MULTI-OBJECTIVE GEOMETRIC PROGRAMMING PROBLEM BASED ON INTUITIONISTIC FUZZY GEOMETRIC PROGRAMMING TECHNIQUE

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Abstract-

The paper aims to give computational algorithm to solve a multi-objective non-linear programming problem using intuitionistic fuzzy geometric programming technique. As the intuitionistic fuzzy optimization technique utilizes degree of membership and degree of non-membership, we made a study of correspondence between linear membership and non-membership functions to see its impact on optimization and to get insight in such optimization process. Also we made a comparative study of optimal solution between intuitionistic fuzzy geometric programming and geometric programming. The developed algorithm has been illustrated by a numerical example.

Keywords-Intuitionistic fuzzy set, Multi-objective non-linear programming, Membership function, Non-membership function, Intuitionistic fuzzy geometric programming.

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1. Introduction

Modeling of most of real life problems involving optimization process turns out to be multi-objective programming problem in a natural way. These objectives are conflicting in nature and hence solution of such problems are in general compromise solutions which satisfy each objective function to a degree of satisfaction and a concept of membership and non-membership arises in such situations. It was Zimmermann [17], [18] who first used the fuzzy set introduced by Zadeh [16] for solving the multi-objective mathematical programming problem. Optimization in fuzzy environment was further studied and was applied in various areas by many researchers such as Tanaka [15], Luhandjula [8], Sakawa [14] etc.In view of growing use of fuzzy set in modeling ofproblems under situations when informationavailable is imprecise,vagueoruncertain,variousextensionoffuzzy setsimmerged. Insuchextensions,Atanassov [2],[3] introducedtheintuitionistic fuzzysetsasapowerful extension offuzzyset.Atanassov inhisstudiesemphasized thatinviewofhandlingimprecision, vagueness or uncertainty ininformation boththedegreeofbelongingand degreeofnon-belonging shouldbeconsideredastwo independent propertiesasthesearenotcomplementofeach other.Thisconceptof membershipand non-membership wasconsideredbyAngelov[1]inoptimizationproblem andgaveintuitionistic fuzzy approach tosolveoptimization problems. JanaandRoy[7]studiedthemultiobjective intuitionisticfuzzylinearprogrammingproblemandapplied ittotransportation problem.Luo[9]appliedtheinclusion degreeofintuitionistic fuzzysettomulticriteriadecision makingproblem.FurthermanyworkerssuchasMahapatra *etal.*,[10],Nachammai[11] andNagoorgani[12]etc.have also studied linear programming problem under intuitionisticfuzzyenvironment.RecentlyDubey*etal.*, [5], [6]studiedlinearprogramming problemintuitionistic fuzzyenvironment usingintuitionisticfuzzynumberand intervaluncertaintyinfuzzynumbers. The motivation of the present study is to give computational algorithm to solve a multi-objective non-linear programming problem using intuitionistic fuzzy geometric programming technique. Also we made a comparative study of optimal solution between intuitionistic fuzzy geometric programming and geometric programming.

2. Some preliminaries

2.1. Definition -1 (Fuzzy set) [17]

Let X be a fixed set. A fuzzy set A of X is an object having the form $\tilde{A}$ = {(x,μ_A (x)), x Є X} where the function $\mu_A(x)$: X → [0, 1] define the truth membership of the element x Є X to the set A

2.2. Definition-2 (Intuitionistic fuzzy set) [16]

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form $\tilde{A}^i$ ={$< X, \mu_A(x), \nu_A(x) > / x \in X$} where $\mu_A(x)$: X→ [0, 1] and $\nu_A(x)$: X→ [0, 1] define the Truth-membership and Falsity-membership respectively , for every element of x∈ X , 0≤ $\mu_A(x) + \nu_A(x)$ ≤1

2.3 Multi-objective geometric programming problem

A multi-objective geometric programming problem can be defined as

Find $x=(x_1,x_2,\ldots\ldots\ldots x_n)^T$,so as to ....(1)

$$\text{Min } f_{10}(x)= \sum_{t=1}^{T_{10}} C_{10t} \prod_{j=1}^{n} x_j^{a_{10tj}}$$

$$\text{Min } f_{20}(x) = \sum_{t=1}^{T_{20}} C_{20t} \prod_{j=1}^{n} x_j^{a_{20tj}}$$

……………………………………..

$$\text{Min } f_{p0}(x) = \sum_{t=1}^{T_{p0}} C_{p0t} \prod_{j=1}^{n} x_j^{a_{p0tj}}$$

$$\text{such that } f_i(x) =\sum_{t=1}^{T_i} c_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le 1 \qquad i=1,2,\ldots\ldots,m$$

$$x_j > 0, \qquad j= 1,2,\ldots\ldots\ldots n$$

Where $c_{k0t}>0$ for all k and t . a_{itj} ,a_{k0tj} are all real ,for all i,k,t,j .

3. Computational Algorithm

To solve the multi-objective non-linear programming problem we use the following steps.

Step-1. Pick the first objective function and solve it as a single objective subject to the constraints. Continue the process k-times for k different objective functions. Find value of objective functions and decision variables.

Step-2. To build membership functions, goals and tolerances should be determined at first. Using the ideal solutions, obtained in step-1 we find the values of all the objective functions at each ideal solution and construct pay-off matrix as follows:

$$\begin{bmatrix} f_1^*(x^1) f_2(x^1) \ldots \ldots \ldots \ldots f_p(x^1) \\ f_1(x^2) f_2^*(x^2) \ldots \ldots \ldots \ldots f_p(x^2) \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ f_1(x^p) f_2(x^p) \ldots \ldots \ldots \ldots f_p(x^p) \end{bmatrix}$$

Step-3. From step-2 we find the upper and lower bounds of each objective functions.

Step-4. Let $U_k{}^{\mu}$ = max $\{f_{r0}(x^k)\}$ and $L_k{}^{\mu}$ = min $\{f_{r0}(x^k)\}$ where $1 \le r \le k$

For membership of objectives.

Step-5. We represents upper and lower bounds for non- membership of objectives as follows:

$U_k{}^{\mu} = U_k{}^{\nu}$ and $L_k{}^{\nu} = L_k{}^{\mu} + t\,(U_k{}^{\mu} - L_k{}^{\mu})$ where $0 < t < 1$

Step-6. Define linearmembership and non-membership functions as follows:

$$\mu_k(f_{k0}(x)) = \begin{cases} 1 & if\ f_{k0}(x) \le L_k{}^{\mu} \\ \frac{U_k{}^{\mu} - f_{k0}(x)}{U_k{}^{\mu} - L_k{}^{\mu}} & if\ L_k{}^{\mu} \le f_{k0}(x) \le U_k{}^{\mu} \\ 0 & if\ f_{k0}(x) \ge U_k{}^{\mu} \end{cases}$$

$\nu_k(f_{k0}(x)) = 1 - \frac{1}{1-t}\mu_k(f_{k0}(x))$

It is obvious that

$$\nu_k(f_{k0}(x)) = \begin{cases} 1 & if\ f_{k0}(x) \le L_k{}^{\mu} \\ \frac{f_{k0}(x) - L_k{}^{\nu}}{U_k{}^{\nu} - L_k{}^{\nu}} & if\ L_k{}^{\mu} \le f_{k0}(x) \le U_k{}^{\mu} \\ 0 & if\ f_{k0}(x) \ge U_k{}^{\mu} \end{cases}$$

and $0 \le \mu_k(f_{k0}(x)) + \nu_k(f_{k0}(x)) \le 1$.

Step-7. Now an intuitionistic fuzzy geometric programming techniquefor multi-objective non-linear programming problem with the linear membership and non-membership functions can be written as

Maximize $(\mu_1(f_{10}(x)),\mu_2(f_{20}(x)),\ldots\ldots\ldots,\mu_p(f_{p0}(x)))$ ……….(2)

Minimize $(\nu_1(f_{10}(x))\ \nu_2(f_{20}(x))\ \ldots\ldots\ldots,\nu_p(f_{p0}(x)))$

subject to $f_i(x) = \sum_{t=1}^{T_i} c_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le 1$ for i=1,2,…………m

$X_j>0$ j= 1,2,…………n.

Using weighted sum method the multi-objective non-linear programming problem (2) reduces to

$\text{Min} V_{MA}(f_{k0}(x))=\sum_{k=1}^{p} w_k(\nu_k(f_{k0}(x))-\mu_k(f_{k0}(x)))$………..(3)

$$= \frac{2-t}{1-t}\sum_{k=1}^{p} w_k \frac{\sum_{t=1}^{T_{k0}} c_{k0t}\prod_{j=1}^{n} x_j^{a_{k0tj}}}{U_k^{\mu}-L_k^{\mu}} - \left\{\left(\frac{2-t}{1-t}\sum_{k=1}^{p} w_k \frac{U_k^{\mu}}{U_k^{\mu}-L_k^{\mu}}\right) - \sum_{k=1}^{p} w_k\right\}$$

subject to $f_i(x) = \sum_{t=1}^{T_i} c_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le 1$ for i=1,2,…………m

$X_j > 0$, j= 1,2,…………n.

Excluding the constant term the above (3) reduces to following geometric programming problem

$$\text{Min}\ V_{MA1}(f_{k0}(x)) = \frac{2-t}{1-t}\sum_{k=1}^{p} w_k \frac{\sum_{t=1}^{T_{k0}} c_{k0t}\prod_{j=1}^{n} x_j^{a_{k0tj}}}{U_k^{\mu}-L_k^{\mu}} \quad \ldots\ldots\ldots\ldots (4)$$

such that $f_i(x) = \sum_{t=1}^{T_i} c_{it}\prod_{j=1}^{n} x_j^{a_{itj}} \le 1$

$x_j > 0$

For k=1,2,……….p; i=1,2,…….m; j=1,2,………n; and pre-determined t Є (0, 1).

Where $V_{MA}(f_{k0}(x)) = V_{MA1}(f_{k0}(x)) - \left\{\left(\frac{2-t}{1-t}\sum_{k=1}^{p} w_k \frac{U_k^{\mu}}{U_k^{\mu}-L_k^{\mu}}\right) - \sum_{k=1}^{p} w_k\right\}$.

Here (4) is a posynomial geometric programming problem with

$$DD = \sum_{k=1}^{p} T_{k0} + \sum_{i=1}^{m} T_i - n - 1.$$

It can be solved by usual geometric programming technique.

Definition M-N Pareto optimal solution

A decision variable x^*Є X is said to be a M-N Pareto optimal solution to the IFGPP (2) if there does not exit another x Є X such that $\mu_k(f_{k0}(x)) \leq \mu_k(f_{k0}(x^*))$, $\nu_k(f_{k0}(x)) \geq \nu_k(f_{k0}(x^*))$ for all k=1,2,……..p. and $\mu_l(f_{l0}(x)) \neq \mu_l(f_{l0}(x^*))$, $\nu_l(f_{l0}(x)) \neq \nu_l(f_{l0}(x^*))$ for at least one l, l=1,2,……..p.

Some basic theorems on M-N Pareto optimal solutions are introduced below.

Theorem 1 The solution of (2) based on weighted sum method IFGP problem (3) is weakly M-N Pareto optimal.

Proof. Let x^*Є X be a solution of the IFGP problem. Let us suppose that it is not weakly M-N Pareto optimal. In this case there exit another x Є X such that$\mu_k(f_{k0}(x))<\mu_k(f_{k0}(x^*))$,$\nu_k(f_{k0}(x))>\nu_k(f_{k0}(x^*))$. for all k=1,2,……..p. Observing that $\mu_k(f_{k0}(x))$ is strictly monotone decreasing function with respect to $f_{k0}(x)$, this implies $\mu_k(f_{k0}(x))>\mu_k(f_{k0}(x^*))$ and $\nu_k(f_{k0}(x))$ is strictly monotone increasing function with respect to $f_{k0}(x)$, this implies $\nu_k(f_{k0}(x))<\nu_k(f_{k0}(x^*))$. Thus we have $\sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x))>\sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x^*))$ and $\sum_{k=1}^{p} w_k\, \nu_k(f_{k0}(x))<\sum_{k=1}^{p} w_k \nu_k(f_{k0}(x^*))$. This is a contradiction to the assumption that x^* is a solution of the IFGP Problem (2). Thus $x^*$is weakly M-N Pareto optimal.

Theorem 2 The unique solution of IFGP problem (3) based on max-additive operator is weakly M-N Pareto optimal.

Proof. Let x^*Є X be a unique solution of the IFGP problem. Let us suppose that it is notweakly M-N Pareto optimal. In this case there exit another x Є X such that $\mu_k(f_{k0}(x)) \leq \mu_k(f_{k0}(x^*))$, $\nu_k(f_{k0}(x)) \geq \nu_k(f_{k0}(x^*))$ for all k=1,2,……..p and$\mu_l(f_{l0}(x)) < \mu_l(f_{l0}(x^*))$, $\nu_l(f_{l0}(x)) > \nu_l(f_{l0}(x^*))$for at least one l. Observing that $\mu_k(f_{k0}(x))$ is strictly monotone decreasing function with respect to $f_{k0}(x)$, this implies $\mu_k(f_{k0}(x)) > \mu_k(f_{k0}(x^*))$ and $\nu_k(f_{k0}(x))$ is strictly monotone increasing function with respect to $f_{k0}(x)$, this implies $\nu_k(f_{k0}(x)) < \nu_k(f_{k0}(x^*))$. Thus we have $\sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x)) \geq \sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x^*))$ and $\sum_{k=1}^{p} w_k\, \nu_k(f_{k0}(x)) \leq \sum_{k=1}^{p} w_k\, \nu_k(f_{k0}(x^*))$. On the other hand, the uniqueness of x^* means that $\sum_{k=1}^{p} w_k \mu_k(f_{k0}(x^*)) < \sum_{k=1}^{p} w_k \mu_k(f_{k0}(x))$ and$\sum_{k=1}^{p} w_k \nu_k(f_{k0}(x^*)) > \sum_{k=1}^{p} w_k \nu_k(f_{k0}(x))$. The two sets inequalities above are contradictory and thus $x^*$is weakly M-N Pareto optimal.

4. Illustrated example

Min $f_1(x_1, x_2) = {x_1}^{-1}{x_2}^{-2}$

Min $f_2(x_1, x_2) = 2{x_1}^{-2}{x_2}^{-3}$

Such that $x_1 + x_2 \leq 1$

Here pay-off matrix is $\begin{bmatrix} 6.75 & 60.78 \\ 6.94 & 57.87 \end{bmatrix}$

Define membership and non-membership functions as follows:

$$\mu_1(f_1(x)) = \begin{cases} 1 & if\ {x_1}^{-1}{x_2}^{-2} \leq 6.75 \\ \frac{6.94 - {x_1}^{-1}{x_2}^{-2}}{0.19} & if\ 6.75 \leq {x_1}^{-1}{x_2}^{-2} \leq 6.94 \\ 0 & if\ {x_1}^{-1}{x_2}^{-2} \geq 6.94 \end{cases}$$

$$\mu_2(f_2(x)) = \begin{cases} 1 & if\ 2\,{x_1}^{-2}{x_2}^{-3} \leq 57.87 \\ \frac{60.78 - 2{x_1}^{-2}{x_2}^{-3}}{2.91} & if\ 57.87 \leq 2\,{x_1}^{-2}{x_2}^{-3} \leq 60.78 \\ 0 & if\ 2\,{x_1}^{-2}{x_2}^{-3} \geq 60.78 \end{cases}$$

$\nu_1(f_1(x)) = 1 - \frac{1}{1-t}\mu_1(f_1(x))$, and $\nu_2(f_2(x)) = 1 - \frac{1}{1-t}\mu_2(f_2(x))$

Table -1: Optimal values of primal, dual variables and objective functions from intuitionistic fuzzy geometric programming problem for equal weights

t	Dual variables	Primal variables		Optimal Objectives		
		x_1	x_2	f_1	f_2	$f_1 + f_2$
0.1	W_{01}=0.6352729, w_{02}=0.36473 W_{11}=1.36473, w_{12}= 2.36473	0.36593	0.63407	6.797161	58.58984	65.38701
02	W_{01}=0.6341507, w_{02}=0.36276 W_{11}=1.36276, w_{12}= 2.36276	0.36579	0.63422	6.796547	58.59312	65.38966
0.3	W_{01}=0.6341507, w_{02}=0.36585 W_{11}=1.36585, w_{12}= 2.36585	0.36601	0.63399	6.797391	58.58641	65.38380
0.4	W_{01}=0.6454384, w_{02}=0.35457 W_{11}=1.35457, w_{12}= 2.35457	0.365197	0.63480	6.795091	58.62182	65.41691
0.5	W_{01}=0.6344708, w_{02}=0.36553 W_{11}=1.36553, w_{12}= 2.36553	0.36599	0.63401	6.797333	58.58727	65.38460
0.6	W_{01}=0.6330181, w_{02}=0.36699 W_{11}=1.36699, w_{12}= 2.36699	0.36609	0.63391	6.797621	58.58298	65.38060

0.7	W_{01}=0.6328063, w_{02}=0.3672 W_{11}=1.3672, w_{12}= 2.3672	0.36611	0.63389	6.797678	58.58212	65.37980
0.8	W_{01}=0.6330903, w_{02}=0.36691 W_{11}=1.36691, w_{12}= 2.36691	0.36609	0.63391	6.797621	58.58298	65.38060
0.9	W_{01}=0.6357255, w_{02}=0.36428 W_{11}=1.36428, w_{12}= 2.36428	0.3659	0.6341	6.797075	58.59114	65.38821

From table 1, it shows that best optimal solution is obtained for t= 0.7

Table -2: Optimal values of primal, dual variables and objective functions from intuitionistic fuzzy geometric programming problem for $w_1 = 0.9$, $w_2 = 0.1$

t	Dual variables	Primal variables		Optimal Objectives		
		x_1	x_2	f_1	f_2	$f_1 + f_2$
0.1	W_{01}=0.8842138 w_{02}=0.1157862 W_{11}=1.1157862 w_{12}=2.1157862	0.3452766	0.6547234	6.756428	59.77535	66.53178
02	W_{01}=0.8850876 w_{02}=0.1149124 W_{11}=	0.3451928	0.6548072	6.756338	59.78141	66.53775

	1.1149124 w_{12}= 2.1149124					
0.3	W_{01}=0.8837129 w_{02} =0.1162871 W_{11}=1.1162871 w_{12}= 2.1162871	0.3453245	0.6546755	6.756479	59.77188	66.52836
0.4	W_{01}=0.8886815 w_{02}=0.1113185 W_{11}=1.1113185 w_{12}= 2.1113185	0.3448476	0.6551524	6.755976	59.80653	66.56250
0.5	W_{01}=0.8838554 w_{02}=o.1161446 W_{11}=1.1161446 w_{12}=2.1161446	0.3453109	0.6546891	6.756464	59.77287	66.52933
0.6	W_{01}=0.8832077 w_{02}=0.1167923 W_{11}=1.1167923 w_{12}= 2.1167923	0.3453728	0.6546272	6.756531	59.76839	66.52492
0.7	W_{01}=0.8831760 w_{02}=0.116824 W_{11}=1.116824 w_{12}= 2.116824	0.3453759	0.6546241	6.756534	59.76817	66.52471
0.8	W_{01}=0.8834861 w_{02} =0.1165139 W_{11}=1.1165139 w_{12}=2.1165139	0.3453462	0.6546538	6.756502	59.77032	66.52682
0.9	W_{01}=0.8845047 w_{02}=0.1154953 W_{11}=1.1154953 w_{12}= 2.1154953	0.3452487	0.6547513	6.756398	59.77737	66.53377

From table 2, it shows that first objective gives better optimal result. And this is happened for t = 0.4

Table -3: Optimal values of primal, dual variables and objective functions from intuitionistic fuzzy geometric programming problem for w_1 = 0.2, w_2 = 0.8

t	Dual variables	Primal variables		Optimal Objectives		
		x_1	x_2	f_1	f_2	$f_1 + f_2$
0.1	W_{01}=0.1827858 w_{02}=0.8172142 W_{11}=1.8172142 w_{12}= 2.8172142	0.3921118	0.6078882	6.901487	59.77535	66.53178
02	W_{01}=0.1840774 w_{02}=0.8159226 W_{11}=1.8159226 w_{12}= 2.8159226	0.3920517	0.6079483	6.901180	59.78141	66.53775
0.3	W_{01}=0.1820522 w_{02} =0.8179478 W_{11}=1.8179478 w_{12}=2.8179478	0.3921460	0.6078540	6.901661	59.77188	66.52836
0.4	W_{01}=0.1895565 w_{02}=0.8104435 W_{11}=1.8104435 w_{12}=2.8104435	0.3917957	0.6082043	6.89	59.80653	66.56250
0.5	W_{01}=0.1822604 w_{02}=0.8177396 W_{11}=1.8177396 w_{12}=2.8177396	0.3921363	0.6078637	6.756464	59.77287	66.52933
0.6	W_{01}=0.1813176 w_{02}=0.8186824	0.3921801	0.6078199	6.756531	59.76839	66.52492

	W_{11}=1.8186824 w_{12}=2.8186824					
0.7	W_{01}=0.1812716 w_{02}=0.8187284 W_{11}=1.8187284 w_{12}=2.8187284	0.3921823	0.6078177	6.756534	59.76817	66.52471
0.8	W_{01}=0.1817218 w_{02} =0.8182782 W_{11}=1.8182782 w_{12}= 2.8182782	0.3921613	0.6078387	6.756502	59.77032	66.52682
0.9	W_{01}=0.1832141 w_{02}=0.8167859 W_{11}=1.8167859 w_{12}=2.8167859	0.3920919	0.6079081	6.756398	59.77737	66.53377

From table-3, it shows that second objective gives better optimal result. And this is happened for t= 0.7

Table-4. Comparison of optimal solutions obtained by various methods for equal weight.

Decision variables & objective functions	Best solution obtained by Fuzzy geometric programming technique	Best solution obtained by Intuitionistic fuzzy geometric programming technique
X_1	0.360836	0.36611
X_2	0.6391634	0.63389
f_1	6.783684	6.797678
f_2	58.82652	58.58212
Sum of objectives	65.610204	65.37980

Table-4. Shows that Intuitionistic fuzzy geometric programming technique gives better optimal result than fuzzy geometric programming technique.

5. Conclusion

In viewofcomparingtheintuitionisticfuzzygeometric programming techniquewithfuzzy geometric programming technique,wealsoobtainedthe solution of the undertaken numerical problem byfuzzy optimization method and took the best result obtained for comparison with present study.

The objectives of the present study is to give the effective algorithm for intuitionisticfuzzygeometric programming method for getting optimal solutions to a multi-objective non-linear programming problem. The merit of the method lies with fact that it gives a set of solutions with various values of t. The decision makers may choose a suitable optimal solution according to the demand of the actual situation. Further the comparisons of results obtained for the undertaken problem clearly show the superiority of intuitionisticfuzzygeometric programming techniqueoverfuzzy geometric programming technique.

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