gamma_A'(z, q)) \\
 \text{(iv)} \quad & (\mu_B \circ \mu_B')((x_1, x_2)(y_1, y_2))_q = r \min(\mu_A'(x_2, q), \mu_A'(y_2, q), \mu_B(x_1 y_1, q)) \\
 & (\gamma_B \circ \gamma_B')((x_1, x_2)(y_1, y_2))_q = r \min(\gamma_A'(x_2, q), \gamma_A'(y_2, q), \gamma_B(x_1 y_1, q)) \\
 & \text{for all } (x_1, x_2)(y_1, y_2) \in E^o - E.
 \end{aligned}$$

Proposition 3.6: Let G_1 and G_2 be the two interval-valued intuitionistic Q-fuzzy graphs. Then Composition of $G_1[G_2]$ is an interval-valued intuitionistic Q-fuzzy graphs.

Proof:

Let $x \in V_1, x_2 y_2 \in E$ then

$$\begin{aligned}
 (\mu_B \circ \mu_B')((x, x_2)(x, y_2))_q &= r \min(\mu_A(x, q), \mu_B'(x_2 y_2, q)) \\
 &\leq r \min\{\mu_A(x, q), r \min(\mu_A'(x_2, q), \mu_A'(y_2, q))\} \\
 &= r \min\{r \min(\mu_A(x, q), \mu_A'(x_2, q)), r \min(\mu_A(x, q), \mu_A'(y_2, q))\} \\
 &= r \min\{(\mu_A \circ \mu_A')(x, x_2), (\mu_A \circ \mu_A')(x, y_2)\} \\
 (\gamma_B \circ \gamma_B')((x, x_2)(x, y_2))_q &= r \max(\gamma_A(x, q), \gamma_B'(x_2 y_2, q)) \\
 &\geq r \max\{\gamma_A(x, q), r \max(\gamma_A'(x_2, q), \gamma_A'(y_2, q))\} \\
 &= r \max\{r \max(\gamma_A(x, q), \gamma_A'(x_2, q)), r \max(\gamma_A(x, q), \gamma_A'(y_2, q))\} \\
 &= r \max\{(\gamma_A \circ \gamma_A')(x, x_2), (\gamma_A \circ \gamma_A')(x, y_2)\}
 \end{aligned}$$

Let $z \in V_2, x_1 y_1 \in E$, then

$$\begin{aligned}
 (\mu_B \circ \mu_B')((x_1, z)(y_1, z))_q &= r \min(\mu_B(x_1 y_1, q), \mu_A'(z, q)) \\
 &\leq r \min\{r \min(\mu_A(x_1, q), \mu_A(y_1, q)), \mu_A'(z, q)\} \\
 &\leq r \min\{r \min(\mu_A(x_1, q), \mu_A'(z, q)), r \min(\mu_A(y_1, q), \mu_A'(z, q))\} \\
 &= r \min\{(\mu_A \circ \mu_A')(x_1, z), (\mu_A \circ \mu_A')(y_1, z)\}
 \end{aligned}$$

$$\begin{aligned}
 (\gamma_B \circ \gamma_B')((x_1, z)(y_1, z))_q &= r \max(\gamma_B(x_1 y_1, q), \gamma_A'(z, q)) \\
 &\geq r \max\{r \max(\gamma_A(x_1, q), \gamma_A(y_1, q)), \gamma_A'(z, q)\} \\
 &\geq r \max\{r \max(\gamma_A(x_1, q), \gamma_A'(z, q)), r \max(\gamma_A(y_1, q), \gamma_A'(z, q))\} \\
 &= r \max\{(\gamma_A \circ \gamma_A')(x_1, z), (\gamma_A \circ \gamma_A')(y_1, z)\}
 \end{aligned}$$

Let $(x_1, x_2)(y_1, y_2) \in E^o - E$, so $x_1 y_1 \in E_1, x_2 \neq y_2$. Then

$$(\mu_B \circ \mu_B')((x_1, x_2)(y_1, y_2))_q = r \min(\mu_A'(x_2, q), \mu_A'(y_2, q), \mu_B(x_1 y_1, q))$$

$$\begin{aligned} &\leq r \min (\mu_A'(x_2, q), \mu_A'(y_2, q)), r \min (\mu_A(x_1, q), \mu_A(y_1, q)) \\ &\leq r \min \{ r \min (\mu_A(x_1, q), \mu_A'(x_2, q)), r \min (\mu_A(y_1, q), \mu_A'(y_2, q)) \} \\ &= r \min \{ (\mu_A \circ \mu_A')(x_1, x_2)_q, (\mu_A \circ \mu_A')(y_1, y_2)_q \} \end{aligned}$$

$$\begin{aligned} (\gamma_B \circ \gamma_B')((x_1, x_2)(y_1, y_2))_q &= r \max (\gamma_A'(x_2, q), \gamma_A'(y_2, q), \gamma_B(x_1 y_1, q)) \\ &\geq r \max (\gamma_A'(x_2, q), \gamma_A'(y_2, q)), r \max (\gamma_A(x_1, q), \gamma_A(y_1, q)) \\ &\geq r \max \{ r \max (\gamma_A(x_1, q), \gamma_A'(x_2, q)), r \max (\gamma_A(y_1, q), \gamma_A'(y_2, q)) \} \\ &= r \max \{ (\gamma_A \circ \gamma_A')(x_1, x_2)_q, (\gamma_A \circ \gamma_A')(y_1, y_2)_q \} \end{aligned}$$

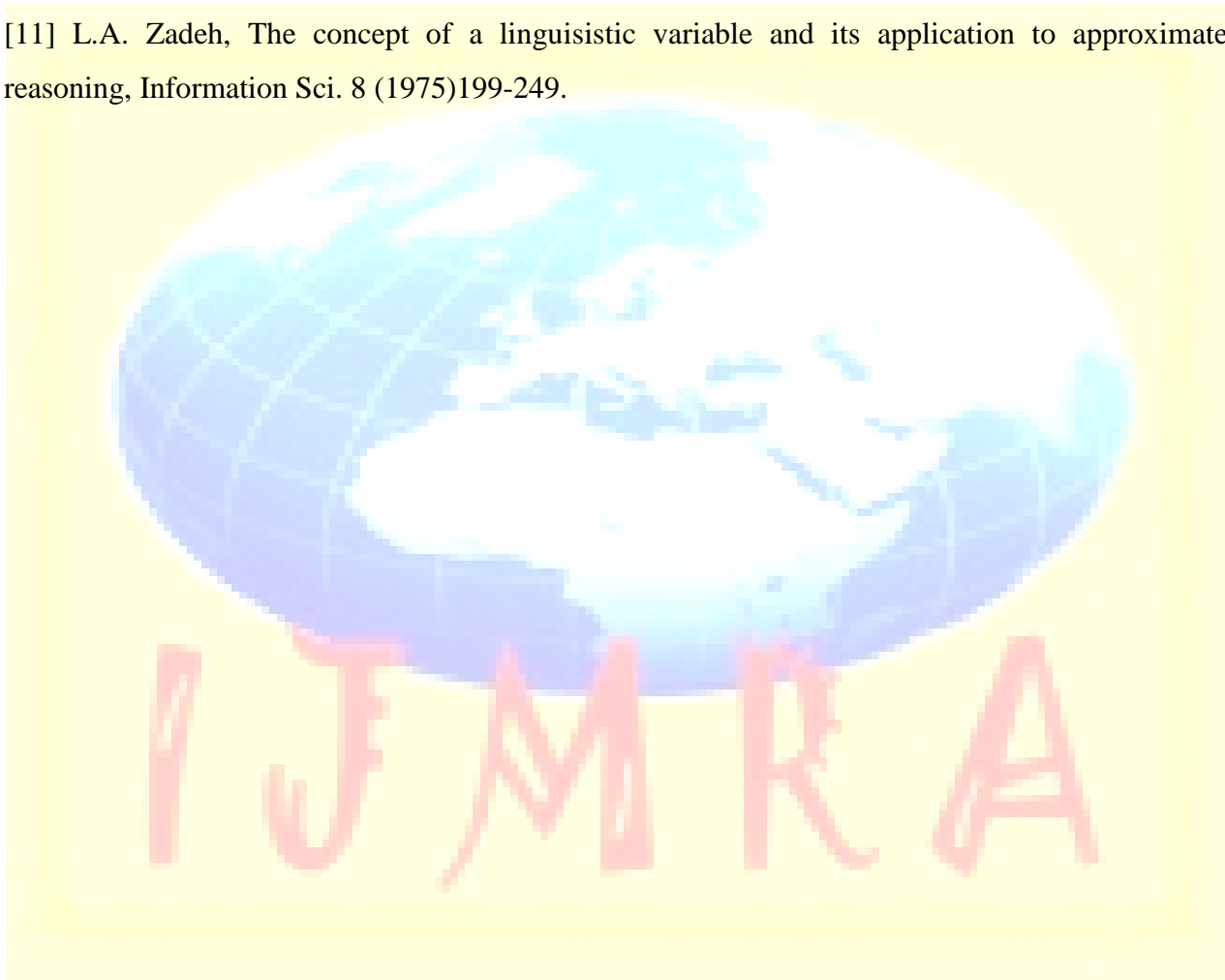
This completes the proof.

4. Conclusion: The interval valued fuzzy model give more precision, flexibility and compatibility to the system as computed to the classical and fuzzy models. It is known that fuzzy graph theory has numerous applications in Modern science and Engineering, especially in the field of information theory, neural networks, expert systems, cluster analysis, medical diagnosis, traffic engineering and control theory. In this paper, we introduce the notion of interval –valued intuitionistic Q-fuzzy graphs and describe various methods of their construction. We also present the concept of interval-valued intuitionistic Q-fuzzy regular graphs.

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Author's Detail:



Dr. R.NAGARAJAN has been working as Associate Professor and Head of Mathematics in J.J.College of Engineering & technology,Trichy. He has 16 years of experience in the field of Teaching. He has completed his M.Sc., from Bharathidasan University, Trichy M.Phil in the field of **Minimal graphoidal cover of a graph** from Alagappa University, Karaikudi. He Received his Ph.D degree in the field of **Fuzzy Techniques in Algebra** from Bharathidasan University, ,Trichy. He has Published more than 50 research articles in International Journals and 4 in National Journals. His area of interests are Fuzzy Algebraic Structures, Fuzzy Topological Structures, Fuzzy Decision making and Fuzzy Optimizations.

