HYPERSURFACE OF A LORENTZIAN PARA-SASAKIAN MANIFOLD

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ABSTRACT

In this paper, we have studied the hypersurface of a Lorentzian para-sasakian manifold. The non-vanishing condition for the scalar function λ on the hypersurface, has also been discussed.

Key words: Differentiable manifold, hypersurface, linear transformation field, immersion.

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1. INTRODUCTION:

Let M be an n-dimensional differentiable manifold. Let there exist a tensor field F of type (1,1), a vector field T, a 1-form A, a Riemannain metric G, s.t.

(1.1) (a) $F^2X = X + A\ \ X\ \ T$

(b) $FT = 0$

(c) $A\ \ T\ \ = -1$

(d) $AoF = 0$

(e) $G\ \ FX, FY\ \ = -G\ \ X, Y\ \ + A\ \ X\ \ \ A\ \ Y$

(f) $A\ \ X\ \ = -G\ \ X, T$

Let D be the Riemannian connexion on M, s.t.

(g) $D_X F\ \ Y = -G\ \ X, Y\ \ T + A\ \ X\ \ \ A\ \ Y\ \ T + \left[X + A\ \ X\ \ T\right] A\ \ Y$

For all vector fields X, Y tangential to M, then M is called a Lorentzian para-sasakian manifold.

Let $\bar{M}$ be the hypersurface of M. Let $b : \bar{M} \longrightarrow M$ be the immersion, i.e. $p \in \bar{M} \Rightarrow b\ \ p\ \ \in M$.

$$B : T_p\ \ \bar{M}\ \ \longrightarrow T_{b\ \ p}\ \ M\ \ ,\ i.e.\ X \in T_p\ \ \bar{M}\ \ \Rightarrow BX \in T_{b\ \ p}\ \ M$$

B. is the differnetial of the immersion.

We have

(1.2) (a) $FBX = B\varphi X + \eta\ \ X\ \ C$,

(b) $FC = B\xi + \lambda C$.

Then $\varphi, \xi, \eta, \lambda$ define respectively a linear transformation field, a vector field, a 1-form and a scalar function on $\bar{M}$. Here C denotes the unit normal vector to $\bar{M}$. Let g be the induced Riemannain metric on $\bar{M}$, then

(1.3) $G(BX, BY) = g(X, Y)$

2. HYPERSURFACE IMMERSED IN A LPS MANIFOLD:

Theorem (2.1)

Define a vector field ξ^1, a 1-form η^1 on $\bar{M}$ as

(2.1) (a) $T = B\xi^1$

(b) $A(BX) = \eta^1(X)$,

then for hypersurface structure $(\varphi, \xi, \xi^1 \eta, \eta^1, \lambda, g)$

we have

(2.2) (a) $\varphi^2 X = X - \eta(X)\xi + \eta^1(X)\xi^1$

(b) $\eta(\varphi X) = -\lambda\eta(X)$

(c) $\varphi\xi = -\lambda\xi$

(d) $\eta(\xi) = 1 - \lambda^2$

(e) $\eta(\xi^1) = 0$

(f) $\varphi\xi^1 = 0$

(g) $\varphi^3 X - \varphi X = \lambda\eta(X)\xi$

(h) $\eta^1 \ \xi \ = 0$

(i) $g \ \varphi X, \varphi Y \ = -g \ X, Y \ - \eta \ X \ \eta \ Y \ + \eta^1 \ X \ \eta^1 \ Y$

(j) $\lambda^4 - \lambda^2 + 2 = 0$

(k) $\eta^1 \ \xi^1 \ = -1$

Proof:

Operating F to both the sides of (1.2) (a),

(2.3) $F^2 BX = FB \varphi X + \eta \ X \ FC$

using (1.1) (a), (1.2) (a), (b) in (2.3)

(2.4) $BX + A \ BX \ T = B\varphi^2 X + \eta \ \varphi X \ C + \eta \ X \ \ B\xi + \lambda C$

using (2.1) (a), (b) in (2.4),

(2.5) $BX + \eta^1 \ X \ B\xi^1 = B\left[\varphi^2 X + \eta \ X \ \xi\right] + \left[\eta \ \varphi X \ + \lambda \eta \ X \ \right] C$

which gives

$$\left.\begin{aligned} &\varphi^2 X = X - \eta \ X \ \xi + \eta^1 \ X \ \xi' \\ &\text{and} \\ &\eta \ \varphi X \ = -\lambda \eta \ X \end{aligned}\right\}$$

which are (2.2) (a) and (2.2) (b).

Operating F to both the sides of (1.2) (b)

(2.6) $F^2 C = FB\xi + \lambda FC$

using (1.1) (a), (1.2) (a), (b) in (2.6),

(2.7) $C + A \; C \;\; T = B\varphi\xi + \eta \;\; \xi \;\; C + \lambda \;\; B\xi + \lambda C$

as C is unit normal to $\bar{M}$ $\therefore$ from (1.1) (f), A(C)=0. Then (2.7) gives.

(2.8) $C = B \;\; \varphi\xi + \lambda\xi \;\; + \left[\eta \;\; \xi \;\; + \lambda^2\right] C$

Thus

$$\left.\begin{aligned} \varphi\xi &= -\lambda\xi \\ \eta \;\; \xi \;\; &= 1 - \lambda^2 \end{aligned}\right\}$$

which are (2.2) (c) and (2.2) (d).

From (2.2) (b), we have

(2.9) $\eta \;\; \varphi^2 X \;\; = \lambda^2 \eta \;\; X$

using (2.2) (a) in (2.9)

(2.10) $\eta \left[X - \eta \;\; X \;\; \xi + \eta^1 \;\; X \;\; \xi^1 \right] = \lambda^2 \eta \;\; X$

(2.11) $\eta \;\; X \;\; - \eta \;\; X \;\; \eta \;\; \xi \;\; + \eta^1 \;\; X \;\; \eta \;\; \xi^1 \;\; = \lambda^2 \eta \;\; X$

using (2.2) (d) in (2.11),

$$\eta \;\; X \;\; - \eta \;\; X \;\; \left[1 - \lambda^2\right] + \eta^1 \;\; X \;\; \eta \;\; \xi^1 \;\; = \lambda^2 \eta \;\; X$$

which gives

$$\eta \;\; \xi^1 \;\; = 0,$$

which is (2.2) (e).

From (1.1) (b), and (2.1) (a)

(2.12) $FB\ \xi^1 = 0$

using (1.2) (a), (2.2) (e) in (2.12), we get

$\varphi\xi^1 = 0$

which is (2.2) (f)

From (2.2) (a), we have

(2.13) $\varphi^3 X = \varphi X - \eta\ X\ \ \varphi\xi + \eta^1\ X\ \ \theta\xi^1$

using (2.2) (c), (2.2) (f) in (2.13),

$\varphi^3 X - \varphi X = \lambda\eta\ \ X\ \ \xi$

which is (2.2) (g)

Replacing X by ξ in (2.2) (a)

(2.14) $\varphi^2\xi = \xi - \eta\ \ \xi\ \ \xi + \eta^1\ \ \xi\ \ \xi^1$

using (2.2) (c), (d) in (2.14),

(2.15) $\lambda^2\xi = \xi - \left[1-\lambda^2\right]\xi + \eta^1\ \ \xi\ \ \xi^1$

which gives

$\eta^1\ \ \xi\ \ = 0$

which is (2.2) (h)

From (1.1) (e),

(2.16) $G\ \ FBX, FBY\ \ = -G\ \ BX, BY\ \ + A\ \ BX\ \ \ A\ \ BY$

using (1.2) (a) in (2.16)

(2.17) $G\left(B\varphi X+\eta(X)C, B\varphi Y+\eta(Y)C\right)=-G(BX,BY)+A(BX)A(BY)$

C being unit normal to $\bar{M}$.

(2.18) $G\left(B\varphi X, B\varphi Y\right)+\eta(X)\eta(Y)=-G(BX,BY)+A(BX)A(BY)$

using (1.3), (2.1) (b) in (2.18)

$$g(\varphi X,\varphi Y)=-g(X,Y)-\eta(X)\eta(Y)+\eta^1(X)\eta^1(Y)$$

which is (2.2) (i)

From (2.2) (i), we have

(2.19) $g(\varphi\xi,\varphi\xi)=-g(\xi,\xi)-\eta(\xi)\eta(\xi)+\eta^1(\xi)\eta^1(\xi)$

using (2.2) (c), (d), (h) in (2.19), we get

$$\lambda^2=-1-\left[1-\lambda^2\right]^2=-2-\lambda^4+2\lambda^2$$

$\lambda^4-\lambda^2+2=0$ which is (2.2) (j).

Again from (2.2) (i)

(2.20) $g(\varphi\xi^1,\varphi\xi^1)=-g(\xi^1,\xi^1)-\eta(\xi^1)\eta(\xi^1)+\eta^1(\xi^1)\eta^1(\xi^1)$

using (2.2) (f), (e) in (2.20)

$$0=-1+\left[\eta^1(\xi^1)\right]^2$$

$$\eta^1(\xi^1)=-1$$

which is (2.2) (k)

Theorem (2.2):

The vector field FC can not be tangential to $\bar{M}$ or $\lambda \neq 0$.

Proof: from (1.2) (b)

(2.21) $g\ \ FC, C\ \ = g\ \ B\xi, C\ \ + \lambda g\ \ C, C$

$0 = 0 + \lambda$

(2.22) $\lambda = 0$

From (2.2) (j) and (2.22), we get

(2.23) 2=0, which is an absurdity

Thus $\lambda \neq 0$.

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