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(α,β)-FUZZY SOFT INT-GROUPS OVER FUZZY SOFT INTERIOR IDEALS

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ABSTRACT:

The aim of the paper is to lay a foundation for providing a soft fuzzy algebraic tool in considering many problems that contain uncertainties. In order to provide these soft fuzzy algebraic structures, the notion of (α,β)-fuzzy soft int-groups which is a generalization of that fuzzy soft groups is provided. By introducing the notion soft fuzzy cosets, soft fuzzy quotient groups based on (α,β)-fuzzy soft interior ideals are established. Finally , isomorphism theorems of (α,β)-fuzzy soft int-groups related to invariant fuzzy soft sets are discussed.

KEY WORDS: soft set, fuzzy set, fuzzy soft int-group, fuzzy soft interior ideal, soft fuzzy coset, soft fuzzy quotient group, invariant fuzzy soft set, extended image set.

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1. INTRODUCTION: The notion of fuzzy set was introduced by L.A.Zadeh[23], and since then this concept has been applied to various algebraic structure. Later several authors such as Booth[5] and Satyanarayana[20] studied the ideal theory of near-rings. Molodtsov[14] initiated the concept of soft sets that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Maji et.al [15]gave the operations of soft sets and their properties. Furthermore, they[16] introduced fuzzy soft sets which combine the strength of both soft sets and fuzzy sets. As a generalization of the soft set theory, the fuzzy soft set theory makes description of the objective world more realistic, practical precise in some cases, making it very promising.

Since the notion of soft groups was proposed by Aktas and Cagman[2], then the soft set theory is used a new tool to discuss algebraic structures. Acar et.al[3] initiated the concepts of soft rings similar to soft groups. Liu et.al further the investigated isomorphism and fuzzy isomorphism theories of soft rings in [13], respectively. Soft sets were also applied to other algebraic structures such as near-rings[17], Γ-modulus and BCK/BCI-algebras[22].Bhakat and Das[6] proposed the concept of (α, β)-fuzzy subgroups.

Cagman et.al[7] studied on soft int-group, which are different from the definitions of soft groups[2]. The new approach is based on the inclusion relation and intersection of sets. It brings the soft set theory, the set theory, and the group theory together. On the basic of soft int-groups, Sezgin et.al[18] introduced the concept of soft intersection near-rings (soft int-near rings) by using intersection operation of sets and gave the applications of soft int near-rings to the near-ring theory. By introducing soft intersection, union products and soft characteristic functions, Sezer[19]made a new approach to the classical ring theory via the soft set theory, with the concepts of soft union rings, ideals and bi-ideal. Jun et.al[10] applied intersectional soft sets to BCK/BCI-algebras[11] an obtained many results. In the present paper , we provide the notion of (α, β)-fuzzy soft int-groups over fuzzy soft interior ideals and the notion of fuzzy coset , soft fuzzy quotient groups based on(α, β)-fuzzy soft int-ideals are established. Finally , isomorphism theorems of (α, β)-fuzzy soft int-groups related to invariant fuzzy soft sets are discussed.

2. PREMINARIES

In this section, we would like to recall some basic notions related to soft sets and soft int-groups. Throughout the paper, G denote arbitrary groups and e,e_1, and e_2 are the identity elements of G,G_1 and G_2 respectively. U is an initial universe and E is a set of parameters under the conditions with respect to U. A and B are subsets of E. The set of all subsets of U is denoted by P(U). Molodtsov[14] defined the concept of soft sets in the following way.

Definition 2.1: [15]A soft set f_A over U is defined as f_A: E→ P(U) such that $f_A(x)$=∅ if $x \notin$ A.

In other words, a soft set U is a parameterized family of subsets of the universe U. For all $\epsilon \in$ A, $f_A(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set f_A. A soft set f_A over U can be presented by the set of ordered pairs: f_A = $\{(x, f_A(x))\ /\ x \in$ E, $f_A(x)$= P (U)}……….(1). Clearly, a soft set is not a set. For illustration, Molodtsov consider several examples in (14).

If f_A is a soft set over U, then the image of f_A is defined by Im(f_A) = $\{f_A(a)/a \in A\}$. The set of all soft sets over U will be denoted by S(U). Some of the operations of soft sets are listed as follows.

Definition 2.2:[16]Let $f_A, f_B \in$ S(U). If $f_A(x) \subseteq f_B(x)$, for all $x \in$ E, then f_A is called a soft subset of f_B and denoted by $f_A \subseteq f_B$. f_A and f_B are called soft equal, denoted by $f_A = f_B$ if and only if $f_A \subseteq f_B$ and $f_B \subseteq f_A$.

Definition 2.3: [18]Let $f_A, f_B \in$ S(U) and let χ be a function from A to B. Then the soft anti-image of f_A under χ denoted by $\chi(f_A)$, is a soft set over U defined by,

$$\chi_{f_A}(b) = \begin{cases} \cap\{ f_A(a)/a \in A, \chi(a) = b\}, if\ \chi^{-1}(b) \neq \emptyset \\ 0 \qquad , \text{ otherwise} \end{cases} \quad \text{……………(2)}$$

for all b∈B. And the soft preimage of f_B underχ, denoted by $\chi^{-1}(f_B)$, is a soft set over U defined by $\chi^{-1}{}_{f_B}$(a) = $f_B(\chi(a))$, for all $a \in A$.

Note that the concept of level sets in the fuzzy set theory, Cagman et.al[7] initiated the concept of lower inclusions soft sets which serves as a bridge between soft sets and crisp sets.

Definition 2.4:[9] Let G be a group and $f_G \in$ S(U). Then f_G is called a soft intersection groupoid over U if $f_G(xy) \supseteq f_G(x) \cap f_G(y)$ for all $x, y \in G$. f_G is called a soft intersection group over U if the soft intersection groupoid satisfies $f_G(x^{-1}) = f_G(x)$ for all$x \in G$.

For the sake of brevity, soft intersection group is abbreviated by soft int-group throughout this paper.

Example: Assume that U=Z is the universal set and G=Z_b is the subset of parameters. We define a soft set f_G by $f_G(0)$ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$f_G(1)$ = {0, 2, 4, 6, 8, 10},$f_G(2)$ = {1, 3, 4, 6, 7},$f_G(3)$ = {0, 2, 3, 6, 9},$f_G(4)$ = {1, 3, 4, 6, 7}

$f_G(5)$ = {0, 2, 4, 6, 8, 10} .It is clear that $f_G(1+z) \not\supseteq f_G(1) \cap f_G(z)$, implying f_G is not a soft int-group over U.

Definition 2.5:[23] Let U be a non-empty set. Then by a fuzzy set on U is meant a function A : U → [0,1].A is called the membership function, A(x) is called the membership grade of x in A. We also write A= {(x, A(x)): x∈U}

Example: Consider U = { a, b, c ,d } and A : U → [0,1] defined by A(a)=0, A(b)=0.7, A(c)=0.4, A(d)=1

Definition 2.6:[2] Let U be an initial universe, E be the set of all parameters and A⊆ E. A pair (F, A) is called a fuzzy soft set over U where F: A ⟶ P (U) is a mapping from A into P (U) , where P (U) denotes the collection of all subsets of U.

Example: Consider the above example, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then

(f_A, E) = {$f_A(e_1)$ = {(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)}, f_A (e_2) = {(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)} is the fuzzy soft set .

Definition 2.7: Let f_G be a fuzzy soft set over U. f_G is called a (α, β)-fuzzy soft int-group of U if (i) $f_G(xy) \cap \alpha \geq f_G(x) \cap f_G(y) \cup \beta$ (ii) $f_G(x^{-1}) \cap \alpha = f_G(x) \cup \beta$. For all$x, y \in G$.

Example:1 Let Z/(3) = $\{\bar{0}, \bar{1}, \bar{2}\}$ be a modulo 3 residue class group, A=$\{\lambda_1, \lambda_2\}$. Define a fuzzy soft set (F,A) over < Z/(3) , + > as ; F(λ_1)($\bar{0}$)=0.3, F(λ_1)($\bar{1}$)=0.6, F(λ_1)($\bar{2}$)=0.8, F(λ_2)($\bar{0}$)=0.4,

F(λ_2)($\bar{1}$)=0.5, F(λ_2)($\bar{2}$)=0.7. It is easy verify that F(λ_1), F(λ_2) are fuzzy subgroups of < Z/(3) , + >. Therefore (F,A) is a fuzzy soft int-groups over < Z/(3) , + >.

Example:2 Let G= {*e* ,*x* ,*y*, *z*} be the group with the binary operation defined below.

*	e	x	y	z
e	e	x	y	z
x	x	z	e	y
y	y	e	z	x
z	z	y	x	e

Let A = $\{h_1, h_2\}$ be the set of parameters. For each parameter $h_1 \in$ A, F(h_1): G→[0,1]. For each parameter we define

F(h_1) = { < *e*,0.6>, <*x*,0.75>, <*y*,0.62>, <*z*,0.31>}

F(h_2) = { < *e*,0.77>, <*x*,0.88>, <*y*,0.92>, <*z*,0.7>}.

Here (F,A) is fuzzy soft int-group.

Lemma 1: Let f_G be a fuzzy soft set over U. If f_G is a (α, β)-fuzzy soft int-group of G, then

(i) $(f_G(x^{-1}) \cap \alpha) \cup \beta \geq (f_G(x) \cap \alpha) \cup \beta$.(ii) $f_G(x^{-1}) \cap \alpha \geq f_G(x) \cup \beta$ for all$x \in G$.

Proof:(i) Assume that f_G is a (α, β)-fuzzy soft int-group of G.

Then for all$x \in G$, we get that

$(f_G(x^{-1}) \cap \alpha) \cup \beta = (f_G(x^{-1}) \cap \alpha \cap \alpha) \cup \beta \geq ((f_G((x^{-1})^{-1}) \cup \beta) \cap \alpha) \cup \beta$

$= (f_G(x) \cap \alpha) \cup \beta$.

(ii)It is straight forward.

Lemma2: Let f_G be a fuzzy soft set over U. If f_G is a (α, β)-fuzzy soft int-group of G, then

(i) $(f_G(e) \cap \alpha) \cup \beta \geq (f_G(x) \cap \alpha) \cup \beta$.(ii) $f_G(e) \cap \lambda \geq f_G(x) \cup \beta$ for all$x \in G$.

Proof: (i) Assume that f_G is a (α, β)-fuzzy soft int-group of G.

Then for all$x \in G$, we get that

$(f_G(e) \cap \alpha) \cup \beta = (f_G(xx^{-1}) \cap \alpha \cap \alpha) \cup \beta \geq ((f_G(x) \cap f_G(x^{-1})) \cup \beta) \cap \alpha) \cup \beta$

$= ((f_G(x) \cap \alpha) \cup \beta) \cap ((f_G(x^{-1}) \cap \alpha) \cup \beta) \geq (f_G(x) \cap \alpha) \cup \beta.$ (By Lemma:1)

(ii)It is straight forward.

Combining lemma:2 and Definition:2.7, we obtain the following characterization of (α, β)-fuzzy soft int-groups.

Theorem 2.1:A fuzzy soft set f_G over U is a (α, β)-fuzzy soft int-group over U if and only if

$f_G(xy^{-1}) \cap \alpha \geq f_G(x) \cap f_G(y) \cup \beta$ for all$x, y \in G$.

Proof: Suppose that f_G is a (α, β)-fuzzy soft int-group over U. Then

$f_G(xy^{-1}) \cap \alpha \geq f_G(x) \cap f_G(y^{-1}) \cup \beta = f_G(x) \cap f_G(y) \cup \beta$ for all$x, y \in G$.

Conversely, suppose that $f_G(xy^{-1}) \cap \alpha \geq f_G(x) \cap f_G(y) \cup \beta$ for all$x, y \in G$.

First, choosing x=e yields $f_G(y^{-1}) \cap \alpha \geq f_G(y) \cup \beta$. Thus,

$f_G(y) \cap \alpha = f_G((y^{-1})^{-1}) \cap \alpha \geq f_G(y^{-1}) \cup \beta$. Hence $f_G(y) \cap \alpha = f_G(y^{-1}) \cup \beta$. Secondly

$f_G(xy) \cap \alpha = f_G(x(y^{-1})^{-1}) \cap \alpha \geq f_G(x) \cap f_G(y^{-1}) \cup \beta = f_G(x) \cap f_G(y) \cup \beta$. Therefore

f_G is a (α, β)-fuzzy soft int-group over U.

Theorem2.2: Let f_G over U is a (α, β)-fuzzy soft int-group over U and $x \in G$. Then

$f_G(xy) \cap \alpha \geq f_G(y) \cup \beta$ for all $y \in G$ if and only if $f_G(x) \cap \alpha = f_G(e) \cup \beta$.

Proof: Let $f_G(xy) \cap \alpha \geq f_G(y) \cup \beta$ for all $y \in G$.

Choosing $y = e$ yields $f_G(x) \cap \alpha \geq f_G(e) \cup \beta$, thus by lemma-2, $f_G(x) \cap \alpha = f_G(e) \cup \beta$.

Conversely, let $f_G(x) \cap \alpha = f_G(e) \cup \beta$. Then

$f_G(xy) \cap \alpha \geq f_G(x) \cap f_G(y) \cup \beta = f_G(e) \cap f_G(y) \cup \beta = f_G(y) \cup \beta$.

Theorem2.3: Let f_G and f_H be (α, β)-fuzzy soft int-group over U. Then $f_G \cap f_H$ is also a (α, β)-fuzzy soft int-group over U.

Proof: Let (x_1, y_1), $(x_2, y_2) \in$ G × H. Then

$f_{GH}\,((x_1, y_1)(x_2, y_2)^{-1}) \cap \alpha = f_{GH}\,(x_1 x_2^{-1}, y_1 y_2^{-1}) \cup \beta = f_G\,(x_1 x_2^{-1}) \cap f_H(y_1 y_2^{-1}) \cup \beta$

$\geq (f_G(x_1) \cap f_G(x_2) \cup \beta) \cap (f_H(y_1) \cap f_H(y_2) \cup \beta) = (f_G(x_1) \cap f_H(y_1) \cup \beta) \cap (f_G(x_2) \cap f_H(y_2) \cup \beta) = f_{GH}\,(x_1, y_1) \cap f_{GH}\,(x_2, y_2) \cup \beta$.Therefore, f_G □ f_H is a (α, β)-fuzzy soft int-group over U.

Note that $f_G \cup f_H$ is not a (α, β)-fuzzy soft int-group over U.

Definition 2.8:[22] Let $f_A, f_B \in$ S (U). Then, ∩-Product and V- Sum of f_A and f_B, denoted by $f_A f_B$ and $f_A \mathrm{V} f_B$, are defined by $f_{AB}(x, y) = f_A(x) \cap f_B(y)$, $f_{A\mathrm{V}B}(x, y) = f_A(x) \cup f_B(y)$ for all x ,$y \in E$ respectively.

Definition 2.9: Let f_G and f_H be (α, β)-fuzzy soft int-groups over U. Then, the product of soft int-groups f_G and f_H is defined as $f_G \times f_H = f_{G\times H}$ where

$f_{G\times H}(x, y) \cap \alpha = f_G(x) \times f_H(y) \cup \beta$ for all $(x, y) \in$ G × H.

Theorem2.4: If f_G and f_H be (α, β)-fuzzy soft int-groups over U, then so is $f_G \times f_H$ over U×U.

Proof: By definition:2.9, let $f_G \times f_H = f_{G\times H}$ where

$f_{G\times H}(x, y) \cap \alpha = f_G(x) \times f_H(y) \cup \beta$ for all $(x, y) \in$ G × H.

Then for all (x_1, y_1), $(x_2, y_2) \in$ G × H,

$f_{G\times H}\,((x_1, y_1)(x_2, y_2)^{-1}) \cap \alpha = f_{G\times H}\,(x_1 x_2^{-1}, y_1 y_2^{-1}) \cap \alpha = f_G\,(x_1 x_2^{-1}) \times f_H(y_1 y_2^{-1}) \cup \beta$

$\geq (f_G(x_1) \cap f_G(x_2) \cup \beta) \times (f_H(y_1) \cap f_H(y_2) \cup \beta) = (f_G(x_1) \times f_H(y_1) \cup \beta) \cap (f_G(x_2) \times f_H(y_2) \cup \beta) = f_{G\times H}\,(x_1, y_1) \cap f_{G\times H}\,(x_2, y_2) \cup \beta$.

Hence, $f_G \times f_H = f_{G\times H}$ is fuzzy soft int-groups over U×U.

Theorem2.5: Let f_H be (α,β)-fuzzy soft int-group over U and h be a homomorphism from G to H. Then $h^{-1}(f_H)$ is (α,β)-fuzzy soft int-group over U.

Proof: Let $x, y \in$ G. Then,

$$h^{-1}(f_H)(xy) \cap \alpha = f_H(h(xy)) \cap \alpha = f_H\big(h(x)h(y)\big) \cap \alpha \geq f_H(h(x)) \cap f_H(h(y)) \cup \beta$$

$$= h^{-1}(f_H)(x) \cap h^{-1}(f_H)(y) \cup \beta.$$

Also,

$$h^{-1}(f_H)(x^{-1}) \cap \alpha = f_H(h(x^{-1})) \cap \alpha = f_H((h(x))^{-1}) \cap \alpha = f_H\big(h(x)\big) \cup \beta = h^{-1}(f_H)(x) \cup \beta$$

Hence, $h^{-1}(f_H)$ is (α,β)-fuzzy soft int-group over U.

Definition 2.8: A fuzzy soft set f_G over U is called (α,β)-fuzzy soft interior ideal of G if it satisfies $f_G(xy) \cap \alpha \geq f_G(x) \cap f_G(y) \cup \beta$, $f_G(xwy) \cap \alpha \geq f_G(x) \cup \beta$ for all $x, y, w \in$ G.

Example: Let S= {0, e,f,a,b} be a set with the following cayley table.

.	0	e	f	a	b
0	0	0	0	0	0
e	0	e	0	a	0
f	0	0	f	0	b
a	0	a	0	0	e
b	0	0	b	f	0

Let h : G⟶ [0,1] be a fuzzy soft set in G defined by h(0) = h(e) =h(f) = 1, h(a) = h(b) = 0. By routine calculations one show that h is an (α,β)-fuzzy soft interior ideals.

Definition 2.9: Let f_G be a (α,β)-fuzzy soft interior ideal of G. Then E Im(f_G) is called the extended image set of f_G, where E Im(f_G) = Im$(f_G) \cup (\alpha,\beta)$.

Now we characterize (α,β)-fuzzy soft interior ideals by upper inclusion.

Theorem 2.6: Let f_G be a fuzzy soft set over U and E Im(f_G) a totally order set in inclusion. Thenf_G is a (α, β)-fuzzy soft interior ideal of G if and only if $\cup(f_G: \lambda)$ is an ideal of G, whenever it is non empty, for each $\lambda \subseteq$ U where $\beta \leq \lambda < \alpha$.

Proof: Assume that f_G is a (α, β)-fuzzy soft interior ideal of G and $\cup(f_G: \lambda)$ is non empty.

It is sufficient to show that $x\ y \in \cup(f_G: \lambda)$. Let $x, y \in \cup(f_G: \lambda)$ it follows that $f_G(x) \geq \lambda$ and $f_G(y) \geq \lambda$. Since f_G is a (α, β)-fuzzy soft interior ideal of G and E Im(f_G) a totally order set, then

$f_G(xy) \cap \alpha \geq f_G(x) \cap f_G(y) \cup \beta \geq \lambda \cup \lambda \cup \lambda = \lambda < \alpha,$

$f_G(xwy) \cap \alpha \geq f_G(x) \cup \beta \geq \lambda \cup \lambda = \lambda < \alpha.$

And thus $f_G(xy) \geq \alpha$. Hence $x\ y \in \cup(f_G: \lambda)$. Therefore $\cup(f_G: \lambda)$ is an ideal of G.

Conversely, assume that $\cup(f_G: \lambda)$ is an ideal of G, whenever it is non empty, for each

$\lambda \subseteq$ U where $\beta \leq \lambda < \alpha$. Suppose that $f_G(xy) \cap \alpha \geq f_G(x) \cap f_G(y) \cup \beta$ does not holds for some $x, y \in$ G; then there exists $x_0, y_0 \in$G such that $f_G(x_0 y_0) \cap \alpha \leq \lambda = (f_G(x_0) \cap f_G(y_0) \cup \beta)$.

There fore $f_G(x_0) \cap f_G(y_0) \supseteq \lambda$; that is $x_0 y_0 \notin \cup(f_G: \lambda)$ which is contradiction. Hence

$f_G(xy) \cap \alpha \geq f_G(x) \cap f_G(y) \cup \beta$, for all $x, y \in$ R. Similarly, we can prove that

$f_G(xwy) \cap \alpha \geq f_G(x) \cup \beta$ for all $x, y, w \in$ G. Thus, f_G is a (α, β)-fuzzy soft interior ideal of G.

Theorem2.7: Let f_G be a fuzzy soft set over U and χ is a group homomorphism from G_1 to G_2. If $f_{G_1}$is a (α, β)-fuzzy soft interior ideal of G_2, then $\chi^{-1}(f_{G_2})$ is a (α, β)-fuzzy soft interior ideal of G_1.

Proof: Let $x_1, x_2 \in G_1$. Then$\chi^{-1}(f_{G_2})(x_1 x_2) \cap \alpha = f_{G_2}(\chi(x_1 x_2)) \cap \alpha = f_{G_2}(\chi(x_1).\chi(x_2)) \cap \alpha$

$\geq f_{G_2}(\chi(x_1)) \cap f_{G_2}(\chi(x_2)) \cup \beta = \chi^{-1}(f_{G_2})(x_1) \cap \chi^{-1}(f_{G_2})(x_2) \cup \beta$

Moreover, we have

$$\chi^{-1}(f_{G_2})(x_1wx_2)\cap\alpha = f_{G_2}(\chi(x_1wx_2))\cap\alpha = f_{G_2}(\chi(x_1).\chi(w).\chi(x_2))\cap\alpha$$

$$\geq (f_{G_2}(\chi(x_1)))\cup\beta = \chi^{-1}(f_{G_2})(x_1)\cup\beta$$

Hence $\chi^{-1}(f_{G_2})$ is a (α,β)-fuzzy soft interior ideal of G_1.

Theorem 2.8: Let f_{G_1} be a fuzzy soft set over U and χ a group epimorphism from G_1 to G_2.If $f_{G_1}$is a (α,β)-fuzzy soft interior ideal of G_1, then $\chi(f_{G_1})$ is (α,β)-fuzzy soft interior ideal of G_2 and $(\chi(f_{G_1})(e_2)\cap\alpha)\cup\beta = (f_{G_1})(e_1)\cap\alpha)\cup\beta$.

Proof: Let $y_1,y_2\in G_2$ and $f_{G_1}$is a (α,β)-fuzzy soft interior ideal of G_1. Since χ is a group epimorphism from G_1 to G_2, then $\chi^{-1}(y_1)\neq\emptyset$ and $\chi^{-1}(y_2)\neq\emptyset$. And thus, there exist

$x_1,x_2\in G_1$ such that $\chi(x_1)=y_1,\chi(x_2)=y_2$. There fore , we have

$\chi(f_{G_1})(y_1y_2)\cap\alpha = \cap\{f_{G_1}(x_1x_2)/\ \chi(x_1x_2)=y_1y_2\}\cap\alpha = \cap\{f_{G_1}(x_1x_2)\cap\alpha/\ \chi(x_1x_2)=y_1y_2\}$

$\geq\cap\{f_{G_1}(x_1)\cap f_{G_1}(x_2)\cup\beta/\ \chi(x_1)=y_1,\chi(x_2)=y_2\} = \cap\{\ f_{G_1}(x_1)/\ \chi(x_1)=y_1\}\cap\{f_{G_1}(x_2)/\ \chi(x_2)=y_2\}\cup\beta = \chi(f_{G_1})(y_1)\cap\chi(f_{G_1})(y_2)\cup\beta$

$\chi(f_{G_1})(y_1wy_2)\cap\alpha = \cap\{f_{G_1}(y_1wy_2)/\ \chi(x_1wx_2)=y_1wy_2\}\cap\alpha$

$= \cap\{f_{G_1}(y_1wy_2)\cap\alpha/\ \chi(x_1wx_2)=y_1wy_2\}$

$\geq\cap\{f_{G_1}(\ y_1)\cap\alpha/\ \chi(x_1)=y_1\} = \cap\{\ f_{G_1}(x_1)/\ \chi(x_1)=y_1\}\cup\beta = \chi(f_{G_1})(y_1)\cup\beta$

Therefore $\chi(f_{G_1})$ is (α,β)-fuzzy soft interior ideal of G_2.

By lemma-2, we have

$$(\chi(f_{G_1})(e_2)\cap\alpha)\cup\beta = (\cap\{f_{G_1}(x)/x\in G_1,\chi(x)=e_2\}\cap\alpha)\cup\beta$$

$$= \cap\{(\ f_{G_1}(x)\cap\alpha)\cup\beta/x\in G_1,\chi(x)=e_2\} = (f_{G_1}(e_1)\cap\alpha)\cup\beta.$$

3. SOFT FUZZY QUOTIENT GROUPS

The main purpose of this section is to give an approach for constructing soft fuzzy quotient groups based on (α,β)-fuzzy soft interior ideals. Such approaches involve the concept of soft fuzzy cosets. In addition, some simple characterizations of soft fuzzy cosets are presented.

Definition 3.1: Let f_G be a (α,β)-fuzzy soft int-group of G over U and $g \in G$. Then, a soft fuzzy coset $g\oplus f_G$ of f_G is defined by $(g\oplus f_G)(x) = (f_G(x-g)\cap\alpha)\cup\beta$, for all $x\in G$.

Main results

Theorem3.1: Let f_G be a (α,β)-fuzzy soft int-group of G over U and $a,b\in G$.Then

$(f_G(ab)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$ if and only if $(f_G(ba)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$.

Proof: Suppose that $(f_G(ba)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$.Since f_G is (α,β)-fuzzy soft int-group, then $(f_G(ab)\cap\alpha)\cup\beta = (f_G(ab)\cap\alpha\cap\alpha)\cup\beta \geq ((f_G(ba)\cup\beta)\cap\alpha)\cup\beta$

$$= (f_G(ba)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$$

By lemma-2, we have,

$(f_G(e)\cap\alpha)\cup\beta \geq (f_G(ab)\cap\alpha)\cup\beta$.Thus, $(f_G(ab)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$.

Conversely, assume that $(f_G(ab)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$. We can prove that

$(f_G(ba)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$ in a similar way.

Proposition3.1: Let f_G be a (α,β)-fuzzy soft int-group of G over U and $a,b\in G$.Then

$a\oplus f_G = b\oplus f_G$ if and only if $(f_G(ab)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$.

Proof: Suppose that $(f_G(ab)\cap\alpha)\cup\beta = (f_G(e)\cap\alpha)\cup\beta$.

Then $(b\oplus f_G)(x) = (f_G(xb)\cap\alpha)\cup\beta = (f_G(xaa^{-1}b)\cap\alpha)\cup\beta$

$= (f_G(xaa^{-1}b)\cap\alpha\cap\alpha)\cup\beta \geq ((f_G(xa)\cap f_G(a^{-1}b)\cup\beta)\cap\alpha)\cup\beta$

$= ((f_G(xa)\cap\alpha)\cup\beta)\cap((f_G(a^{-1}b)\cap\alpha)\cup\beta) = ((f_G(xa)\cap\alpha)\cup\beta)\cap((f_G(e)\cap\alpha)\cup\beta)$

$\geq (f_G(xa) \cap \alpha) \cup \beta = (a \oplus f_G)(x)$ for all $x \in G$.

Therefore $b \oplus f_G \geq a \oplus f_G$. Similarly we can show that $a \oplus f_G \geq b \oplus f_G$. Hence $b \oplus f_G = a \oplus f_G$.

Conversely, assume that $a \oplus f_G = b \oplus f_G$.

It follows that $(f_G(ab) \cap \alpha) \cup \beta = (b \oplus f_G)(a) = (a \oplus f_G)(a) = (f_G(e) \cap \alpha) \cup \beta$.

Based on the above proposition, we give a property related to soft fuzzy cosets as follows.

Proposition3.2: Let f_G be a (α, β)- fuzzy soft interior ideal over U and $a, b, x, y \in G$. If

$x \oplus f_G = a \oplus f_G, y \oplus f_G = b \oplus f_G$, then $xy \oplus f_G = ab \oplus f_G, xwy \oplus f_G = a \oplus f_G$.

Proof: Suppose $x \oplus f_G = a \oplus f_G$,$y \oplus f_G = b \oplus f_G$.

Then, $(f_G(xa) \cap \alpha) \cup \beta = (f_G(e) \cap \alpha) \cup \beta$ and $(f_G(yb) \cap \alpha) \cup \beta = (f_G(e) \cap \alpha) \cup \beta$, by proposition 3.1.Since f_G is a (α, β)-fuzzy soft interior ideal, then

$$(f_G((xyab)) \cap \alpha) \cup \beta = (f_G((xa.yb)) \cap \alpha) \cup \beta = (f_G((xa.yb)) \cap \alpha \cap \alpha) \cup \beta$$

$$\geq((f_G(xa) \cap f_G(yb) \cup \beta) \cap \alpha) \cup \beta$$

$$= (f_G(xa) \cap \alpha) \cap (f_G(yb) \cap \alpha) \cup \beta = (f_G(e) \cap \alpha) \cup \beta$$

On the other hand, it follows from lemma:2 that $(f_G(e) \cap \alpha) \cup \beta \geq(f_G((xyab)) \cap \alpha) \cup \beta$.

Hence $(f_G((xyab)) \cap \alpha) \cup \beta = (f_G(e) \cap \alpha) \cup \beta$, and so $xy \oplus f_G = ab \oplus f_G$.

More over $(f_G((xwyab)) \cap \alpha) \cup \beta = (f_G(xawy.aywb) \cap \alpha \cap \alpha) \cup \beta$

$$= (f_G(xay) \cap \alpha) \cup \beta = (f_G(xay) \cap \alpha \cap \alpha) \cup \beta$$

$$\geq ((f_G(xa) \cup \beta) \cap \alpha)= (f_G(xa) \cap \alpha) \cup \beta$$

$$= (f_G(e) \cap \alpha) \cup \beta.$$

According to lemma-2, we get that $(f_G(e) \cap \alpha) \cup \beta \geq(f_G((xyab)) \cap \alpha) \cup \beta$.

Therefore, $(f_G(e) \cap \alpha) \cup \beta \geq(f_G((xyab)) \cap \alpha) \cup \beta$; that is $xy \oplus f_G = ab \oplus f_G$.

In view of proposition 3.2, we have the following result.

Proposition3.3: Let f_G be a (α,β)- fuzzy soft interior ideal over U. Then $(G/f_G\,,.)$ is a group, where $G/f_G \triangleq \{\, a\oplus f_G/a \in G\}$, $(x\oplus f_G)\ (y\oplus f_G)\ \triangleq xy\oplus f_G$, and $(x\oplus f_G)w\ (y\oplus f_G)\ \triangleq xy\oplus f_G$, for all $x, y \in G$.

Proof: It is straight forward.

Definition 3.2: Let f_G be a (α,β)- fuzzy soft interior ideal over U. Then $(G/f_G\,,.)$is called a soft fuzzy quotient group.

Theorem3.2: Let f_G be a (α,β)- fuzzy soft interior ideal over U. Then $G/{f_G}^{*} \cong G/f_G$.

Proof: Assume that h*:* $G \rightarrow G/f_G$ such that $h(x) = x\oplus f_G$, for all $x \in G$. It is easy to see that

h is a surjective homomorphism from G to G/f_G.

Since Kerl(h) = $\{\, x \in G/h(x) = e\oplus f_G\} = \{\, x \in G/x\oplus f_G = e\oplus f_G\} = \{\, x \in G/(f_G(x) \cap \alpha) \cup \beta$ $= (f_G(e) \cap \alpha) \cup \beta\} = {f_G}^{*}$.Therefore, $G/{f_G}^{*} \cong G/f_G$.

Definition 3.3: Let $\chi : G_1 \rightarrow G_2$ be a group homomorphism. A fuzzy soft set of f_{G_1} over U is called invariant fuzzy soft set with respect to χ if $\chi(x_1) = \chi\,(x_2)$ implies $f_{G_1}\,(x_1) = f_{G_1}(x_2)$ for all $x_1, x_2 \in G_1$.

Proposition 3.4: Let $\chi : G_1 \rightarrow G_2$ be a group homomorphism and f_{G_2} a fuzzy soft set of G_2 over U. Then, $\chi^{-1}(f_{G_2})$ is an invariant fuzzy soft set with respect to χ.

Proof: Let $x_1, x_2 \in G_1$ such that $\chi(x_1) = \chi\,(x_2)$. Then $\chi^{-1}(f_{G_2})(x_1) = f_{G_2}(\chi(x_1))= f_{G_2}(\chi(x_2))$ $= \chi^{-1}(f_{G_2})(x_2)$.Hence, $\chi^{-1}(f_{G_2})$ is an invariant fuzzy soft set with respect to χ.

Next, we establish isomorphism theorem on (α,β)-fuzzy soft int-groups.

Theorem3.3: [Isomorphism theorem]Let $\chi : G_1 \rightarrow G_2$ be an epimorphism and let (α,β)-fuzzy soft interior ideal f_{G_1}be a invariant fuzzy soft set with respect to χ. Then $G_1/f_{G_1} \cong G_2/\chi(f_{G_1})$.

Proof: Let $\chi : G_1 \rightarrow G_2/\chi(f_{G_1})$ be a mapping such that $\chi(x) = \chi(x) \oplus f_{G_1}$, for all $x \in G_1$. Obviously, χ is an epimorphism. Since f_{G_1}be a invariant fuzzy soft set with respect to χ, Ker(χ) = $\{x \in G_1 / \chi(x) = e_2 \oplus \chi(f_{G_1})\}$= $\{x \in G_1 / \chi(x) \oplus \chi(f_{G_1}) = e_2 \oplus \chi(f_{G_1})\}$

= $\{x \in G_1 / \chi(f_{G_1})(\chi(x) \cap \alpha) \cup \beta\}$ = $\{ \chi(f_{G_1})(e_2) \cap \alpha) \cup \beta\}$ = $\{ x \in G_1 / x \oplus f_{G_1} = e_1 \oplus f_{G_1}\}$ = ${f_{G_1}}^*$ Therefore, $G_1/{f_{G_1}}^* \cong G_2/\chi(f_{G_1})$. By theorem 3.2 we have $G_1/{f_{G_1}}^* \cong G_1/f_{G_1}$. Hence $G_1/f_{G_1} \cong G_2/\chi(f_{G_1})$.

Proposition3.5: Let $\chi : G_1 \rightarrow G_2$ be an epimorphism and let (α, β)-fuzzy soft interior ideal f_{G_2} be a invariant fuzzy soft set with respect to χ. Then $G_1/\chi^{-1}(f_{G_2}) \cong G_2/f_{G_2}$.

Proof: It follows from the theorem 3.2 and proposition 3.4 that $\chi^{-1}(f_{G_2})$ is a (α, β)- fuzzy soft interior ideal of G_1 and $\chi^{-1}(f_{G_2})$ is a invariant fuzzy soft set with respect to χ. Since χ is an epimorphism, then $\chi(\chi^{-1}(f_{G_1}))$= f_{G_2}. By theorem3.3, we get $G_1/\chi^{-1}(f_{G_2}) \cong G_2/f_{G_2}$.

Conclusion: In this paper, using fuzzy soft sets and intersection of sets, we have defined (α, β)-fuzzy soft int-group that is new type of fuzzy soft group on a fuzzy soft set and then make theoretical studies of (α, β)-fuzzy soft int-groups and (α, β)-fuzzy soft interior ideal in more detail and improved several results. We have focused on (α, β)-fuzzy soft int-groups and (α, β)-fuzzy soft interior ideal, anti-image of fuzzy soft set and investigate these notions with respect to fuzzy soft int-groups and fuzzy soft interior ideal .

Future work: To extend our work, further research can be done to study the properties of fuzzy soft int-group in other algebraic structures such as modules, rings and fields.

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