(ψ, χ)-BIPOLAR FUZZY GROUP AND (ψ, χ)- BI-POLAR FUZZY D-IDEALS

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Abstract:

In this paper, we apply the notion of bipolar-valued fuzzy set to groups. We introduce the concept of (ψ, χ)-bipolar fuzzy groups / (fuzzy d-ideals of groups and investigate several properties. We give relations between a (ψ, χ)-bipolar fuzzy group and (ψ, χ)-bipolar fuzzy d-ideal. We provide a condition for (ψ, χ)-bipolar fuzzy groups to be a(ψ, χ)- bipolar fuzzy d-ideal. We also give characterizations of (ψ, χ)-bipolar fuzzy ideal. We consider the concept of strongest (ψ, χ)-bipolar fuzzy relations on (ψ, χ)-bipolar fuzzy d-ideals of a group and discuss some related properties.

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Index terms: (ψ, χ)-bipolar fuzzy group, (ψ, χ)- bipolar fuzzy d-ideal, (α,β)- cut, strongest (ψ, χ)-bipolar fuzzy relation.

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Section-1 **Introduction:** Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. There are several kinds of fuzzy sets extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets etc. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1]. Bipolar-Valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property and its counter property. In a bipolar valued fuzzy set the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0,1] indicate that elements some what satisfy the property, and the membership degrees on [-1,0) indicate that elements somewhat satisfy the implicit counter property. In the definition of bipolar-valued fuzzy sets, there are two kinds of representations so called canonical representation and reduced representation. In this paper, we use the canonical representation of bipolar valued fuzzy sets. In this paper, we apply the notion of bipolar-valued fuzzy set to groups. We introduce the concept of (ψ, χ)-bipolar fuzzy groups / fuzzy d-ideals of groups and investigate several properties. We give relations between a (ψ, χ)-bipolar fuzzy group and (ψ, χ)-bipolar fuzzy d-ideal. We provide a condition for (ψ, χ)-bipolar fuzzy groups to be a(ψ, χ)- bipolar fuzzy d-ideal. We also give characterizations of (ψ, χ)-bipolar fuzzy ideal. We consider the concept of strongest (ψ, χ)-bipolar fuzzy relations on (ψ, χ)-bipolar fuzzy d-ideals of a group and discuss some related properties.

2.Preliminaries

2.1 Definition: Let 'S' be a set. A fuzzy set in S is a function $\mu : S \rightarrow [0,1]$

2.2Definition: Let 'G' be a non-empty set. A bipolar-Valued Fuzzy set A in G is an object having the form $A = \{(x, \mu_A^+(x), \mu_A^-(x) / x\varepsilon G\}$ where $\mu_A^+ : G \rightarrow [0,1]$ and $\mu_A^- : G \rightarrow [-1,0]$ are mapping. The positive membership degree $\mu_A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to 'A' and the negative membership degree $\mu_A^-(x)$ denotes the satisfaction degree of x to some implicit counter property of A.

2.3 Definition: A bipolar fuzzy set 'A' in X is called a (ψ, χ)-bipolar Fuzzy group (BPFG)of X if it satisfies

(BPFG$_1$) $\mu_A^+(xy) \cap \psi \geq T\{\mu_A^+(x), \mu_A^+(y)\} \vee \chi$
(BPFG$_2$) $\mu_A^-(xy) \cap \psi \leq S\{\mu_A^-(x), \mu_A^-(y)\} \vee \chi$
(BPFG$_3$) $\mu_A^+(x^{-1}) \cap \psi \geq \mu_A^+(x) \vee \chi$ and
(BPFG$_4$) $\mu_A^-(x^{-1}) \cap \psi \leq \mu_A^-(x) \vee \chi$ for all x,yεX.

2.4 Definition: For a bipolar fuzzy set 'A' and (β,α) ε [-1.0] x [0,1], we define

$A_t^x = \{x\varepsilon X / \mu_A^+(x) \geq \alpha\}$, $A_S^- = \{x\varepsilon X / \mu_A^+(x) \geq \alpha\}$ which are called the positive α-cut and negative β-cut of A respectively.

2.5 Definition: A bipolar fuzzy set 'A' in X is called a (ψ, χ)- bipolar fuzzy d-ideal of X if it satisfies;

(BPFDI$_1$) $\mu_A^+(x) \cap \psi \geq T\{\mu_A^+(xy), \mu_A^+(y)\} \vee \chi$

(BPFDI$_2$) $\mu_A^-(x) \cap \psi \leq S\{\mu_A^-(xy), \mu_A^-(y)\} \vee \chi$

(BPFDI$_3$) $\mu_A^+(e) \cap \psi \geq \mu_A^+(x) \vee \chi$ and $\mu_A^-(e) \cap \psi \geq \mu_A^-(x) \vee \chi$ and for all x,y ε X.

2.6 Definition: Let λ and μ be two fuzzy subsets in X. The Cartesian Product of $\lambda^+\times\mu^+ : X \times X \rightarrow [0,1]$ is defined by $\lambda^+\times\mu^+(x,y) = T\{\lambda^+(x), \mu^+(y)\}$ and $\lambda^+\times\mu^+ : X \times X \rightarrow [0,1]$ is defined by $\lambda^+\times\mu^+(x,y) = S\{\lambda^-(x), \mu^-(y)\}$ for all x,y ε X.

2.7 Definition: Let f : X → Y be a mapping of group's and 'μ' be a bipolar fuzzy set of y. The map μ^f is the pre image of μ_1 and μ_2 under f. so $\mu_1^{+f}(x) = \mu^{+f}(x)$, $\mu_2^{-f}(x) = \mu^{-f}(x)$

2.8 Definition: Let 'A' be a bipolar fuzzy set in a X, the strongest (ψ, χ)- bipolar fuzzy relation on X that is fuzzy relation on A is μ_A given by,

$\mu_A^+(x,y) \cap \psi = T\{A^+(x), A^+(y)\} \vee \chi$

$\mu_A^-(x,y) \cap \psi = S\{A^-(x), A^-(y)\} \vee \chi$ for all x,y ε X.

3.Main Results

Proposition 3.1: If ϕ is a (ψ, χ)-bipolar fuzzy group of X, then $\mu_\varphi^+(e) \cap \psi \geq \mu_\varphi^+(x) \vee \chi$ and $\mu_\varphi^-(e) \cap \psi \leq \mu_\varphi^-(x) \vee \chi$ for all x ε X.

Proof: Let x ε X, then

$\mu_\varphi^+(e) \cap \psi = \mu_\varphi^+(x\, x^{-1}) \cap \psi \geq T\{\mu_\varphi^+(x), \mu_\varphi^+(x^{-1})\} \vee \chi \geq T\{\mu_\varphi^+(x), \mu_\varphi^+(x)\} \vee \chi \geq \mu_\varphi^+(x) \vee \chi$

and $\mu_\varphi^-(e) \cap \psi = \mu_\varphi^-(x\, x^{-1}) \cap \psi \leq S\{\mu_\varphi^-(x), \mu_\varphi^-(x^{-1})\} \vee \chi \leq S\{\mu_\varphi^-(x), \mu_\varphi^-(x)\} \vee \chi \leq \mu_\varphi^-(x) \vee \chi$

This completes the proof.

Proposition 3.2: Let 'ϕ' be a (ψ, χ)- bipolar fuzzy group of X, then the following assertations are valid.

(i) $(\forall\, \alpha\, \varepsilon\, [0,1]\, (\phi_\alpha^+ \neq \phi \Rightarrow \phi_t^+$ is a group of X)

(ii) $(\forall\, \beta\, \varepsilon\, [-1,0]\, (\phi_\beta^- \neq \phi \Rightarrow \phi_\beta^-$ is a group of X)

Proof: Let t ε [0,1] be such that $\phi_t^+ \neq \phi$. If x,y ε ϕ_t^+, then $\mu_\phi^+(x) \cap \psi \geq t \vee \chi$ and $\mu_\phi^+(y) \cap \psi \geq t \vee \chi$. It follows that $\mu_\phi^+(xy)\ ) \cap \psi \geq T\{\mu_\phi^+(x), \mu_\phi^-(y)\} \vee \chi \geq t \vee \chi$

Corollary 3.3: If ϕ is a(ψ, χ) -bipolar fuzzy group of X, then the sets $\phi^+_{\mu\varphi+(e)}$ and $\phi^-\ \mu_\varphi^-(e)$ are group of X.

Proof: Straight forward.

Proposition 3.4: Let $\phi = (X, \mu_\phi^+, \mu_\phi^-)$ be a (ψ, χ)-bipolar fuzzy d-ideal of X. If the inequality $xy \leq z$ holds in X, then

$$\mu_\phi^+(x) \cap \psi \geq T\{\mu_\phi^+(y), \mu_\phi^+(z)\} \vee \chi$$

$$\mu_\phi^-(x) \cap \psi \leq S\{\mu_\phi^+(y), \mu_\phi^-(z)\} \vee \chi$$

Proof: Let x, y, z ε X be such that $xy \leq z$, then $(xy)z = 0$, and so

$\mu_\phi^+(x) \cap \psi \geq T\{\mu_\phi^+(xy), \mu_\phi^+(y)\} \vee \chi \geq T\{T\{\mu_\phi^+(xy)z, \mu_\phi^+(z)\}, \mu_\phi^+(y)\} \vee \chi = T\{T\{\mu_\phi^+(e), \mu_\phi^+(z)\}, \mu_\phi^+(y)\} \vee \chi = T\{\mu_\phi^+(y), \mu_\phi^+(z)\} \vee \chi$ and

$\mu_\phi^-(x) \cap \psi \leq S\{\mu_\phi^-(xy), \mu_\phi^-(y)\} \vee \chi \leq S\{S\{\mu_\phi^-(xy)z, \mu_\phi^-(z)\}, \mu_\phi^-(y)\} \vee \chi$

$= S\{S\{\mu_\phi^-(e), \mu_\phi^-(z)\}, \mu_\phi^-(y)\} \vee \chi = S\{\mu_\phi^-(y), \mu_\phi^-(z)\} \vee \chi$

This completes the proof.

Proposition 3.5: Let ϕ be a(ψ, χ) -bipolar fuzzy d-ideal of X. If the inequality $x \le y$ holds in X, then $\mu_\phi^+(x) \cap \psi \ge \mu_\phi^+(y) \vee \chi$ and $\mu_\phi^-(x) \cap \psi \le \mu_\phi^-(y) \vee \chi$.

Proof: Let x, y ε X be such that $x \le y$, then $\mu_\phi^+(x) \cap \psi \ge T\{\mu_\phi^+(xy), \mu_\phi^+(y)\} \vee \chi = T\{\mu_\phi^+(e), \mu_\phi^+(y)\} \vee \chi = \mu_\phi^+(y) \vee \chi$ $\mu_\phi^-(x) \cap \psi \le S\{\mu_\phi^-(xy), \mu_\phi^-(y)\} \vee \chi$ $= T\{\mu_\phi^-(e), \mu_\phi^-(y)\} \vee \chi = \mu_\phi^-(y) \vee \chi$

This completes the proof.

Proposition 3.6: In a group X, every (ψ, χ)-bipolar fuzzy d-ideal of X is (ψ, χ)-bipolar fuzzy group of X.

Proof: Let 'ϕ' be a(ψ, χ) bipolar fuzzy d-ideal of a group X. Since $xy \le x$ for all x,y ε X, it follows from Proposition (3.5) that

$\mu_\phi^+(xy) \cap \psi \ge T\{\mu_\phi^+(x)$ and $\mu_\phi^-(x) \cap \psi \le \mu_\phi^-(x) \vee \chi$, so from Proposition 3.1

$(BPFG_1)$ $\mu_\phi^+(xy) \cap \psi \ge T\{\mu_\phi^+(x) \vee \chi \ge T\{\mu_\phi^+(xy), \mu_\phi^+(y)\} \vee \chi = T\{\mu_\phi^+(x), \mu_\phi^+(y)\} \vee \chi$

and

(BPFG2) $\mu_\phi^-(xy) \cap \psi \le \mu_\phi^-(x) \vee \chi \le S\{\mu_\phi^-(xy), \mu_\phi^-(y)\} \vee \chi \le S\{\mu_\phi^-(x), \mu_\phi^-(y)\} \vee \chi$

$\mu_\phi^+(x^{-1}) \cap \psi \ge T\{\mu_\phi^+(xy), \mu_\phi^+(x)\} \vee \chi = T\{\mu_\phi^+(e), \mu_\phi^+(y)\} \vee \chi \ge \mu_\phi^+(x) \vee \chi$ $\mu_\phi^-(x^{-1}) \cap \psi \le S\{\mu_\phi^-(xy), \mu_\phi^-(y)\} \vee \chi \le S\{\mu_\phi^-(e), \mu_\phi^-(y)\} \vee \chi \le \mu_\phi^-(x) \vee \chi$. Hence ϕ is (ψ, χ)-bipolar fuzzy group. The converse of the theorem is not true in general.

Proposition 3.7: Let 'ϕ' be a (ψ, χ) -bipolar fuzzy group of a group X such that Proposition 3.2 holds for all x, y, z ε X satisfying the inequality xy ε z then ϕ is a (ψ, χ)- bipolar fuzzy d-ideal of X.

Proof: Recall from Proposition 3.1; that $\mu_\varphi^+(e) \cap \psi \ge \mu_\varphi^+(x) \vee \chi$ and $\mu_\varphi^-(e) \cap \psi \le \mu_\varphi^-(x) \vee \chi$ for all x ε X. Since $x(xy) \le y$ for all x, y ε X, it follows that Proposition 3.2,

$$\mu_\phi^+(x) \cap \psi \ge T\{\mu_\phi^+(xy), \mu_\phi^+(y)\} \vee \chi \text{ and}$$

$$\mu_\phi^-(x) \cap \psi \le S\{\mu_\phi^-(xy), \mu_\phi^-(y)\} \vee \chi$$

Hence ϕ is a (ψ, χ)-bipolar fuzzy d-ideal of X.

Proposition 3.8: Let λ and μ be (ψ, χ)- bipolar fuzzy d-ideal of X, then λ×μ is also (ψ, χ)- bipolar fuzzy d-ideal of X.

Proof: For any $(x_1, x_2), (y_1, y_2) \varepsilon X \times X$, we have

(BFd$_1$) $(\lambda^+ \times \mu^+)(x_1, x_2) \cap \psi = T\{\lambda^+(x_1), \mu^+(x_2)\} \cap \psi$

$\geq T\{ T\{\lambda^+(x_1, y_1), \lambda^+(y_1)\}, T\{\mu^+(x_2, y_2), \mu^+(y_2)\}\} \vee \chi$

$= T\{ T\{\lambda^+(x_1, y_1), \mu^+(x_2, y_2)\}, T\{\lambda^+(y_1), \mu^+(y_2)\}\} \vee \chi$

$= T\{(\lambda^+ \times \mu^+)((x_1, x_2), (y_1, y_2)\} \vee \chi$

$(\lambda^- \times \mu^-)(x_1, x_2) \cap \psi = S\{\lambda^-(x_1), \mu^-(x_2)\} \cap \psi$

$\leq S\{ S\{\lambda^-(x_1,y_1), \lambda^-(y_1)\}, S\{\mu^-(x_2,y_2), \mu^-(y_2)\} \vee \chi$

$= S\{ S\{\lambda^-(x_1,y_1), \mu^-(x_2,y_2)\}, S\{\lambda^-(y_1), \mu^-(y_2)\} \vee \chi$

$= S\{ (\lambda^- \times \mu^-)(x_1, x_2)(y_1, y_2), (\lambda^- \times \mu^-)(y_1, y_2)\} \vee \chi$

$(\lambda^+ \times \mu^+)(x_1^{-1}, x_2^{-1}) \cap \psi$

$= T\{\lambda^+(x_1^{-1}), \mu^+(x_2^{-1})\} \cap \psi \geq T\{ T\{\lambda^+(x_1,y_1), \lambda^+(y_1)\}, T\{\mu^+(x_2,y_2), \mu^+(y_2)\} \vee \chi$

$= T\{ T\{\lambda^+(x_1,y_1), \mu^+(x_2,y_2)\}, T\{\lambda^+(y_1), \mu^+(y_2)\} \vee \chi$

$= T\{ (\lambda^+ \times \mu^+)(x_1, x_2)(y_1, y_2), (\lambda^+ \times \mu^+)(y_1, y_2)\} \vee \chi$

$(\lambda^- \times \mu^-)(x_1^{-1}, x_2^{-1}) \cap \psi = S\{\lambda^-(x_1^{-1}), \lambda^-(x_2^{-1})\} \cap \psi$

$\leq S\{ S\{\lambda^-(x_1,y_1), \lambda^-(y_1)\}, S\{\mu^-(x_2,y_2), \mu^-(y_2)\} \vee \chi$

$= S\{ S\{\lambda^-(x_1,y_1), \mu^-(x_2,y_2)\}, S\{\lambda^-(y_1), \mu^-(y_2)\} \vee \chi$

$\leq S\{ (\lambda^- \times \mu^-)(x_1, x_2, y_1, y_2), (\lambda^- \times \mu^-)(y_1, y_2)\} \vee \chi$

Hence λ×μ is (ψ, χ)-bipolar fuzzy d-ideal of X.

Proposition 3.9: Let f : X → Y be a homomorphism of groups. If ‘μ’ is a (ψ,χ)- bipolar fuzzy d-ideal of y, then μ^f is(ψ, χ)- bipolar fuzzy d-ideal of X.

Proof: For any x ε X, we have

$\mu^{+f}(x) \cap \psi = \mu^+(f(x)) \cap \psi \geq \mu^+(e) \vee \chi = \mu^+(f(e)) \vee \chi = \mu^{+f}(e) \vee \chi$

$\mu^{-f}(x) \cap \psi = \mu^-(f(x)) \cap \psi \leq \mu^-(e) \vee \chi = \mu^-(f(e)) \vee \chi = \mu^{-f}(e) \vee \chi$

Let x, y ε X

$T\{ \mu^{+f}(xy), \mu^{+f}(y) \} \cap \psi = T\{\mu^{+}(f(xy), \mu^{+}(f(y) \} \cap \psi = T\{\mu^{+}(f(x).f(y)), \mu^{+}(f(y)\} \cap \psi \leq \mu^{+}f(x) \vee \chi = \mu^{+f}(x). \vee \chi$

$S\{\mu^{-f}(xy), \mu^{-f}(y)\} \cap \psi = S \{ \mu^{-}f(xy), \mu^{-}(f(x)\} \cap \psi = S \{ \mu^{-}(f(x),f(x)), \mu^{-}(f(x) \} \cap \psi \geq \mu^{-}(f(x) \vee \chi = \mu^{-f}(x) \vee \chi$

Hence μ^{f} is(ψ,χ)- bipolar fuzzy d-ideal of X.

Proposition 3.10: Let f : X → Y be an epimorphism of groups. If μ^{f} is (ψ, χ)-bipolar fuzzy d-ideal of X, then $\mu(\psi, \chi)$ - bipolar fuzzy d-ideal of Y.

Proof: Let y ε Y, there exists x ε X such that f(x) = y, then

$\mu^{+}(y) \cap \psi = \mu^{+}(f(x)) \cap \psi = \mu^{+f}(x) \cap \psi \leq \mu^{+f}(e) \vee \chi = \mu^{+}(f(e) \vee \chi = \mu^{+}(e) \vee \chi$

$\mu^{-}(y) \cap \psi = \mu^{-}(f(x)) \cap \psi = \mu^{-f}(x) \cap \psi \geq \mu^{-f}(e) \vee \chi = \mu^{-}(f(e) \vee \chi = \mu^{-}(e) \vee \chi$

Let x, y ε Y, then there exists a, b ε X, such that f(a) = x and f(b) = y. It follows that

$\mu^{+}(x) \cap \psi = \mu^{+}(f(a) \cap \psi = \mu^{+f}(a) \vee \chi$ and $\mu^{-}(x) \cap \psi = \mu^{-}(f(a) \cap \psi = \mu^{-f}(a) \vee \chi$

$\geq T\{ \mu^{+f}(ab), \mu^{+f}(b)\} \vee \chi = T\{ \mu^{+}(f(ab), \mu^{+}(f(b)\} \vee \chi = T\{ \mu^{+}(f(a).f(b)), \mu^{+}(f(b)\} \vee \chi$

$= T\{ \mu^{+}(xy), \mu^{+}(y)\} \vee \chi$

Also

$\leq S\{ \mu^{-f}(ab), \mu^{-f}(b)\} \vee \chi = S\{ \mu^{-}(f(ab), \mu^{-}(f(b)\} \vee \chi = S\{ \mu^{-}(f(a).f(b)), \mu^{-}(f(b)\} \vee \chi$

$= S\{ \mu^{-}(xy), \mu^{-}(y)\} \vee \chi$

Hence μ is a(ψ,χ) -bipolar fuzzy d-ideal of y.

Proposition 3.11: Let 'A' be a bipolar fuzzy set in a group X and μ_A be the strongest (ψ, χ)-bipolar fuzzy relation on X, then A is a (ψ, χ)-bipolar fuzzy d-ideal of X if and only if μ_A is a (ψ, χ)-bipolar fuzzy d-ideal of X × X.

Proof: Suppose that 'A' is a(ψ, χ) -bipolar fuzzy d-ideal of X, then

$\mu_A^{+}(e, e) \cap \psi = T \{ A^{+}(e), A^{+}(e)\} \cap \psi$

$\geq T \{ A^{+}(x), A^{+}(y)\} \vee \chi = \mu_A^{+}(x, y) \vee \chi$ for all (x, y) ε X × X.

$\mu_A^{-}(e, e) \cap \psi = S \{ A^{-}(e), A^{-}(e)\} \cap \psi \leq S \{ A^{-}(x), A^{-}(y)\} \vee \chi = \mu_A^{-}(x, y) \vee \chi$ for all (x, y) ε X × X.

For any $x = (x_1, x_2)$ and

$y = (y_1, y_2)$ ε X × X.

$\mu_A^+(x) \cap \psi = \mu_A^+(x_1, x_2) \cap \psi$

$= T\{ A^+(x_1), A^+(x_2)\} \cap \psi \geq T\{T\{A^+(x_1,y_1), A^+(y_1)\}, T\{A^+(x_2, y_2), A^+(y_2)\} \} \vee \chi$

$= T\{ T\{A^+(x_1, y_1), A^+(x_2, y_2)\}, T\{A^+(y_1), A^+(y_2) \} \} \vee \chi$

$= T\{ \mu_A^+(x_1, y_1), (x_2, y_2)), \mu_A^+(y_1, y_2)\} \vee \chi = T\{ \mu_A^+(xy), \mu_A^+(y)\} \vee \chi$

$\mu_A^-(x) \cap \psi = \mu_A^-(x_1, x_2) \cap \psi$

$= S\{ A^-(x_1), A^-(x_2)\} \cap \psi \leq S\{S\{A^-(x_1, y_1), A^-(y_1)\}, S\{A^-(x_2, y_2), A^-(y_2)\} \} \vee \chi$

$= S\{S\{A^-(x_1, y_1), A^-(x_2, y_2)\}, S\{A^-(y_1), A^-(y_2) \} \} \vee \chi$

$= S\{ \mu_A^-(x_1, y_1), (x_2, y_2)), \mu_A^-(y_1, y_2)\} \vee \chi = S\{ \mu_A^-(xy), \mu_A^-(y)\} \vee \chi$

Hence μ_A is a (ψ, χ)-bipolar fuzzy d-ideal of $X \times X$. Conversely, suppose that μ_A is a (ψ, χ)-bipolar fuzzy d-ideal of $X \times X$. Then,

$T\{A^+(e), A^+(e)\} \cap \psi = \mu_A^+(e, e) \cap \psi$

$\geq \mu_A^+(x, y) \vee \chi = T\{A^+(x), A^+(y)\} \vee \chi \ \forall (x, y) \in X \times X.$

$S\{ A^-(e), A^-(e)\} \cap \psi = \mu_A^-(e, e) \cap \leq \mu_A^-(x, y) \vee \chi = S\{ A^-(x), A^-(y)\} \vee \chi$

for any $x = (x_1, y_1)$ and

$y = (y_1, y_2) \in X \times X$., we have

$T\{A(x_1), A(x_2)\} \cap \psi = \mu_A(x_1, x_2) \cap \geq T\{\mu_A((x_1, x_2), (y_1, y_2)), \mu_A(y_1, y_2)\} \vee \chi$

$= T\{\mu_A(x_1y_1, x_2y_2)), \mu_A(y_1, y_2)\} \vee \chi = T\{ T\{A(x_1, y_1), A(x_2, y_2)\}, T\{A(y_1), A(y_2)\} \vee \chi$

$= T\{ T\{ A(x_1, y_1), A(y_1), T\{A(x_2, y_2), A(y_2)\} \vee \chi$

Putting $x_1 = x_2 = 0$, we have

$\mu_A(x_1) \cap \psi \geq T\{\mu_A(x_1, y_1), \mu_A(y_1)\} \vee \chi$

Likewise, $\mu_A(x_1y_1) \geq T\{\mu_A(x_1), \mu_A(x_2)\}$

$S\{A(x_1), A(x_2)\} \cap \psi = \mu_A(x_1, x_2) \vee \chi \leq S\{\mu_A((x_1, x_2), (y_1, y_2)), \mu_A(y_1, y_2)\} \vee \chi$

$= S\{\mu_A(x_1y_1, x_2y_2)), \mu_A(y_1, y_2)\} \vee \chi = S\{ S\{A(x_1, y_1), A(x_2, y_2)\}, S\{A(y_1), A(y_2)\} \vee \chi$

$= S\{ S\{ A(x_1, y_1), A(y_1), S\{A(x_2, y_2), A(y_2)\} \vee \chi$

Putting $x_1 = x_2 = 0$, we have

$\mu_A(x_1) \cap \psi \leq S\{\mu_A(x_1, y_1), \mu_A(y_1)\} \vee \chi$

Likewise, $\mu_A(x_1y_1) \cap \psi \leq S\{\mu_A(x_1), \mu_A(x_2)\} \vee \chi$. Hence A is a (ψ, χ)-bipolar fuzzy d-ideal of X.

Proposition 3.12: Let ϕ be a bipolar fuzzy set in X, then ϕ is a (ψ, χ)-bipolar fuzzy d-ideal of X if and only if it satisfies the following assertions.

$(\forall \alpha \,\varepsilon\, [0,1] \quad (\phi_t^+ \neq \phi \Rightarrow \phi_t^+$ is an ideal of X)

$(\forall \beta \,\varepsilon\, [-1,0] \quad (\phi_s^- \neq \phi \Rightarrow \phi_\beta^-$ is an ideal of X)

Proof: Assume that ϕ is a(ψ, χ)- bipolar fuzzy d-ideal of X. Let (s,t) ε [-1, 0] ε [0,1] be such that $\phi_t^+ \neq \phi$ and $\phi_s^- \neq \phi$.

Obviously, $e \,\varepsilon\, \phi_t^+ \cap \phi_s^-$.

Let x, y ε X be such that $xy \,\varepsilon\, \phi_t^+$ and $y \,\varepsilon\, \phi_t^+$, and

Let a, b ε X be such that $ab \varepsilon \phi_s^-$ and $b \,\varepsilon\, \phi_s^-$, then

$\mu_\phi^+(xy) \cap \psi \geq t \vee \chi$, $\mu_\phi^+(y) \cap \psi \geq t \vee \chi$, $\mu_\phi^-(ab) \cap \psi \leq s \vee \chi$ and $\mu_\phi^-(b) \cap \psi \leq s \vee \chi$.

It follows from Proposition 3.1

$\mu_\phi^+(x) \cap \psi \geq T \{ \mu_\phi^+(xy), \mu_\phi^+(y)\} \geq t \vee \chi$ and

$\mu_\phi^-(a) \cap \psi \leq S \{ \mu_\phi^-(ab), \mu_\phi^-(b) \leq s \vee \chi$

so that $x \,\varepsilon\, \phi_t^+$ and $a \,\varepsilon\, \phi_s^-$. Therefore ϕ_t^+ and ϕ_s^- are ideals of X.

Conversely, suppose that the condition (corollary) is valid. For any x ε X, let $\mu_\varphi^+(x) \cap \psi = t \vee \chi$ and $\mu_\varphi^-(x) \cap \psi = s \vee \chi$, then $x \,\varepsilon\, \phi_t^+ \cap \phi_s^-$, and so ϕ_t^+ and ϕ_s^- are non-empty. Since ϕ_t^+ and ϕ_s^- are ideal of X, $e \,\varepsilon\, \phi_t^+ \cap \phi_s^-$. Hence $\mu_\phi^+(e) \cap \psi \geq t \vee \chi = \mu_\phi^+(x) \vee \chi$ and $\mu_\phi^-(e) \cap \psi \leq s \vee \chi = \mu_\phi^-(x) \vee \chi$ for all x ε X.

If there exists $x^1, y^1, a^1, b^1 \,\varepsilon\,$ X such that $\mu_\phi^+(x^1) \cap \psi \leq T\{ \mu_\phi^+(x^1y^1), \mu_\phi^+(y^1)\} \vee \chi$ and $\mu_\phi^-(a^1) \cap \psi \geq S\{ \mu_\phi^-(a^1b^1), \mu_\phi^-(b^1)\} \vee \chi$ then by taking

$t_0 = ½ \{ \mu_\phi^+(x^1) + T\{ \mu_\phi^+(x^1y^1), \mu_\phi^+(y^1)\}$

$S_0 = ½ \{ \mu_\phi^-(a^1) + S\{ \mu_\phi^-(a^1b^1), \mu_\phi^-(b^1)\}$

We have,

$\mu_\phi^+(x^1) \cap \psi < t_0 \leq T\{ \mu_\phi^+(x^1y^1), \mu_\phi^+(y^1)\} \vee \chi$

$\mu_\phi^-(a^1) \cap \psi < s_0 \leq S\{\mu_\phi^-(a^1b^1), \mu_\phi^-(b^1)\} \vee \chi$

Hence $x^1 \notin \phi_{t0}^+$, $x^1, y^1 \,\varepsilon\, \phi_{t0}^+$, $y^1 \,\varepsilon\, \phi_{t0}^+$, $a^1 \notin \phi_{s0}^-$ and $b^1 \,\varepsilon\, \phi_{s0}^-$. This is a contradiction and thus ϕ is a(ψ, χ)- bipolar fuzzy d-ideal of X.

Conclusion: K.J. Lee [6] introduces the notion of bipolar fuzzy sub-algebra and bipolar fuzzy ideals of BCK/BCI-algebra. In this paper, we provide a condition for a (ψ, χ)-bipolar fuzzy group and (ψ, χ)-bipolar fuzzy d-ideal. We give relations between a (ψ, χ)-bipolar fuzzy group and (ψ, χ)-bipolar fuzzy d-ideal. We consider the concept of strongest (ψ, χ)-bipolar fuzzy relation and discuss some related properties.

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