THE STUDY OF SUNSPOTS AND K-INDEX DATA IN THE PERSPECTIVE OF PROBABILITY DISTRIBUTIONS

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Abstract:

As we know that sunspots cycles varies with the variation of the magnetic field of the Sun, the average sunspots cycle is 11-years while one solar magnetic cycle ranges over 22-years. This means that each solar magnetic cycle contains two sunspots cycles. After each 11 years the magnetic poles change their polarity i.e. the north pole becomes south pole and vice versa. This paper utilizes the sunspots data from 1747 to 2011 (24 cycles) and *K*-index data from 1932 to 2011. We will analyze the sunspots data and *K*-index data using probability distributions. We will stress upon the study of variations of the type of probability distributions. These include Normal, Gamma, Lognormal, Hypersecant and Chi-square distributions which are tested with the help of Kolmogorov-Smirnov *D*-test. Further more, we will cycle wise compare the probability distributions of Sunspots and *K*-index data (geomagnetic indices). We compared the two sunspots cycles data with a single *K*-index cycle data in the perspective of probability distributions. The adequacy of the distribution of sunspots and *K*-index cycle represents the possibility of suitable probability distribution for the next cycles.

Key words: Probability Distributions, Sunspots Cycles (SC), Kolmogorov-Smirnov test

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1. INTRODUCTION:

Sunspots have been observed through telescopes for almost 400 years and early observed, such as Galileo, Scheiner and Hevelius, were already able to distinguish the dark central core of a spot (the umbra) from the fuzzier annulus (the penumbra) that surrounds it [1]. After extensive observations of sunspots, Wolf found in 1852 the period of sunspot cycles to be 11.1 years [14]. A few years later, Carrington (1858) discovered that sunspots drift in latitude towards the equator within a sunspot cycle, which was confirmed one year later by Wolf (Kiepenheuer, 1953; Wolf, 1859) [2 & 13]. It is also important to note that the eleven years Sunspots cycle and ozone layer depletion over arctic region have significant correlation [3-4]. The Sun has an approximately 22-years magnetic cycle. It means that there are two sunspot cycles in each solar magnetic period. The ratio between sunspot cycle and frequency of magnetic field is 2:1 [9, 10 &11] . At the solar poles the dipolar magnetic field reverses in each 11-years and during this reversal very few sunspots are visible some time none can be seen on the sun and the magnetic activity is known as minimum. During the maximum in the cycle, the number of sunspots visible on the sun can be more than 100 at one time. A typical sunspot has a lifetime of a few weeks [5]. With the help of early records Wolf derived a more accurate value over eleven years for the average of cycle length. All the sunspot cycles not attain the same maximum intensity through the study of sunspots cycles it is observed that the peak intensity occurs progressively later for weaker cycles. The study of cycle growth provides to predict the strength and date of maximum approximately for next cycle [15].

Geomagnetic disturbances caused by the solar wind flowing out of coronal holes. We can see a corona hole in x-ray pictures of the sun where the corona appears dark [16]. Coronal holes associated with those parts of the magnetic field where the field lines of magnetic fields are open while the bright regions of x-ray are associated with closed field lines of magnetic field, open field lines extend out to great distances from the Sun while closed lines form loops that return to the Sun [12]. According to the Lorentz force equation

$$F = q\left(E + \frac{v}{c}B\right)$$

where F is the force exerted in an electric field **E** and magnetic field **B** on a charged particle of velocity **v** states that the force due to the magnetic field is always perpendicular to both the direction of the velocity vector and the field (the cross product).

The *K*-index is the geomagnetic index which is one of the more valuable from the point of view of the Earth's climate. It consists of a scale which varies from zero to nine (very quiet to extremely disturbed). It can be measured through each observatory at three hour intervals. *K*p is an average of *K*-indices which is standardized and determined by all stations' data. A storm is considered to be a major storm if equals or exceeds forty. Between 1932 and 1989 more than thousand of these storms occurred. Number of these storms mentions a separate index which is known as Ap index as introduce by Allen in 1978 [9].

The C-figure is the arithmetic mean of the subjective classification by all observatories of each day's magnetic activity on a scale of 0 to 2 (quiet to storm). The *K*-index expressed in thirds of a unit (e.g $5-$ is 4 2/3, 5o is 5 0/3, and $5+$ is 5 1/3) [18].The aa index is the average of the measurements of two antipodal observatories, Greenwich and Melbourne characterizing the magnetic activity on earth given in Gammas [17].

2. MATERIAL AND METHODS:

Probability distributions have been found to be useful in the study of temperature data [6]. We will use these distributions to study the solar activity, particularly, *K*-indices and other solar activity indices. Such studies will not only be useful in the study of solar phenomena but also will be helpful to investigate the solar-terrestrial relations. In this study we will analyze Sunspots data and *K*-indices data. For this purpose we will first estimate determine the type of probability distribution suitable for each Sunspots Cycles (SC). In this regard we have used sunspots data starting from 1747 to 2011. These conclude 23 sunspots cycles. Cycle 24 is in progress and cannot be analyze completely. However, we have given a partial analysis of this cycle as well. Similarly we have analyzed the data regarding *K*-indices. The *K*-index data from 1932 to 2011 (Procured from world data center) divided corresponding to the SCs in between this duration (One magnetic cycle corresponding to two SCs). On this data various distributions (e.g ND, CSD) will be applied. The most suitable distribution for each of the partitions will then be determined. A comparison of the distributions of SC data and *K*-index data will then be made. All the computations are made with the help of statistical software Easy Fit (EF).

3. **PROBABILITY DISTRIBUTION APPROACH**

In any experimental analysis in any branch of science one often encounters the problems where the standard probability distributions are applicable. These probability distributions are often helpful in generating the random numbers [8].

The study of sunspots and their cycles is very helpful to observe the change and variation on the earth climate. There is a marked correlation between sunspots cycles and earth climate (rainfall and temperature) [6]. It is well known that the study of statistical distributions of climatic parameters generally gives more insights of the physical processes. In general, Sunspots and *K*-index data are probability distribution.

In assessing the nature of the distribution obeyed by real-time data, we check for the deviation from normality using KST [Kolmogorov-Smirnov *D*-test]. This tests that whether the statistic

$$D = \max|F(x) - G(x)| \qquad (1.1)$$

exceed a critical value in the K-S table or not. $G(x)$ and $F(x)$ are respectively the sample cumulative distribution and the predetermined cumulative distribution corresponding to a given sample of size n [7]. We know that the normal distribution is the most important distribution of continuous variables applied to symmetrically distributed data. Mathematically it can be expressed as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, (-\infty < x < +\infty), \qquad (1.2)$$

Where, σ is the standard deviation and μ is the mean of the sample.

Similarly if the random variable Y follows gamma distribution with parameters α and β , then the likelihood of Y is expressed as,

$$g(y) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} Y^{\alpha-1} e^{-\beta y}, (y \geq 0, \alpha > 0, \beta > 0) \qquad (1.3a)$$

Where, α *is* shape parameter, β is scale parameter and

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt$$

$$E[Y] = \frac{\alpha}{\beta}, \ \mathrm{Var}(Y) = \frac{\alpha}{\beta^2} \qquad (1.3b)$$

The Normal Distribution (ND) can be converted to Gamma Distribution (GD) and (GD) can be converted to Log-Normal Distribution (LND) [7]. The mathematical expression for LND is given by

$$h(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2}, \quad (-\infty < x < +\infty), \qquad (1.4a)$$

$$E[X] = e^{\mu + \frac{\sigma^2}{2}}, \ \mathrm{Var}(X) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right) \qquad (1.4b)$$

In equations (2.3b) and (2.4b), $E[X]$ and Var (X) represent mean and variance of samples respectively [6, 7 and 8]. A lognormal process being one in which the random variable of interest results from the product of many independent random variables multiplied together, whereas the normal process is one in which the random variable of interest results from the sum of many independent random variables [8].

Where the variable x > 0 and both the parameters μ and $\delta > 0$ belongs to set of real numbers. The LND is used sometimes as a first approximation to the Landau distribution which describes the energy loss by ionization of a heavy charged particle [7, 8].The Chi-square Distribution (CSD) can be approximated by the Normal Distribution (ND) and is given by the following.

$$f(x;n) = \frac{\frac{x^{\frac{n}{2}}}{2}}{2\sigma} \frac{\exp^{-\frac{x}{2}}}{2\Gamma(\frac{n}{2})} \qquad (1.5a)$$

In CSD the variable x $\geq$ 0 and positive integer represents the number of degrees of freedom [8].

The Hyper Secant Distribution (HSD) is defined by its mean and standard deviation similar to ND. The HSD is used for those data which follows symmetric distribution as in a ND. The mathematical expression for HSD with their parameters is given by

$$f(x) = \frac{\mathrm{sech}(y)}{2\sigma} \ \mathit{where} \quad y = \frac{\pi}{2\sigma}(x - \mu) \qquad (1.5b)$$

where mean and mode μ and δ^2 is the variance with $\delta > 0$. The skewness and kurtosis of HSD are 0 and 5 respectively [9]. If the natural log of variable x is Gamma distributed then it is said to be a

Log-Gamma Distribution (LGD), when Gamma variable = 0 then standard Log-Gamma Distribution has minimum value equal to 1. The LGD is given by

$$f(x) = \frac{(\ln[x-\gamma+1]^{\alpha-1}(x-\gamma+1)^{-(\frac{1+\beta}{\beta})}}{\beta^{\alpha}\Gamma(\alpha)} \qquad (1.6)$$

where x ≥ γ and α > 0, β>0 and the mean of LGD is $(1-\beta)^{-\alpha} + \gamma - 1$ [7,8]. Γ and β are the parameters of the distribution.

4. Results and Discussions

There are 23 sunspots cycles (SC) (from 1747 to 2011) as depicted in table: 1.1. This means that in the two centuries, from 1801 to 1900 and from 1901 to 2000 there are approximately nine SC in each century. So the total number of SCs in both he centuries is 18. The remaining 5 cycles occurred before the year 1800 and SC 24 is in continuation. Table: 1.1 shows the mean and standard deviation and duration of SC with parameters of tested probability distributions. The mean, standard deviation and distributions of each of the 23 SC and corresponding K-index data are depicted in Table: 1.1.

4.1 Probability distributions of Sunspots Cycles

From table 1.1 we observe that different cycles follow different distributions. The distributions vary from normal to gamma and gamma to lognormal. This behavior indicates that the distributions become increasingly heavy tailed. Figure: 1.1 represent the plot of total sunspots data and Figure 1.1 represents plot of each cycle from 1747 to 2011. The data as mentioned earlier was obtained from World Data center (WDC). Cycles 1, 5 and 7 follow the normal distribution (ND) while SC 6 follows the Hyper Secant Distribution (HSD) and the remaining nineteen SCs follow Log-Gamma Distribution (LGD). The probability distribution of total SC from 1747 to 2011 shows ND. Details are given in Table: 1.1. It depicts the distributions of all 23 solar cycles. It indicates that approximately 13 % of the solar cycles follow ND, approximately 4% follow HSD, 83% follow LGD. HSD is symmetric like ND but it has more peaked ness than the ND. The variation in mean and standard deviation represents the change in the strength of SCs.

4.2 Probability Distributions of *K*-indices data

According to the available data (1932- 2011) of *K*-indices, each set of distributed data (associated with each SC) have different distributions. Table: 1.2 depicts the alternation with mean, standard deviation and parameters of *K*-indices data sets of almost same duration. Data sets 1 and 3 follows ND while data sets 2 and 4 follows the CSD (see figure: 1.2) which shows the alternation between ND and CSD.

4.3 A comparison of Probability Distributions of the Sunspot and *K*-indices data

Finally, we will consider the data set 1932 to 2011 divided to 4 sets and calculated the probability distributions of sunspot cycles and *K*-indices data. Figure 1.3 give a comparison of the probability distributions of the total K-index data and the SC data from 1932 to 2011 *K*-index follows the ND (figure:1.3 and 1.3.1).

The probability distribution of 22 years K-index and SCs shows that 1st cycle of both the K-index and SCc follow the ND. The last 4th cycle of K-index and SCs follow the CSD which is in continuation. The complete cycle from 1932 to 2011 of both the K-index and SC represent that they follow the same probability distribution LGD. Table 1.3 depicts the probability distributions of four 22 years K-index and SCs along with the probability distribution of total K-index and SC. The 2nd 22 years SC follows the LGD while K-index follows the CSD. The 3rd SC follows the LGD and K-index follows the ND. The variation in SCs shows same distribution while K-index cycles represents increase from CSD to ND.

5. CONCLUSION AND OUTLOOK

In section 1 we applied the probability distributions on 23 sunspots cycles and observed that different cycles have different probability distributions. Overall, 19 solar cycles follow LGD. As we know the minimum duration of SC so far is 9 years while at the maximum a solar cycle lasted for 14 years, the average being 11 years table:1.1 depicted. Variations of mean and standard deviation helped to study the fluctuating behaviors (up and down change) of the sunspots cycles starting from 1747 and ending in 2011. SC 2 is yet incomplete.

In the second section we have distributed the *K*-index data against 2 SCs. *K*-index data sets 1 and 3 which follows ND. whereas *K*-index data sets 2 and 4 follows the CSD (see figure:1.2).

In the third part of this paper we have computed the probability distributions of 22 SCs data from 1932 to 2011 and compare the results with the *K*-index data. The 22years SCs are respectively ND, LGD, LGD and CSD. The results shows that in the 1st SC the ascending and descending phase of this solar activity probably same. The 2nd and 3rd SCs represents that the ascending phase of sunspots activity is quicker than the descending phase. If descending phase of SC slower as compare to the

ND then it may be prolong. The 4th SC follows the CSD which shows much slower descending phase of SC as compare to the ND (see figure 1.3). The comparison of probability distribution of SCs and *K*-index data represents that 1st and the 4th cycles have the same probability distribution. The variation in 2nd and the 3rd cycles shows that there is a maintain in the probability distribution of 2nd and 3rd SCs but rise in the *K*-index data Table 1.3 depicts the results. For the more accurate forecasts of SCs and *K*-index data sets, time series analysis and Hurst exponent calculations will be attempted in the next communication.

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Table:1.1 Probability distribution of sunspots cycles (1-24)

S/No	Cycle	Duration	Years	Mean (μ)	St.dev (δ)	Distribution	Statistic value (KST)	Parameters
1		1747.01-1753.12	---	58.09	26.86	Log-Gamma	0.20564	$\alpha=9.7297E^{+7}$ $\beta=7.6752E^{-8}$
2	1	1754.01-1766.06	12	38.89	25.67	Normal	0.09932	$\sigma=2.6387$ $\mu=1760.5$
3	2	1766.07-1775.02	9	61.37	38.21	Log-Gamma	0.12342	$\alpha=4.8566E^{+7}$ $\beta=1.5399E^{-7}$
4	3	1775.03-1784.05	9	66.75	52.44	Log-Gamma	0.15137	$\alpha=5.5769E^{+7}$ $\beta=1.3419E^{-7}$
5	4	1784.06-1798.05	14	60.28	45.34	Log-Gamma	0.13322	$\alpha=2.3098E^{+7}$ $\beta=3.2426E^{-7}$
6	5	1798.06-1807.12	9	28.55	17.54	Normal	0.10834	$\sigma=1.9989$ $\mu=1803.3$
7	6	1808.01-1822.01	14	17.16	17.66	Hypersecant	0.11953	$\sigma=2.6783$ $\mu=1816.3$
8	7	1822.02-1833.06	11	34.93	27.65	Normal	0.10979	$\sigma=2.1522$ $\mu=1828.7$
9	8	1833.07-1843.02	10	67.20	48.11	Log-Gamma	0.14297	$\alpha=5.0080E^{+7}$ $\beta=1.5009E^{-7}$
10	9	1843.03-1855.09	12	55.27	39.45	Log-Gamma	0.1143	$\alpha=2.9716E^{+7}$ $\beta=2.5314E^{-7}$
11	10	1855.1-1867.01	12	48.26	31.90	Log-Gamma	0.13263	$\alpha=3.5636E^{+7}$ $\beta=2.1127E^{-7}$
12	11	1867.02-1878.08	11	53.93	46.80	Log-Gamma	0.12863	$\alpha=4.5529E^{+7}$ $\beta=1.6549E^{-7}$
13	12	1878.09-1889.11	11	34.05	26.84	Log-Gamma	0.10054	$\alpha=4.4232E^{+7}$ $\beta=1.7048E^{-7}$
14	13	1889.12-1901.04	12	40.31	30.15	Log-Gamma	0.12973	$\alpha=3.9729E^{+7}$ $\beta=1.8995E^{-7}$
15	14	1901.05-1912.02	11	34.12	26.05	Log-Gamma	0.10832	$\alpha=4.7389E^{+7}$ $\beta=1.5938E^{-7}$
16	15	1912.03-1924.01	12	37.84	34.92	Log-Gamma	0.12354	$\alpha=5.0977E^{+7}$ $\beta=1.4828E^{-7}$
17	16	1924.02-1933.08	9	42.74	28.05	Log-Gamma	0.09771	$\alpha=5.3085E^{+7}$ $\beta=1.4249E^{-7}$
18	17	1933.09-1944.04	11	56.38	40.87	Log-Gamma	0.12025	$\alpha=4.9872E^{+7}$ $\beta=1.5178E^{-7}$
19	18	1944.05-1954.01	10	77.47	53.79	Log-Gamma	0.12085	$\alpha=5.8543E^{+7}$ $\beta=1.2939E^{-7}$
20	19	1954.02-1964.07	10	91.00	71.21	Log-Gamma	0.13054	$\alpha=5.6869E^{+7}$ $\beta=1.3329E^{-7}$
21	20	1964.08-1976.07	12	58.78	37.80	Log-Gamma	0.10587	$\alpha=3.6282E^{+7}$ $\beta=2.0907E^{-7}$
22	21	1976.08-1986.06	10	83.63	57.15	Log-Gamma	0.11993	$\alpha=5.8063E^{+7}$ $\beta=1.3074E^{-7}$
23	22	1986.07-1996.1	10	75.96	58.72	Log-Gamma	0.12037	$\alpha=6.0314E^{+7}$ $\beta=1.2594E^{-7}$
24	23	1996.11-2009.08	13	52.72	43.89	Log-Gamma	0.11131	$\alpha=4.4413E^{+7}$ $\beta=1.7115E^{-7}$
25	24	2009.09-…..	---	27.68	22.41	Chi-Squared (2P)	0.36538	$\nu=5$ $\gamma=1996.0$
26	1-24	1749-2011	251	51.90	44.264	Normal	0.11513	s=78.477 m=1889.1

Table: 1.2Probability distribution of K-index data sets (1-4)

S/No	Cycle	Duration	Mean	St.dev	Distribution	Statistic (KST)	Parameters
1	1	1932.01-1953.12	14.583	6.3928	Normal	0.09222	σ=616.15 μ=1.9438E+5
2	2	1954.01-1976.06	14.344	6.1936	Chi-Squared (2P)	0.12712	ν=2.2439E+5 γ=-27948.0
3	3	1976.07-1996.09	15.519	6.2465	Normal	0.09459	σ=563.7 μ=1.9863E+5
4	4	1996.10-20…..	11.006	5.7854	Chi-Squared (2P)	0.08148	ν=63750 γ=1.3652E+5
	1- 4	1932-2011	14.103	6.3685	Log-Gamma	0.07545	α=1.2358E+6 β=9.8650E-6

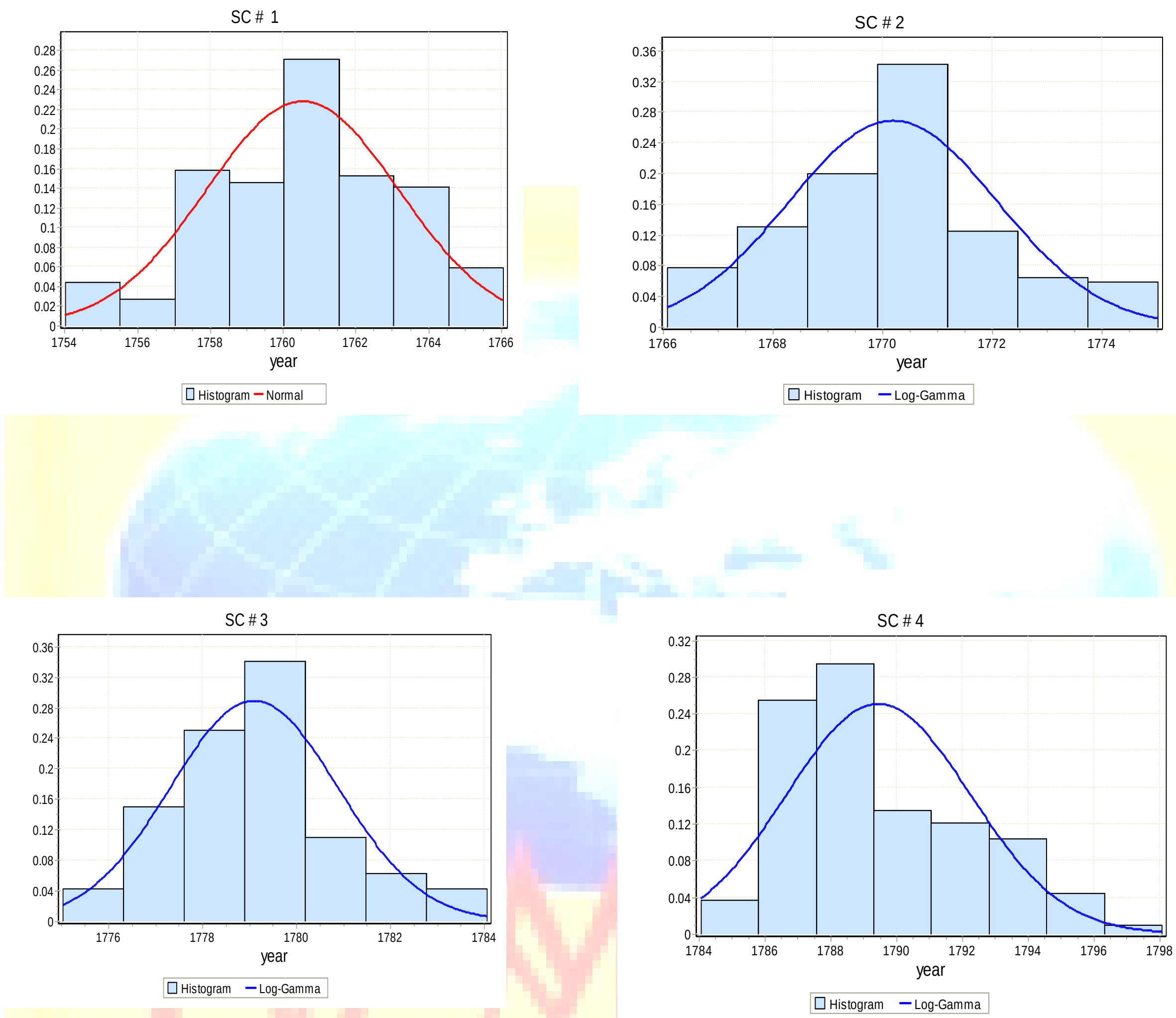
Table: 1.3Probability distribution of sunspots 22 years cycles (1-4)

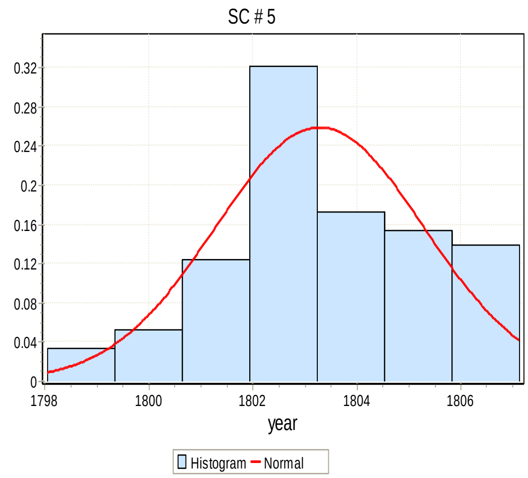
S/No	Cycle	Duration	Mean	St.dev	Distribution	Statistic (KST)	Parameters
1	1	1932.01-1953.12	62.4	49.003	Normal	0.17722	σ=5.4889 μ=1943.8
2	2	1954.01-1976.06	73.811	58.108	Log-Gamma	0.19182	α=6.1782E+6 β=1.2273E-6
3	3	1976.07-1996.09	79.72	57.957	Log-Gamma	0.15923	α=8.1445E+6 β=9.3236E-7
4	4	1996.10-20…..	48.834	42.399	Chi-Squared (2P)	0.10115	ν=6 γ=1995.9
	1- 4	1932-2011	67.446	54.026	Log-Gamma	0.09385	α=5.2173E+5 β=1.4540E-5

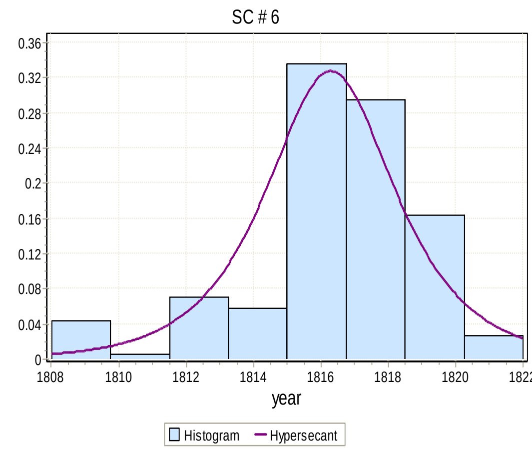
Table: 1.3 A comparison of sunspots and K-index cycles

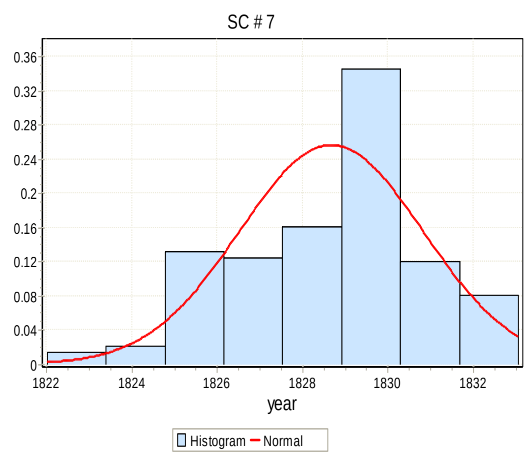
S.No	Cycle	Duration	Mean SC	Mean K index	St.dev SC	St.dev K index	Distribution SC	Distribution K-index	Statistic (KST) SC	Statistic (KST) K index	Parameters SC	Parameters K-index	Parameters SC
1	1	1932.01-1953.12	62.4	14.583	49.003	6.3928	Normal	Normal	0.17722	0.09222	σ=5.4889 μ=1943.8	σ=616.15 μ=1.9438E+5	σ=5.4889 μ=1943.8
2	2	1954.01-1976.06	73.811	14.344	58.108	6.1936	Log-Gamma	Chi-Squared (2P)	0.19182	0.12712	α=6.1782E+6 β=1.2273E-6	ν=2.2439E+5 γ=-27948.0	α=6.1782E+6 β=1.2273E-6
3	3	1976.07-1996.09	79.72	15.519	57.957	6.2465	Log-Gamma	Normal	0.15923	0.09459	α=8.1445E+6 β=9.3236E-7	σ=563.7 μ=1.9863E+5	α=8.1445E+6 β=9.3236E-7
4	4	1996.10-20…..	48.834	11.006	42.399	5.7854	Chi-Squared (2P)	Chi-Squared (2P)	0.10115	0.08148	ν=6 γ=1995.9	ν=63750 γ=1.3652E+5	ν=6 γ=1995.9
	1- 4	1932-2011	67.446	14.103	54.026	6.3685	Log-Gamma	Log-Gamma	0.09385	0.07545	α=5.2173E+5 β=1.4540E-5	α=1.2358E+6 β=9.8650E-6	α=5.2173E+5 β=1.4540E-5

Figure: 1.1(This figures shows the 24 sunspots cycles with different peakness and duration of total duration 1747-2011)




SC # 5
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1798
1800
1802
1804
1806
year
Histogram
Normal


SC # 6
0.36
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1808
1810
1812
1814
1816
1818
1820
1822
year
Histogram
Hypersecant


SC # 7
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0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1822
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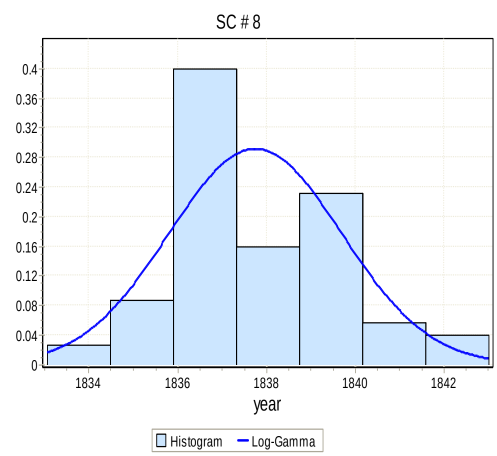
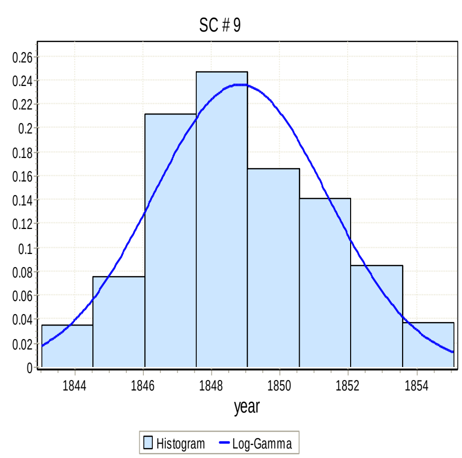
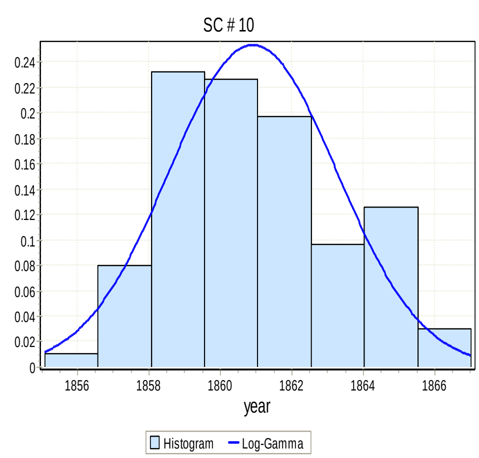
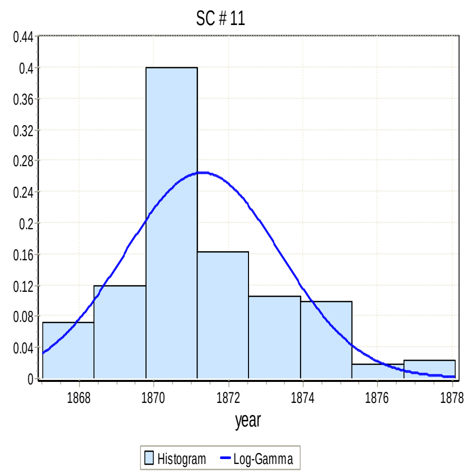
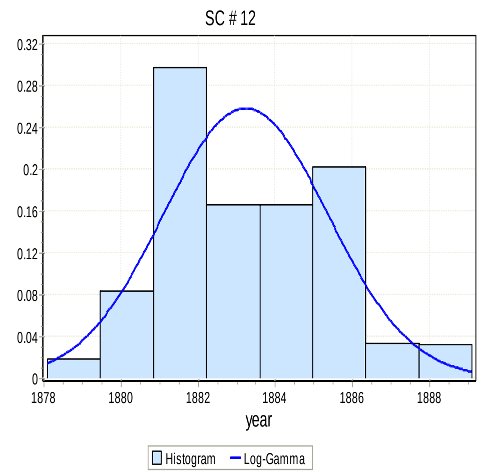
1830
1832
year
Histogram
Normal

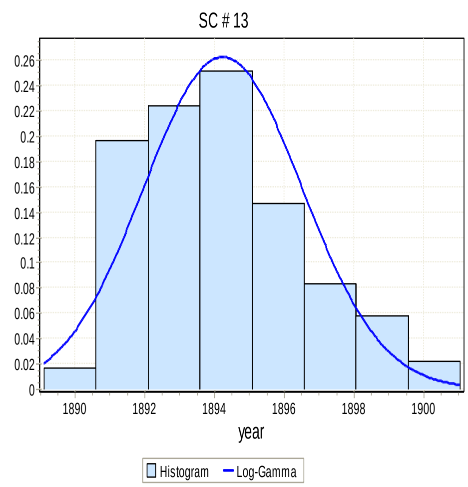

SC # 8
0.4
0.36
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1834
1836
1838
1840
1842
year
Histogram
Log-Gamma

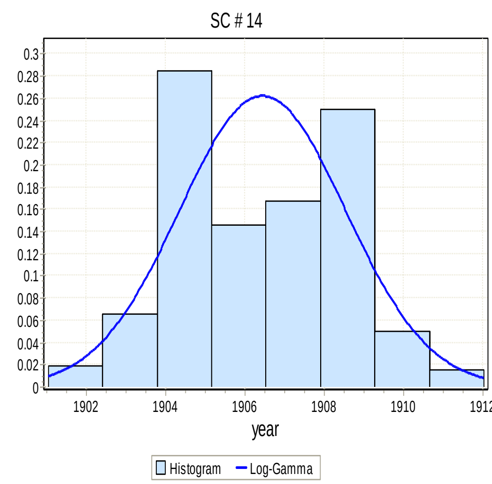

SC # 9
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1844
1846
1848
1850
1852
1854
year
Histogram
Log-Gamma

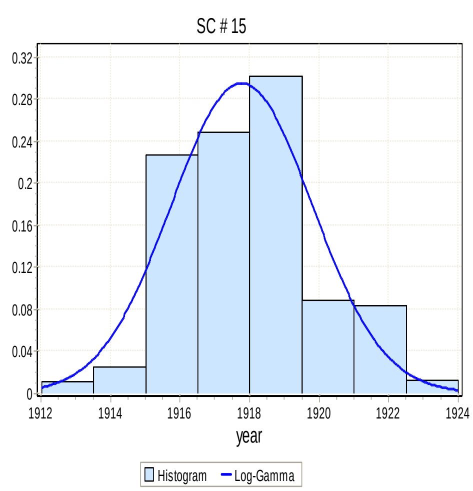

SC # 10
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1856
1858
1860
1862
1864
1866
year
Histogram
Log-Gamma

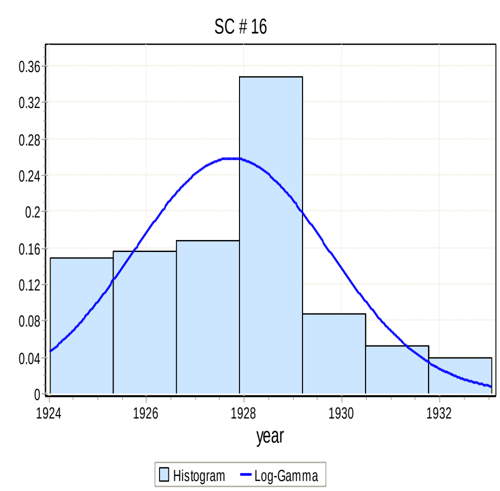

SC # 11
year
Histogram
Log-Gamma

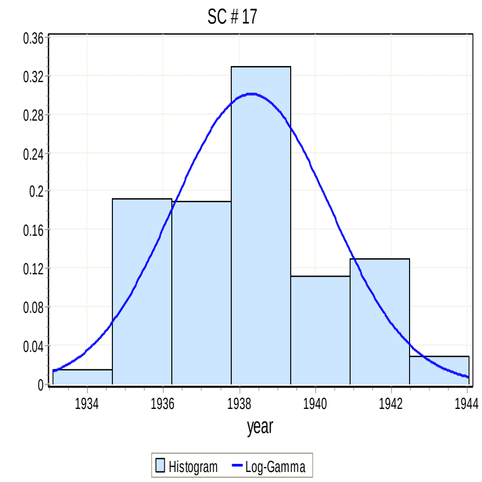

SC # 12
year
Histogram
Log-Gamma

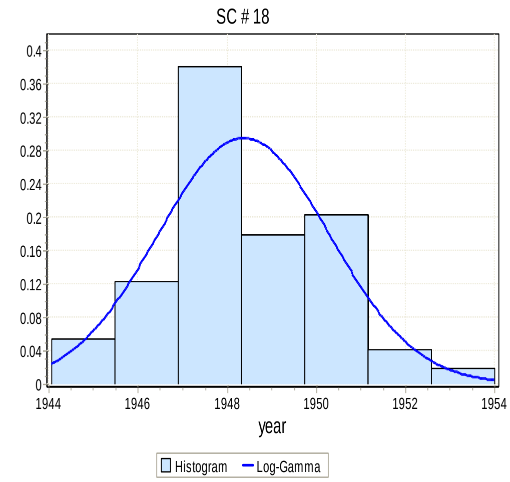

SC # 13
year
Histogram
Log-Gamma

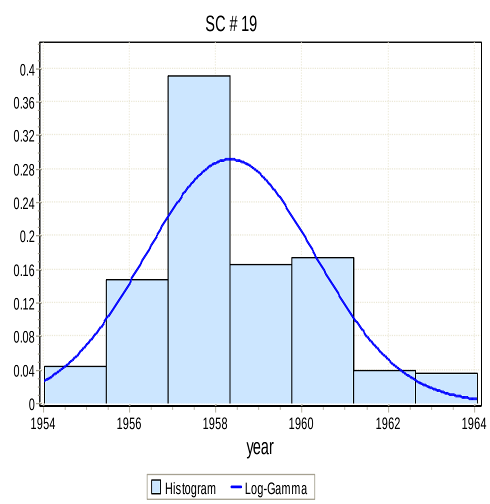

SC # 14
year
Histogram
Log-Gamma

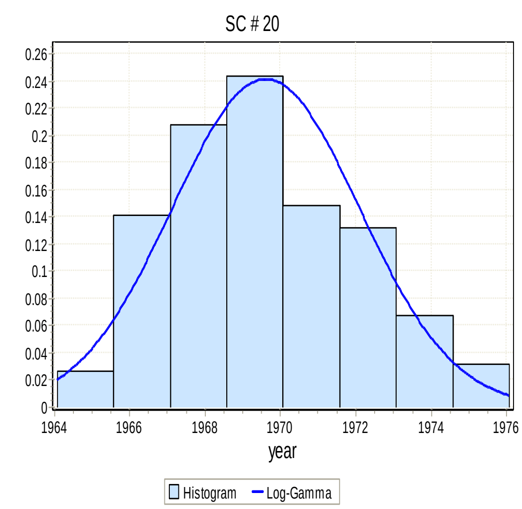

SC # 15
year
Histogram
Log-Gamma

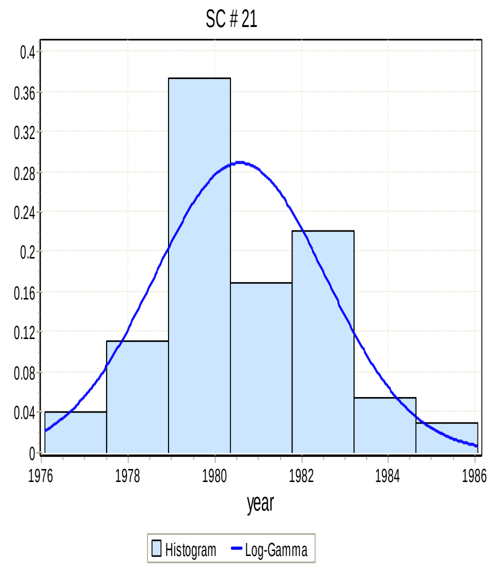

SC # 16
year
Histogram
Log-Gamma

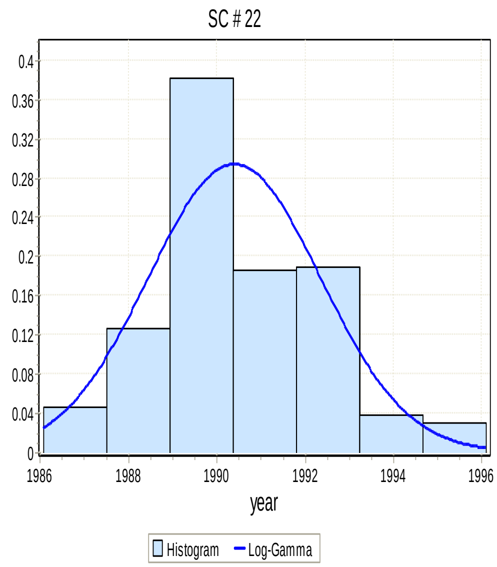

SC # 17
0.36
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1934
1936
1938
1940
1942
1944
year
Histogram
Log-Gamma

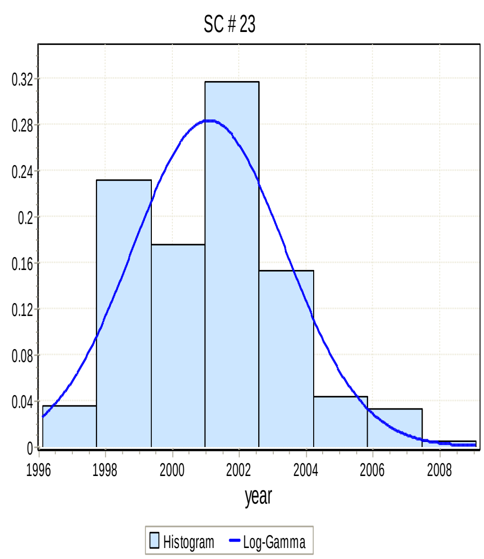

SC # 18
0.4
0.36
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1944
1946
1948
1950
1952
1954
year
Histogram
Log-Gamma


SC # 19
0.4
0.36
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1954
1956
1958
1960
1962
1964
year
Histogram
Log-Gamma


SC # 20
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1964
1966
1968
1970
1972
1974
1976
year
Histogram
Log-Gamma


SC # 21
0.4
0.36
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1976
1978
1980
1982
1984
1986
year
Histogram
Log-Gamma


SC # 22
0.4
0.36
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1986
1988
1990
1992
1994
1996
year
Histogram
Log-Gamma


SC # 23
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
1996
1998
2000
2002
2004
2006
2008
year
Histogram
Log-Gamma

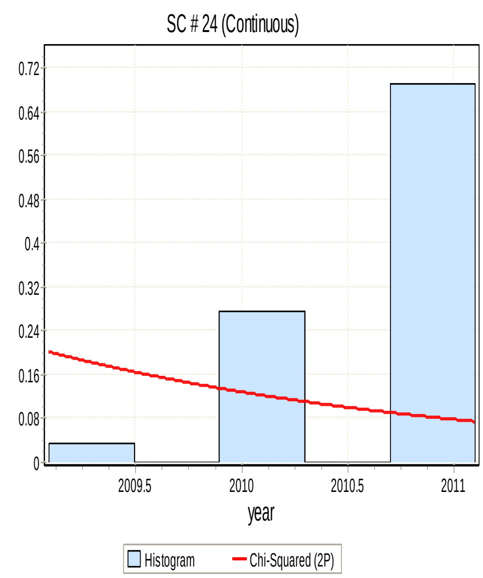

SC # 24 (Continuous)
0.72
0.64
0.56
0.48
0.4
0.32
0.24
0.16
0.08
0
2009.5
2010
2010.5
2011
year
Histogram
Chi-Squared (2P)

Figure: 1.2 (This figure contains four k-index data 1-4, k-index data sets 1 and 3 follows ND and k-index data sets 2 and 4 follows CSD)

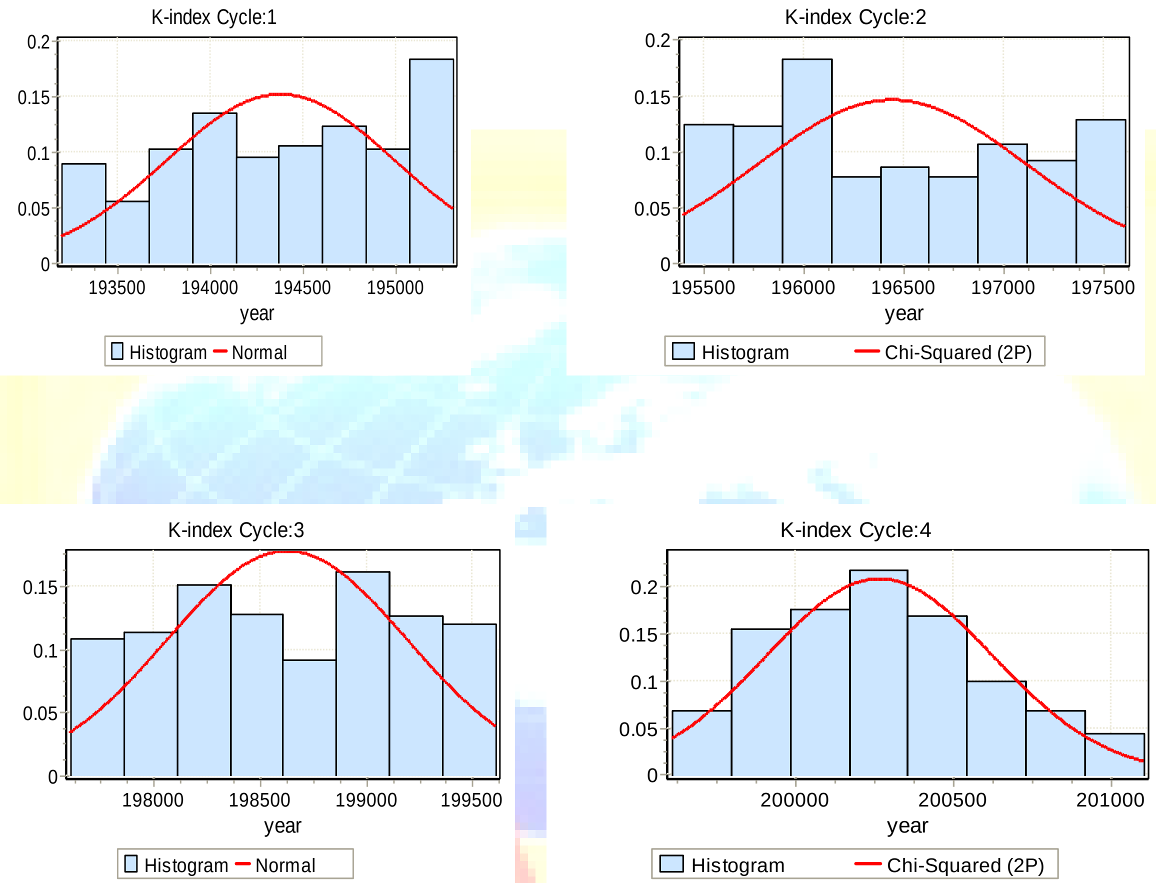


Figure: 1.3 (This figure contains four data sets of 22-years SCs)

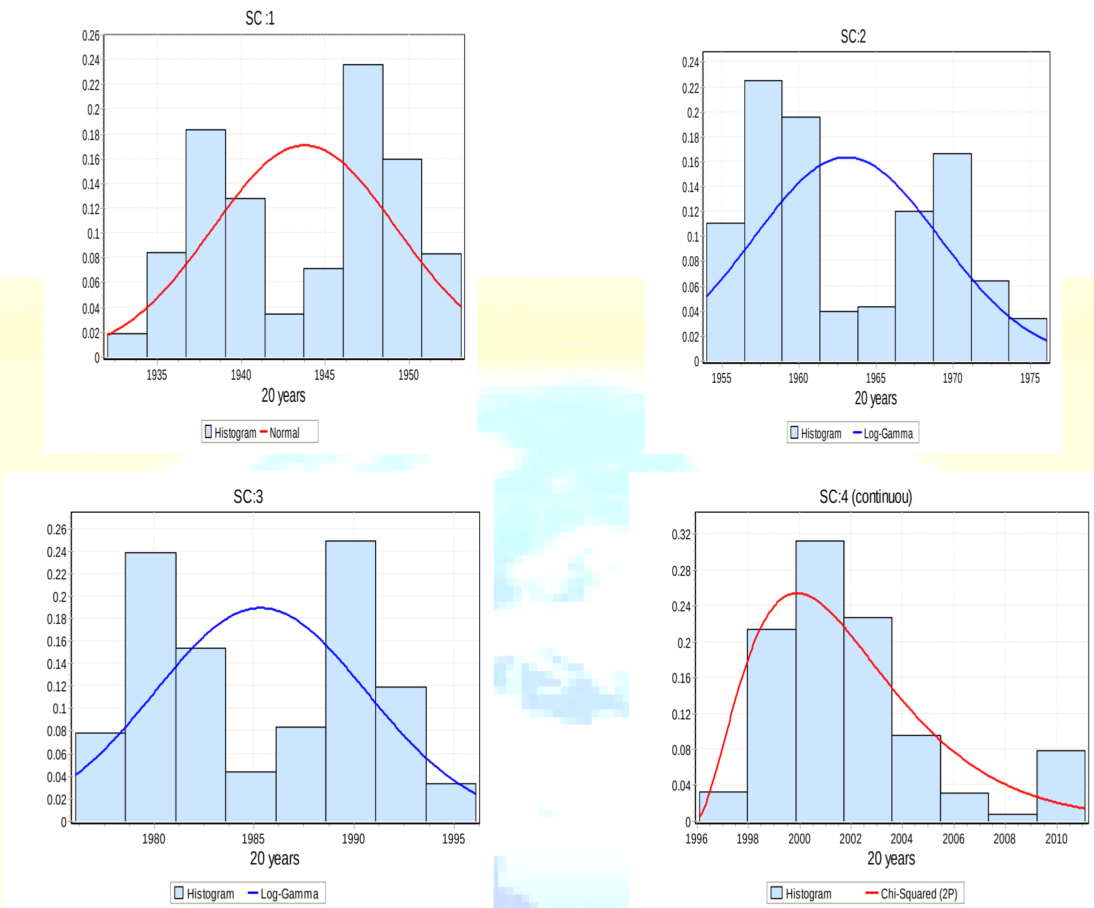


Figure: 1.3.1 (Sunspots cycles (1932-2011) and (1749-2011))