INFLUENCE OF OVERLAPPING STENOSIS ON BLOOD FLOW THROUGH AN ARTERY IN THE PRESENCE OF MAGNETIC FIELD

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ABSTRACT

The aim of the present investigation is to develop a mathematical model to study the blood flow in an arterial having overlapping stenosis under the externally applied transverse magnetic field. Blood is modeled as Herschel-Bulkley fluid to represent the non-Newtonian character of blood in small blood vessels. Flow characteristics like for wall shear stress, volumetric flow rate, axial velocity, and core velocity have been expressed analytically and shown graphically. These expressions reveal considerable alterations in flow characteristics due to stenosis shape and magnetic field. The magnetic field perpendicular to the flow of blood is incorporated which significantly controls the flow patterns. The study provides an insight into the effects of magnetic field on flow rate of blood, wall shear stress, axial and core velocities of blood.

Keywords - *Overlapping stenosis, magnetic field, wall shear stress, flow rate, axial velocity.*

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INTRODUCTION

One of the major causes of deaths in the world is cardiovascular diseases, mainly, atherosclerosis (medically called stenosis). These diseases are directly associated with the nature and behaviour of blood flow and vessels. Now a days magnetic therapy is broadly employed for treatment of these kind of diseases. The blood is considered as magnetohydrodynamics (MHD) fluid which will help in controlling blood pressure and has probable curative use in the diseases of heart and blood vessel. In case of necrosis, when blood flow to a tissue is reduced or obstructed, local exposure of a magnetic field could potentially result in maintaining blood flow and relaxation of blood vessel. Magnetic therapy may also be helpful for the reperfusion of ischemic tissue or during sepsis. Applying appropriate magnetic field can be effective to the conditions like headaches, travel sickness, poor circulation, muscles sprains, strains and joints pain.

As per the existing literature, Kolin [1] gave the idea of applying electromagnetic field in Bio-mathematical research first time. Haldar et al [2] studied the effect of an externally applied homogeneous magnetic field on the flow characteristics of blood through a single constricted blood vessel in the presence of erythrocytes. Magnetic effect on pulsatile flow in a constricted axisymmetric tube is discussed by Amos el al [3]. A mathematical model of bio-magnetic fluid dynamics (BFD), suitable for the description of the Newtonian blood flow under the action of an applied magnetic field, is proposed by Tzirtzilakis [4]. Mishra et al [5] discussed stenosis of bell shaped geometry to investigate the various flow characteristics of blood through an arterial segment in a pathological state. In the environment of a uniform transverse magnetic field, treating blood as electrically conducting fluid, Mustapha et al [6] analyzed the flow of blood through irregular shaped multiple stenosed arteries. The model is consistent with the principles of ferrohydrodynamics and magnetohydrodynamics and takes into account both magnetization and electrical conductivity of blood. A mathematical model for pulsatile flow of blood through a stenosed porous medium with periodic body acceleration under the influence of a uniform transverse magnetic field has been developed by Das et al [7], considering the blood to be a Newtonian and incompressible fluid. Since blood is a suspension of red cells which contains hemoglobin, having iron oxide in composition, it is quite obvious that blood can be assumed as electrically conducting fluid which exhibits the characteristics of magnetohydrodynamics (MHD) flow. If a magnetic field is applied to a moving and electrically conducting fluid, electric as well as magnetic fields will be induced. When these fields interact with each other, a body force known as Lorentz force is produced, which slows down the motion of fluid. Such analysis may be useful for pumping of blood and magnetic resonance imaging (MRI). During surgical procedures, flow of blood can be controlled using magnetic field. Rathod et al [8] studied the pulsatile flow of blood through a porous medium under the influence of periodic body acceleration in presence of magnetic field by considering blood as a couple stress, incompressible, electrical conducting fluid. Utilizing the Herschel-Bulkley fluid model, Jain et al [9] examined the effect of mild stenosis on blood flow, in an irregular axisymmetric artery with oscillating pressure gradient. A mathematical model for magnetohydrodynamics (MHD) blood flow in a stenosed artery under porous medium is developed by Jain et al [10], considering the cosine shaped geometry of the stenosis. Varshney et al [11] proposed a mathematical model for the non-Newtonian flow of blood in overlapping stenosed artery in the presence of transverse magnetic field. Das et al [12] studied the outcomes of a uniform transverse magnetic field on pulsatile flow of blood containing particles through a rough thin walled elastic tube. Shit et al [13] explored the effect of externally imposed body acceleration and magnetic field on peristaltic flow of blood through a stenosed arterial segment. Singh et al [14] formulated a mathematical model to study the effects of shape parameter and stenosis length on the resistance to flow and wall shear stress under stenotic conditions by considering, laminar, steady, one dimensional, non-Newtonian and fully developed flow of blood through axially symmetric but radially non-symmetric stenosed artery. Assuming blood as non-Newtonian fluid (Casson fluid) and artery as circular tube, Bali et al [15] investigated the response of external applied magnetic field on the flow of blood through a multiple stenosed artery. Singh et al [16] examined the effect of magnetic field on flow characteristics through an axially non-symmetric but radially symmetric stenosed arterial segment.

In all of the above studies, stenosis is assumed to be symmetric or single throat. Assuming the pressure variation only along the axis of the tube, investigators [17-18] developed a mathematical model to explore the effects of an overlapping stenosis on blood flow characteristics in an arterial segment without considering magnetic field. The main objective of the present work is to study the effect of transverse magnetic field on the blood flow in an artery having a double throat overlapping stenosis, characterizing blood as Herschel-Bulkley fluid by properly accounting for yield stress of blood in small blood vessels.

II. THE PROBLEM AND ITS SOLUTION

Let us consider an arterial segment having overlapping stenosis with two stenosis throats as shown in Fig 1. The mathematical expression for the radius of the artery may be written as

$$\frac{\bar{R}(\bar{z})}{R_0} = 1 - \frac{32\delta}{R_0 \bar{l}_0^4}\left[\frac{11}{32}(\bar{z}-\bar{d})\bar{l}_0^3 - \frac{47}{48}(\bar{z}-\bar{d})^2\bar{l}_0^2 + (\bar{z}-\bar{d})^3\bar{l}_0 - \frac{1}{3}(\bar{z}-\bar{d})^4\right]; \quad \bar{d} < z < \bar{d} + \bar{L}_0$$

$$= 1; \quad otherwise \tag{1}$$

where $\bar{L}_0 = 3\bar{l}_0/2$ is the length of the stenosis, $\bar{d}$ indicates location of the stenosis R_0 is the radius of the artery outside the stenotic region, $\bar{R}(\bar{z})$ is the radius of the stenosed portion of the arterial segment with $\bar{z}$ measured along the axis of the artery. δ is the maximum height of the stenosis into the lumen, appears at the two specific locations: $\bar{z} = \bar{d} + \bar{l}_0/2$ and $\bar{z} = \bar{d} + \bar{l}_0$. The height of the stenosis at $\bar{z} = \bar{d} + 3\bar{l}_0/4$, called critical height is $3\delta/4$. The length of the arterial segment is taken to be l.

The equation of motion for flow of blood is given by (Singh et al [16])

$$-\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\tau}) + \mu_0 M \frac{\partial \bar{H}}{\partial \bar{z}} = 0$$

(2)

where $\bar{H}$ is magnetic field intensity, $(\partial\bar{H}/\partial\bar{z})$ magnetic field gradient, $(-\partial\bar{p}/\partial\bar{z})$ is called pressure gradient as $\bar{p}$ stands for pressure at any point $(\bar{z},\bar{r})$ with $\bar{r}$ measured along radius of the artery and $\bar{z}$ is the axial coordinate, μ_0 denotes magnetic permeability, M magnetization, $\bar{\tau}$ shearing stress.

The constitutive equation in one dimensional form for Herschel-Bulkley fluid is expressed as

$$-\frac{\partial \bar{u}}{\partial \bar{r}} = \frac{(\bar{\tau}-\bar{\tau}_0)^n}{k}; \quad \bar{\tau} \geq \bar{\tau}_0$$

$$= 0; \quad \bar{\tau} < \bar{\tau}_0$$

(3)

where $\bar{u}$ stands for axial velocity of blood, $\bar{\tau}_0$ the yield stress, n the flow behaviour index of blood and k the viscosity coefficient of blood.

Equations (2) and (3) are to be solved subject to the boundary conditions

$\bar{u} = 0$ at $\bar{r} = \bar{R}(\bar{z})$ (velocity slip condition)

(4)

$\bar{u} = \bar{u}_c$ at $\bar{r} = \bar{R}_c$

(5)

where $\bar{R}_c$ is the radius of the core region and $\bar{u}_c$ is the velocity in the core region.

$\bar{\tau}$ is finite at $\bar{r} = 0$ (regularity condition)

(6)

The following non-dimensional variables are now introduced as:

$$d = \frac{\bar{d}}{l},\ l_0 = \frac{\bar{l}_0}{l},\ z = \frac{\bar{z}}{l},\ r = \frac{\bar{r}}{R_0},\ R = \frac{\bar{R}}{R_0},\ R_c = \frac{\bar{R}_c}{R_0}, u = \frac{\bar{u}}{u_0},\ u_c = \frac{\bar{u}_c}{u_0},\ p = \frac{\bar{p}}{\rho u_0^2},\ \tau = \frac{\bar{\tau}}{\rho u_0^2},\ H = \frac{\bar{H}}{H_0}$$

where H_0 is the external transverse uniform constant magnetic field.

Equations (1) – (6) reduce to the following non-dimensional form:

A. The geometry of the stenosis:

$$R(z) = 1 - \frac{32\delta}{R_0 l_0^4}\left[\frac{11}{32}(z-d)l_0^3 - \frac{47}{48}(z-d)^2 l_0^2 + (z-d)^3 l_0 - \frac{1}{3}(z-d)^4\right];\quad d < z < d + L_0$$

$$= 1;\quad otherwise$$

(7)

B. Equation of motion:

$$-\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) + A_1\frac{\partial H}{\partial z} = 0$$

(8)

C. Constitutive equation of Herschel-Bulkley fluid:

$$-\frac{\partial u}{\partial r} = \frac{(\tau - \tau_0)^n}{A_2};\quad \tau \geq \tau_0$$

$$= 0;\quad \tau < \tau_0$$

(9)

D. Boundary conditions:

$u = 0$ at $r = R(z)$

(10)

$u = u_c$ at $r = R_c$

(11)

τ is finite at $r = 0$

(12)

where $A_1 = \dfrac{\mu_0 M H_0}{\rho u_0^2}$, $A_2 = \dfrac{\mu}{\rho^n u_0^{2n-1} R_0}$

Integrating (8) and (9) and using boundary condition (10) – (12), the expressions for axial velocity u and core velocity u_c are obtained as

$$u = \frac{1}{2^n (n+1) A_2}\left(\frac{\partial p}{\partial z} - A_1 \frac{\partial H}{\partial z}\right)^n \left[(R-R_c)^{n+1} - (r-R_c)^{n+1}\right]$$

(13)

$$u_c = \frac{1}{2^n (n+1) A_2}\left(\frac{\partial p}{\partial z} - A_1 \frac{\partial H}{\partial z}\right)^n (R-R_c)^{n+1}$$

(14)

The volumetric flow rate, Q is formulated as

$$Q = \int_0^{R_c} 2\pi r u_c dr + \int_{R_c}^{R} 2\pi r u dr$$

(15)

Integrating (15) and using (13) and (14), the expression for the volumetric flow rate Q may be given as

$$Q = \alpha C^n R^{n+3}\left[1 + \frac{2}{n+2}\beta + \frac{2}{(n+2)(n+3)}\beta^2\right](1-\beta)^{n+1}$$

(16)

where $C = \left(\dfrac{\partial p}{\partial z} - A_1 \dfrac{\partial H}{\partial z}\right)$, $\alpha = \dfrac{\pi}{2^n (n+1) A_2}$, $\beta = \dfrac{R_c}{R} = \dfrac{\tau_0}{\tau_R}$

When $\tau_0/\tau_R \ll 1$ (16) reduces to

$$Q = \alpha C^n R^{n+3}\left(1 - \frac{n+2}{n+3}\beta\right)^n$$

(17)

Formulating the wall shear stress as

$$\tau_R = -k\left(\frac{\partial u}{\partial r}\right)_{r=R}$$

(18)

On differentiating (13) and using it in (18), the wall shear stress τ_R may be expressed as

$$\tau_R = \frac{k}{2^n A_2}\left(\frac{\partial p}{\partial z} - A_1 \frac{\partial H}{\partial z}\right)^n \left[(R - R_c)^n\right]$$

(19)

III. RESULTS AND DISCUSSIONS

All this section, numerical results have been made available to explore the effects of magnetic field, overlapping stenosis on the axial velocity, core velocity, volumetric flow rate and wall shear stress etc. The present problem with extended ideas considered herein is difficult to handle but computations and graphical representations with MATLAB 7.0 make it easier. Fig. 2 reveals the variation in axial velocity of blood along radial distance, velocity curve shifts towards the origin as radial distance increases. It can be seen through Fig. 2 that magnetic field controls the velocity of blood remarkably. The deviation in axial velocity with stenosis height and induced magnetic field intensity is shown in Fig. 3. Axial velocity of blood increases with decreasing stenosis height and magnetic field. Fig. 4 illustrates that like axial velocity, core velocity shows similar variations with radial distance and magnetic field.

Fig. 5 depicts that the volumetric flow rate remains constant in non-stenotic region, starts decreasing as blood enter into the stenotic region, it becomes least at the peak value of stenosis height and again starts escalating and reaches to same constant value when blood comes to the normal region. The flow rate shows same movements for both of the throats. It can be observed that magnetic field can be applied to manage flow rate. Changes in flow rate due to the size of stenosis are shown in the Fig. 6. One may see that the volumetric flow rate diminishes with growing stenosis height.

Fig. 7 demonstrates the augmentation in the wall shear stress due to increasing stenosis height but the wall shear stress can be taken under control using magnetic field. The shear stress is greater at the stenosis throats than at the other parts of stenosis.

IV. FIGURES

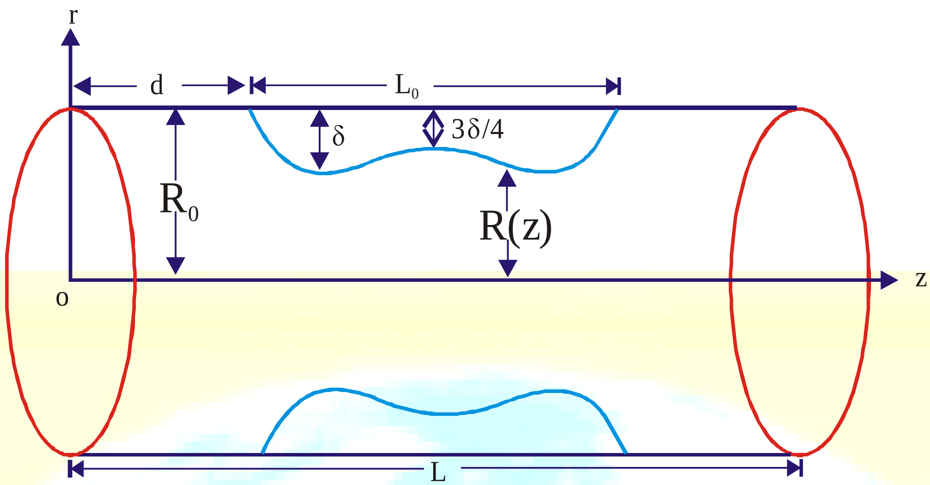

Figure 1. Geometry of overlapping stenosis in an arterial segment

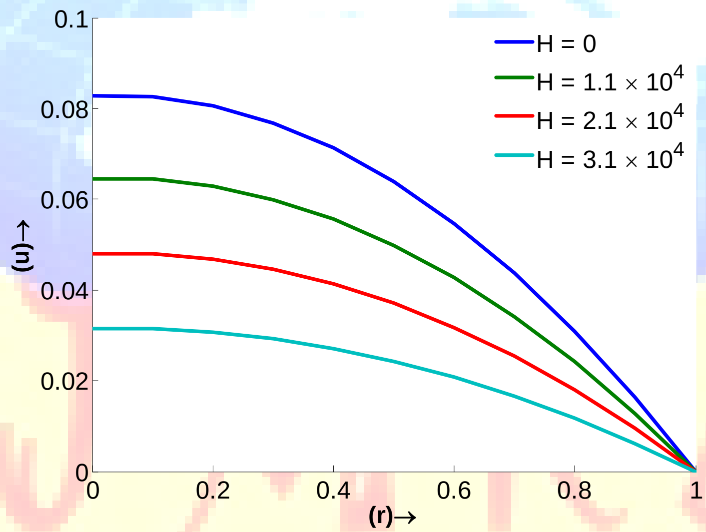

Figure 2. Variation of axial velocity along radial distance for different values of magnetic field intensity (*H*)

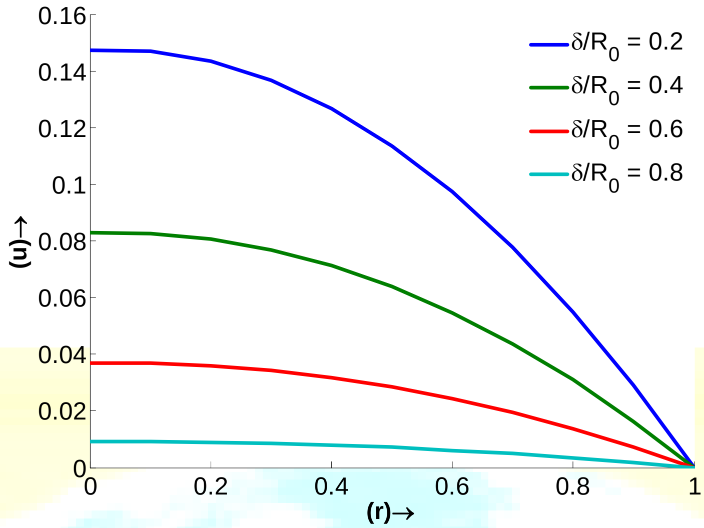

Figure 3. Variation of axial velocity of blood along radial distance for different values of stenosis height (δ/R_0)

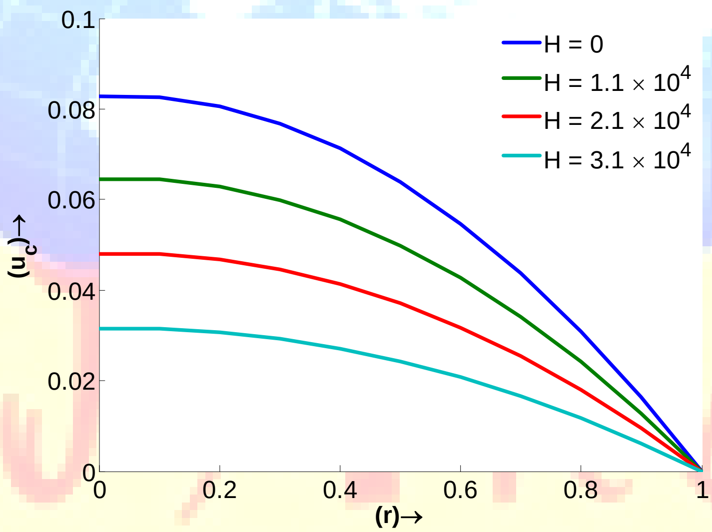

Figure 4. Variation of core velocity of blood along radial distance for different values of magnetic field intensity (H)

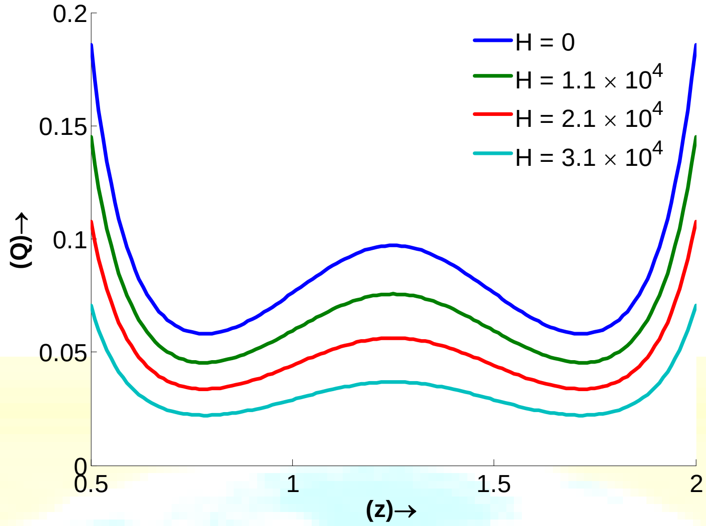

Figure 5. Variation of volumetric flow rate along axial distance for different values of magnetic field intensity (*H*)

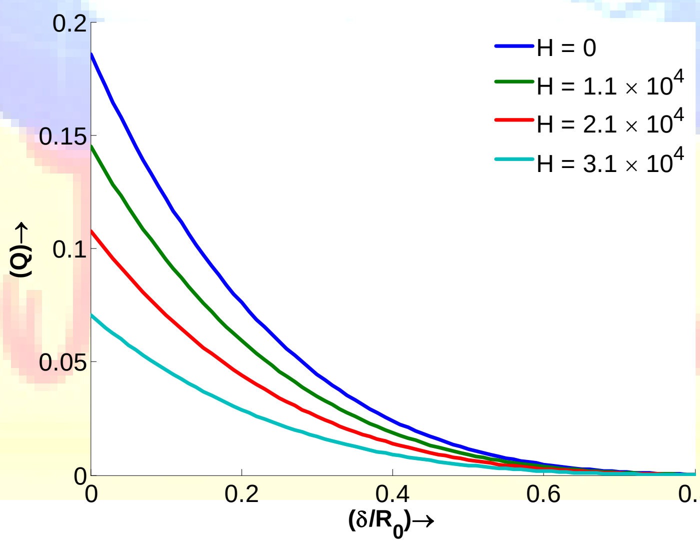

Figure 6. Variation of volumetric flow rate with stenosis height for different values of *H*

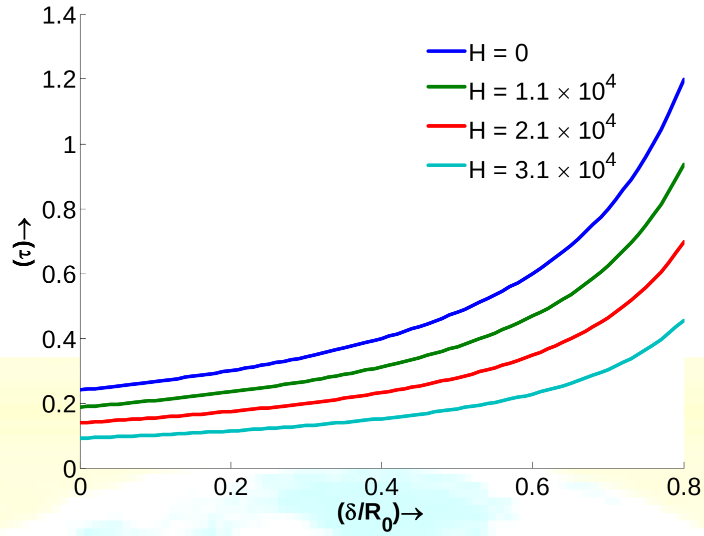

Figure 7. Variation of wall shear stress with stenosis height for different values of *H*

V. CONCLUSION

In the present analysis, we have developed a mathematical model to investigate the influence of slip velocity and magnetic field on velocity profile, volumetric flow rate and wall shear stress. Blood is characterized as Herschel-Bulkley fluid model. It is observed that magnetic field reduces the flow characteristics amazingly. Also the height of stenosis significantly affects the velocity, wall shear stress and flow rate. Knowing about slip effect, appropriate magnetic field can be applied to manage and control the flow behaviour of blood. These investigations may be useful for the medical practitioners to treat the hypertension patients through magnetic therapy and to understand the flow of blood under stenotic conditions.

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