CONTROL CHARTS FOR WAITING TIME USING METHOD OF WEIGHTED VARIANCE AND POWER TRANSFORMATION FOR (M/M/S) : (∞ : FCFS) MODEL

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Abstract

In this paper to monitor the waiting time of the (M/M/S) : (∞ : FCFS) queuing model , control chart for the random waiting time is constructed using method of weighted variance and Nelson's power transformation. The performance measure average run length for these charts is obtained and compared.

Keywords : False alarm rate, Type II error, Average queue length, Average run length, average queue length, average waiting time , Weibull distribution

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1 Introduction

Various types of control charts for the random queue length N and waiting time i.e. W_s for the (M/M/1):(∞/FCFS) queuing model are constructed by Khaparde M.V. and Dhabe S.D. In this paper control charts for random waiting time for
(M/M/s) : (∞ / FCFS) queuing model are constructed .

2 (M/M/S) : (∞/FCFS) queuing model

Notations

Let P_n denote steady state probability of having exactly n customers in the system

λ = mean arrival rate , μ = mean service rate per busy server

s = number of parallel servers , ρ = Traffic intensity λ/s_μ

W_s = waiting time per customer in the system

W_Q = waiting time per customer in the queue

f (W_s) = density function of waiting time of the customer in the system.

f(W_Q)= density function of waiting time of the customer in the queue

Multichannel queuing theory deals with the condition in which there are several service stations in parallel and each element in the waiting line can be served by more than one station. Each service facility is prepared to deliver the same type of service. The new arrival selects one station without any external pressure. When a waiting line is formed, a single line usually breaks down into shorter lines in front of each service station. The arrival rate λ and service rate μ are mean values from Poisson distribution and exponential distribution respectively. Service discipline is first come first serve and customers are taken from a single queue i.e. any empty channel is filled by the next customer in line.

When n < s, there is no queue because all arrivals are being serviced, and the rate of servicing will be nμ as only n channels are busy, each at the rate of μ. When
n = s, all channels will be working and when n > s, there will be (n – s) persons in the queue and rate of service will be sμ as all the s channels are busy.

3 Construction of control charts

For any queuing system, average queue length and average waiting time are the main observable characteristics. Customers want to have waiting time in the system as minimum as possible i.e.queue length should be small. Haim Shore (1999) has made pioneering attempt of extending the application of statistical process control to queuing systems. He obtained control limits for the random queue length N for (M/M/s) queuing model. These control limits are explicitly expressed in terms of mean, standard deviation and skewness of the distribution of r.v.N. This control chart monitors the stability of the queuing system in terms of N. If an out of control signal is generated it will indicate a change in the parameter arrival rate or service rate which determine N.

To monitor the waiting time of the customers in the queuing system ,the following control charts for random waiting time are constructed .

3.1 The following two control charts for W_Q are constructed .

i) Control chart sW_Q^1 - This is simple Shewhart control chart and

i) Control chart sW_Q^2 - This chart is constructed using method of weighted variance.

3.2 The following three control charts for r.v.W_s are constructed which are referred to as sW_s^1, sW_s^2 and sW_s^3

i) Control chart sW_s^1 - This is simple Shewhart control chart

ii) Control chart sW_s^2 - This chart is constructed using method of weighted variance.

iii) Control chart sW_s^3 -This chart is constructed using Nelson's transformation.

4 Control charts for r.v.W_Q for (M/M/s : ∞/FCFS) model

In this section ,Shewhart control chart for W_Q for (M/M/s : ∞/FCFS) model is constructed

Waiting time distribution of W_Q

Construction of control limits for W_Q , needs the distribution of W_Q , its expectation and variance. The r.v.W_Q denote the waiting time of customer in the queue. Assuming that the queue discipline is FCFS, from queuing theory ,the distribution f(x) of W_Q is given by

$$f(x)dx = 1 - \frac{(\lambda/\mu)^s}{(s-1)!\left(s - \frac{\lambda}{\mu}\right)} p_0 = 1 - \frac{(\lambda/\mu)^s}{s!(1-\rho)} p_0, \quad x = 0$$

$$= \frac{(\lambda/\mu)^s}{(s-1)!} \mu e^{-(s\mu-\lambda)x} p_0, \qquad x > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.1$$

4.1 Moments of W_Q

First two raw moments of $\mathbf{W_Q}$ are obtained

$$\mu_1^1 = E[W_Q] = [W_Q = 0]P[W_Q = 0] + \int_{\epsilon}^{\infty} x f(x)dx, \quad \text{as } \epsilon \to 0$$

Now

$$\int_{\epsilon}^{\infty} x f(x)dx = \int_{\epsilon}^{\infty} \frac{(\lambda/\mu)^s p_0 \mu}{(s-1)!} \{x e^{-(s\mu-\lambda)x}\} dx \to \left\{\frac{(\lambda/\mu)^s \mu p_0}{(s-1)!}\right\} \frac{1}{(s\mu-\lambda)^2}$$

$$= \frac{(\lambda/\mu)^s}{(s\mu)(s!)(1-\rho)^2} p_0 = \frac{p_s}{s\mu(1-\rho)^2}$$

$$E[W_Q] = \frac{P[N \geq s]}{s\mu(1-\rho)_Q} = \frac{(\lambda/\mu)^s}{(s-1)!} p_0 \mu \frac{1}{(s\mu-\lambda)2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.1.1$$

$$\mu_2^1 = E[W_Q^2] = \int_{\epsilon}^{\infty} x^2 f(x)dx$$

on simplification

$$E[W_Q^2] = \frac{(\lambda/\mu)^s}{(s-1)!} p_0 \mu \frac{2}{(s\mu-\lambda)^3}$$

Let σ^2 denote variance of W_Q

$$\sigma^2 = V[W_Q] = \mu_2^1 - (\mu_1^1)^2$$

$$\sigma^2 = V[W_Q] = E[W_Q^2] - \{E[W_Q]\}^2$$

$$= \frac{p_0\mu}{(s-1)!} \frac{(\lambda/\mu)^s}{(s\mu-\lambda)^2} \left[\frac{2}{(s\mu-\lambda)} - \frac{(\lambda/\mu)^s p_0 \mu}{(s-1)!(s\mu-\lambda)^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.1.2$$

5 Control chart sW_Q^1

Knowing $E[W_Q]$ and $V[W_Q]$,the 3 sigma control limits for W_Q are given by

$$UCL = E[W_Q] + 3\sqrt{V(W_Q)}$$
$$CL = E[W_Q] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5.1$$
$$LCL = E[W_Q] - 3\sqrt{V(W_Q)}$$

False alarm rate

Let α_u denote type I error probability generated in the upper tail or false alarm rate which is given by $\alpha_u = P[W_Q > UCL]$

where UCL is obtained from 5.1 and $P[W_Q > UCL]$ is obtained using expression 4.1

6 Control chart sW_Q^2

Control chart for the r.v. W_Q using method of weighted variance

In order to obtain control limits for W_Q using this method, the probability P_{WQ} defined as follows ,is needed

$$P_{W_Q} = P\left[W_Q \le E(W_Q)\right]$$
$$= \int_0^{E[W_Q]} f(x)dx \qquad = \int_0^{E[W_Q]} \frac{(\lambda/\mu)^s p_0 \mu}{(s-1)!} e^{-(s\mu-\lambda)x} dx$$

Solving the integral and substituting for $E[W_Q]$,

$$P_{W_Q} = \frac{\mu p_0 (\lambda/\mu)^s}{(s-1)!(s\mu-\lambda)} \left\{1 - e^{\frac{-\mu p_0(\lambda/\mu)}{(s\mu-\lambda)}}\right\} \ldots\ldots\ldots\ldots\ldots 6.1$$

If the underlying population is symmetric then $P_{WQ} = 0.5$ and the chart for weighted variance reduce to Shewhart chart. However, if the underlying population is skewed to the right then $P_{WQ} > 0.5$ and the distance of UCL from the Center Line (CL) is larger than that of LCL similarly if the underlying population is skewed to the left then $P_{WQ} < 0.5$ and the distance of the LCL from the CL is larger than that of UCL.

6.1 Control limits using method of weighted variance

The 3 sigma control limits using method of weighted variance are given by:

$$UCL = E[W_Q] + 3\sqrt{V(W_Q)}.\sqrt{2P_{W_Q}}$$
$$CL = E[W_Q] \qquad \ldots\ldots\ldots\ldots\ldots\ldots 6.1.1$$
$$LCL = E[W_Q] - 3\sqrt{V(W_Q)}.\sqrt{2(1-P_{W_Q})}$$

Where $E[W_Q]$, $V[W_Q]$ and P_{WQ} are obtained using 4.1.1, 4.1.2 and 6.1

6.2 False alarm rate (FAR)

Let α_u denote type I error probability generated in the upper tail or false alarm rate which is given by $\alpha_u = P[W_Q > UCL]$

where UCL is obtained from 6.1.1 and the corresponding probability can be obtained from expression 4.1

7 Control charts for the r.v.W_s for (M/M/S : ∞/FCFS) model

The distribution of r.v.W_Q and its moments are obtained in section 4. In this section ,control limits for the r.v.W_s using Shewhart method and method of weighted variance are to be constructed. In order to obtain control limits for the r.v.W_s, the expressions for $E[W_s]$ and $V[W_s]$ are required.

Let W_s denote the waiting time of the customer in the system.

$$\therefore W_s = W_Q + \frac{1}{\mu} \ldots\ldots\ldots\ldots\ldots 7.1$$

where W_Q is the waiting time of the customer in the queue and $(1/\mu)$ is the service rate of individual channel.

$$E[W_s] = E\left[W_Q + \frac{1}{\mu}\right] = E[W_Q] + \frac{1}{\mu} \ldots\ldots\ldots\ldots 7.2$$

and

$$V[W_s] = V\left[W_Q + \frac{1}{\mu}\right] == V[W_Q] \ldots\ldots\ldots\ldots 7.3$$

where, $E(W_Q)$ and $V(W_Q)$ are obtained from 4.1.1 and 4.1.2.

7.1 Control chart sW_s^1

Control limits for W_s using Shewhart method

The 3 sigma control limits for r.v.W_s are

$$UCL = E[W_s] + 3\sqrt{V(W_s)}$$
$$CL = E[W_s]$$
$$LCL = E[W_s] - 3\sqrt{V(W_s)}$$

where E[W_s] and V[W_s] are obtained using expressions 7.2 and 7.3.

7.2 False alarm rate

It is of interest to know the probability that the time of waiting in the line plus the service time exceeds time t. This probability is denoted by P [W_s > t]. This probability is given by

$$P[W_s > t] = e^{-ut}\left[1 + \frac{W}{s} + \frac{1 - e^{-\mu st\left[1-\left(\frac{\lambda}{\mu s}\right)-\left(\frac{1}{s}\right)\right]}}{\left[1-\left(\frac{\lambda}{\mu s}\right)-\left(\frac{1}{s}\right)\right]}\right] \text{.................... ....7.2.1}$$

Where W is the probability that a customer has to wait in line, which is the sum of all probabilities that all service facilities are being used or that s or more customers are in line.

$$W = \frac{P_0}{s!}\left(\frac{\lambda}{\mu}\right)^s \sum_{n=0}^{\infty}\left(\frac{\lambda}{\mu s}\right)^n W = \left(\frac{\lambda}{\mu}\right)^s \frac{P_0}{s!\left(1 - \frac{\lambda}{\mu s}\right)}$$

Let α_u denote false alarm rate given by α_u = P[W_s > UCL] ,replacing t by UCL in expression 5.3.1 the expression for false alarm rate α_u is obtained and is given by

$$\alpha_u = P[W_S > UCL] = e^{-\mu UCL}\left[1 + \frac{W}{S} + \frac{1 - e^{-\mu UCL\left[1-\frac{\lambda}{\mu s}-\frac{1}{s}\right]}}{\left[1-\left(\frac{\lambda}{\mu s}\right)-\left(\frac{1}{s}\right)\right]}\right] \text{.................... ....7.2.2}$$

7.3 Numerical analysis of Control chart sW_s^1

In order to study effect of ρ on control limits, one set of values of λ, μ and s is selected. For this set of values of ρ, p_0, LCL, UCL, α_u and ARL are obtained and are displayed in table 1 From this table, it is observed that keeping λ and μ fixed, if s is increased, the value of α_u increases which results in the corresponding decrease in the values of ARL. We also observe that for some combination of λ, μ and s, α_u turns out to be 0. This will mean that in a queuing system with those particular combinations of λ, μ and s, there are no chances of system going out of control, which means that system is performing very well.

Table 1
Lower and Upper Control limits and the associated values of α_u for r.v. Ws for M/M/S queue using sW_s^1 chart with L = 3

Sr. No	λ	μ	s	p_o	variance	ρ	LCL	CL	UCL	α_u	ARL
1	20	15	2	0.2000	0.0078	0.666667	0	0.1200	0.3853	0	--
2	20	15	3	0.2542	0.0005	0.444444	0.0050	0.0738	0.1427	0.0025	400
3	20	15	4	0.2621	0.000006	0.333333	0.0441	0.0679	0.0917	0.0339	30
4	20	15	5	0.2633	0.000008	0.266667	0.0582	0.0668	0.0755	0.0715	14
5	20	15	6	0.2635	0.000001	0.222222	0.0635	0.0667	0.0698	0.0936	11
6	10	15	2	0.5000	0.0007	0.3333	0.0	0.0750	0.1579	0.0026	385
7	10	15	3	0.5121	0.00005	0.222222	0.00	0.0675	0.0892	0.0508	20
8	10	15	4	0.5133	0.00004	0.166667	0.0607	0.0667	0.0728	0.0951	11
9	10	15	5	0.5134	0	0.133333	0.0650	0.0666	0.0683	0.1131	9
10	100	35	3	0.0111	0.03968	0.952381	0	0.2108	0.8084	0	-
11	100	35	4	0.0464	0.00043	0.714286	0	0.0398	0.1025	0	-
12	100	35	5	0.0546	0.00006	0.571429	0.0071	0.0312	0.0553	0.0016	625
13	100	35	6	0.0567	0.000013	0.47619	0.0185	0.0293	0.0401	0.0175	57
14	100	35	7	0.0572	0.000003	0.408163	0.0237	0.0287	0.0337	0.0428	23
15	16	15	2	0.5333	0.0003084	0.533333	0	0.0931	0.2597	0.000003	3333333
16	16	15	3	0.3390	0.000240	0.355556	0.0238	0.0703	0.1167	0.0121	82
17	16	10	2	0.1111	0.057284	0.8	0	0.2777	0.9958	0	-
18	16	10	3	0.1871	0.002411	0.533333	0	0.1195	0.2668	0.0002	5000
19	16	10	4	0.1992	0.000301	0.4	0.0517	0.1037	0.1557	0.0165	61
20	12	6	3	0.1111	0.019204	0.666667	0	0.2407	0.6564	0	-

8 Control chart sW_s^2

Control limits for W_s using method of weighted variance

To obtain control limits, the method of weighted variance needs the probability P_{WS}, where P_{WS} is given by

$$P_{W_S} = P[W_s \le E(W_s)] \\ = 1 - P[W_s > E(W_s)] \qquad \ldots\ldots 8.1$$

This probability can be obtained using7.2.1. The control limits of P_{Ws} using method of weighted variance are given by :

$$UCL = E[W_s] + 3\sqrt{V(W_s)}\sqrt{2P_{Ws}}$$
$$CL = E[W_s] \quad \ldots\ldots\ldots\ldots\ldots\ldots 8.2$$
$$LCL = E[W_s] - 3\sqrt{V(W_s)}\sqrt{2(1-P_{Ws})}$$

8.1 False alarm rate

Let α_u denote the false alarm rate which is given by $\alpha_u = P[W_s > UCL]$

Where UCL is obtained using 8.2 and $P[W_s > UCL]$ is obtained using expression 7.2.1

8.2 Numerical analysis

In order to study effect of ρ on control limits. the same set of values of λ, μ and s as in chart sW_s^1 are selected. For this set UCL, CL and LCL, P_{Ws}, P_0, α_u and ARL are obtained and are displayed in table 2. From this table it is observed that if we keep λ and μ fixed and s is increased then α_u increases and consequently the associated ARL decreases very rapidly.

If ARL of this chart is compared with the ARL of chart sW_s^1 then the increase in values of ARL can be noticed. This means that the performance of this chart is better than performance of control chart sW_s^1. This improvement in ARL is due to the presence of factor P_{Ws} in control limits which takes into account skewness of the underlying distribution of W_s for that particular combination of λ, μ and s.

Table 2

Lower and Upper Control limits and the associated values of α_u for r.v. Ws for M/M/s queue using method of weighted variance sW_s^2 with L = 3

Sr. No	λ	μ	s	p_o	Pws	ρ	LCL	CL	UCL	α_u	ARL
1	20	15	2	0.2000	0.9975	0.666667	0.1013	0.1200	0.4957	0	-
2	20	15	3	0.2542	0.9411	0.444444	0.0502	0.0738	0.1683	0.0008	1250
3	20	15	4	0.2621	0.9130	0.333333	0.0580	0.0679	0.1001	0.0245	41
4	20	15	5	0.2633	0.9020	0.266667	0.0630	0.0668	0.0784	0.0642	16
5	20	15	6	0.2635	0.8956	0.222222	0.0652	0.0667	0.0708	0.0903	11
6	10	15	2	0.5000	0.9186	0.3333	0.0415	0.0750	0.1873	0.0008	1250

7	10	15	3	0.5121	0.8880	0.222222	0.0573	0.0675	0.0964	0.0392	26
8	10	15	4	0.5133	0.8830	0.166667	0.0638	0.0667	0.0747	0.0889	11
9	10	15	5	0.5134	0.8804	0.133333	0.0650	0.0666	0.0688	0.1111	9
10	100	35	3	0.0111	1.0000.	0.952381	0.2108	0.2108	1.0560	0	-
11	100	35	4	0.0464	0.9954	0.714286	0.0388	0.0398	0.1283	0	-
12	100	35	5	0.0546	0.9672	0.571429	0.0251	0.0312	0.0648	0.00052	1923
13	100	35	6	0.0567	0.9449	0.47619	0.0257	0.0293	0.0441	0.0115	18
14	100	35	7	0.0572	0.9306	0.408163	0.0269	0.0287	0.0356	0.0360	28
15	16	15	2	0.5333	0.9777	0.533333	0.0580	0.0931	0.3260	0	-
16	16	15	3	0.3390	0.9176	0.355556	0.0514	0.0703	0.1332	0.0062	161
17	16	10	2	0.1111	0.9999	0.8	0.2762	0.2777	1.2932	0	0
18	16	10	3	0.1871	0.9651	0.533333	0.0806	0.1195	0.3242	0.000032	31250
19	16	10	4	0.1992	0.9277	0.4	0.0840	0.1037	0.1746	0.0098	102
20	12	6	3	0.1111	0.9927	0.666667	0.1906	0.2407	0.8265	0	-

9 Control Chart sW_S^3

Nelson's control chart for W_s for M/M/s model

Khaparde M. V. and Dhabe S. D. have constructed control chart using power transformation for r.v.W_s for (M/M/1: ∞/FCFS) model.Like (M/M/1 : ∞/FCFS) model, in (M/M/s: ∞/FCFS) model also the distribution of W_s is exponential. But this exponential distribution is a special case of Weibull W $\left(\frac{1}{s\mu-\lambda},1\right)$ distribution.

Using the transformation $Y=(W_s)^{\frac{1}{3.6}}=W_s^{0.277}$,Y transforms to Weibull

$$W\left[\left(\frac{1}{s\mu-\lambda}\right)^{0.2777},3.6\right]$$

and thus follows approximate normal distribution. The mean of Y is given by

$$E(Y)=\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}\Gamma\left(1+\frac{1}{3.6}\right)=(0.901)\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9.1$$

This expression is used to set the center line of the control chart for Y. The standard deviation of Y is given by.

$$\sqrt{V(Y)}=\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\left\{\Gamma\left(1+\frac{1}{3.6}\right)\right\}^{2}}$$

$$=\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}(0.278)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9.2$$

9.1 Control limits for W_s using Nelson Chart

Using the above approximation the control limits for W_s are given by

$$UCL=E[W_s]+L\sqrt{V(W_s)}$$

$$=(0.901)\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}+L\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}(0.278)\ldots\ldots\ldots\ldots\ldots 9.1.1$$

$$CL=(0.901)\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}.$$

$$LCL=(0.901)\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}+L\left(\frac{1}{s\mu-\lambda}\right)^{0.2777}(0.278)\ldots\ldots\ldots\ldots\ldots 9.1.2$$

where L is the distance of control limits from the center line. Taking L = 3, we get 3 sigma control limits.

9.2 Derivation and definition of α

Let α be the probability of type I error then $\alpha = \alpha_u + \alpha_l$

where α_u and α_l are the risk probabilities generated in the upper and lower tail respectively and are defined as

$\alpha_u = P[W_s > UCL]$; $\alpha_l = P[W_s < LCL]$

The distribution of W_s is exponential. But after using transformation the distribution of W_s is not complete symmetrical about its mean. Therefore the probability of W_s exceeding upper control limit is obtained from C.D.F. of Weibull distribution.

$\therefore\ \alpha_u = P[W_s > UCL] = 1 - P[W_s < UCL] = 1 - F[UCL]$

where F(.) is the distribution function of 2 parameter Weibull distribution $W(\eta, \nu)$.

$$F[UCL] = 1 - \exp\left\{-\left(\frac{UCL}{\eta}\right)^{\nu}\right\} \quad \ldots\ldots 9.2.1$$

but $\nu = 3.6$

$$= 1 - \exp\left\{-\left(\frac{UCL}{\eta}\right)^{3.6}\right\}$$

$$\alpha_l = P[W_s \le LCL] = \exp\left\{\left(-\frac{LCL}{\eta}\right)^{3.6}\right\} \ldots\ldots 9.2.2.$$

where $\eta = \left(\frac{1}{s\mu - \lambda}\right)^{0.2777}$

9.3 Numerical analysis

We have selected same set of values of λ, μ and s as that of control chart sW_s^1 and sW_s^2, the control limits α_u and α_l are obtained. These are given in table 3 .

For this chart α_u = 0.000732 and α_l = 0.000059 for all values of λ, μ and s.

If we are interested in detecting the shift in upper as well as lower direction then ARL is given by

$$ARL = \frac{1}{\alpha_u + \alpha_l} = \frac{1}{(0.000732) + (0.000059)} = \frac{1}{0.000791} = 1264.2225$$

This ARL remains same for all values of λ, μ and s. The main difference that can be observed in the ARL of this chart and the ARL of earlier two charts is that, in the chart sW_s^1 and sW_s^2, if we keep λ, μ fixed then ARL decreases with increase in value of s (the number of servers) but in this chart it remains same.

Using Nelson's chart if we want to detect the shift in the upward direction only then we have to consider value of α_u only and in that case

$$ARL = \frac{1}{\alpha_u} = \frac{1}{0.000732}$$
$$= 1366.1202$$
$$\cong 1366$$

10 Conclusion- The comparison of the above three control charts for waiting time on the basis of ARL reveals that since ARL is highest for the third control chart sW_S^3 where Nelson transformation is used ,Therefore it is the best chart.

Table 3
Lower and Upper Control limits and the associated values of α_u and α_l for r.v. Ws for M/M/s queue using Nelson transformation sW_s^3, with L = 3

Sr. No.	λ	μ	s	ρ	UCL	CL	LCL	α_u	α_l
1	20	15	2	0.666667	0.915376	0.475362	0.035349	0.000732	0.000059
2	20	15	3	0.444444	0.709727	0.368567	0.027407	0.000732	0.000059
3	20	15	4	0.333333	0.622884	0.323469	0.024054	0.000732	0.000059
4	20	15	5	0.266667	0.570165	0.296092	0.022018	0.000732	0.000059
5	20	15	6	0.222222	0.533231	0.276912	0.020592	0.000732	0.000059
6	20	15	7	0.190476	0.505242	0.262377	0.019511	0.000732	0.000059
7	10	15	3	0.222222	0.646416	0.335689	0.024962	0.000732	0.000059
8	10	15	4	0.166667	0.585458	0.304033	0.022608	0.000732	0.000059
9	10	15	5	0.133333	0.544319	0.282669	0.02102	0.000732	0.000059
10	100	35	3	0.952381	1.109674	0.576263	0.042852	0.000732	0.000059
11	100	35	4	0.714286	0.622884	0.323469	0.024054	0.000732	0.000059
12	100	35	5	0.571429	0.523112	0.271657	0.020201	0.000732	0.000059
13	100	35	6	0.47619	0.470332	0.244247	0.018163	0.000732	0.000059
14	100	35	7	0.408163	0.4356	0.226211	0.016821	0.000732	0.000059
15	16	15	2	0.533333	0.833719	0.432957	0.032195	0.000732	0.000059
16	16	15	3	0.355556	0.68107	0.353685	0.026301	0.000732	0.000059
17	16	10	2	0.8	1.180613	0.613102	0.045591	0.000732	0.000059
18	16	10	3	0.533333	0.833719	0.432957	0.032195	0.000732	0.000059
19	16	10	4	0.4	0.717819	0.372769	0.02772	0.000732	0.000059
20	12	6	3	0.666667	1.054889	0.547813	0.040736	0.000732	0.000059

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