STUDENTS' LEARNING SKILL OF MATHEMATICS – A FUZZY MODELING APPROACH

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Abstract

The quality of teaching and learning mathematics have been one of the major challenges of the educators. Mathematics is the subject that has the lot of skills, concepts with variety of topics. This paper presents a mathematical model to analyze the students' mathematical learning skill using fuzzy logic. In this analyses the degree of satisfaction is already defined by experts with respect to levels of performance. From this, the degree of satisfaction of a mathematical topic is calculated and the result is calculated based on all the topics in mathematics. The obtained results from the proposed approach are compared with the conventional non fuzzy approach and the comparative results are given.

Keywords: Fuzzy performance sheet, Fuzzy set, Linguistic term, Students' learning skill.

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I Introduction

In modern world of science and technology mathematics plays a very important role. Mathematics is the quite necessary tool for all curriculum, so called as " Queen of sciences". It is based on the both logic and reality of the world. But how far this subject is concentrated by the students is a question mark. Understanding of pure mathematics papers have always been challenging criteria for the mathematics students also.

In higher educational institution, evaluation usually occurs when the lectures evaluate their students based on final examination , tests, quizzes, assignment, group project thesis, oral presentation and so on. This paper is focused the mathematical skill of the students for some topic of mathematics. The result from the experts are usually vague rather than crisp, a result should be expressed by using fuzzy sets which has the capability of representing vague data. Here , we refer [1-6] the few papers handling fuzzy sets and a multi-attribute method together for the performance evaluation.

This paper is divided into five sections. The following section proposes the fuzzy based [7-9] student evaluation method with block diagram. In third section, the proposed fuzzy mathematical modeling [10,11] are given in detail. The fourth section explains the proposed fuzzy modeling with a suitable case study. The comparison of fuzzy performance evaluation with the non-fuzzy evaluation approach is discussed in section five. Finally the conclusion are given.

II Fuzzy Student Evaluation

Figure 1 shows the structure of a student evaluation method. In this model the input parameters are evaluators satisfaction level, allotted marks to topics and given learning skill parameter. The output produced from the model is total marks and/or students rating.

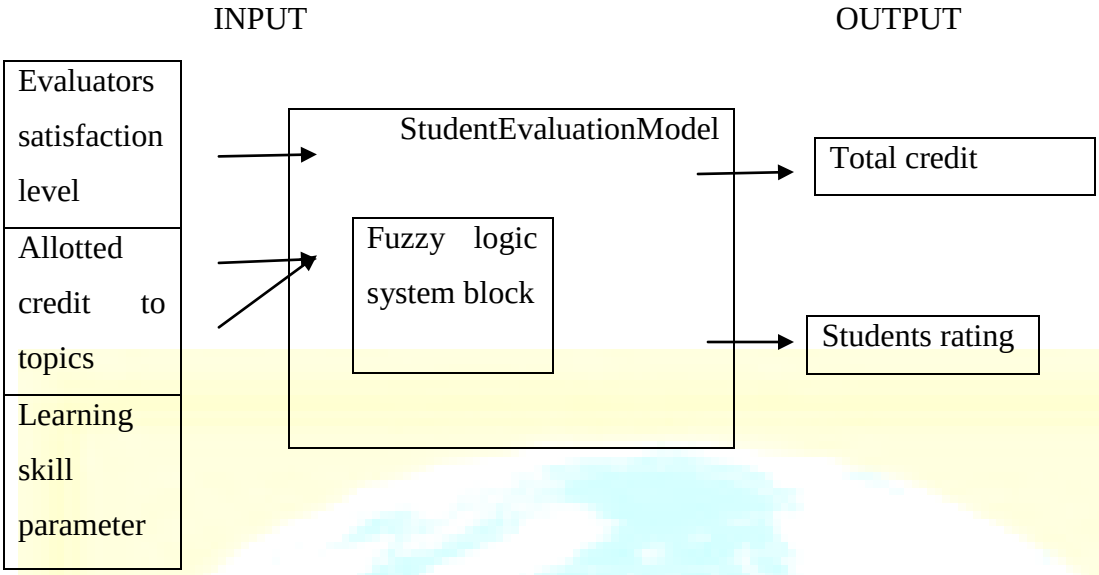

Figure 1 : Structure of the student evaluation model

III Fuzzy Mathematical Modeling

Let there are nine satisfaction levels to evaluate the students learning quality. They are E(excellent), VG(very good), G(good), MG(more or less good), F(fair), MB(more or less bad), B(bad), VB(very bad), EB(extremely bad). The degrees of the satisfaction levels are shown in the following table:

Satisfaction levels	Degrees of satisfaction	Maximum degree of satisfaction
E	91%-100!	1.00
VG	81%-90%	0.9
G	71%-80%	0.8
MG	61%-70%	0.7
F	51%-60%	0.6
MB	41%-50%	0.5
B	25%-40%	0.4
VB	10%-24%	0.24
EB	0%-95	0.09

Table 1:Degrees of satisfaction according to performance level

Let K be a set of satisfaction levels, where K = {E,VG,G,MG,F,MB,B,VB,EB} and assume H be a mapping function which maps a satisfaction level to the maximum degree of

satisfaction of the corresponding satisfaction level, where H:K→[0,1]. From the above table, H(Excellent) = 1.0 i.e.,H(E) = 1.0. similarly, H(VG) = 0.9, H(G) = 0.8, H(MG) = 0.7, H(F) = 0.6, H(MB) = 0.5, H(B) = 0.4, H(VB) = 0.24, H(EB) = 0.09.

So, in the proposed method, the degrees of satisfaction is defined in advance with respect to levels of performance and from which the maximum degree of satisfaction per level is obtained. A fuzzy performance sheet is given below.

Topic no.	Satisfaction level									Degree of satisfaction
	E	VG	G	MG	F	MB	B	VB	EB	
T_1	0	0.7	0.8	0.5	1.0	0	0	0	0	
T_2										
T_3										
….	…	…	…	…	…	…	…	…	…	…
T_n										Total marks=

Step 1: let the fuzzy mark of the topic T_1 of a student's performance evaluated by the evaluator is shown in below table: Table 2: Example of a fuzzy performance sheet

At the bottom of the sheet there is a box which tells the total marks. The first column shows the serial numbers of the topic, in any row, the columns from the second to the tenth shows the fuzzy mark awarded to the answer of the corresponding topic in first column, where the fuzzy mark is represented as a fuzzy set in the universe of discourse K. The last column indicates the degree of satisfaction evaluated by the given method awarded to each topic. The box at the bottom shows the total marks awarded to a student.

From the above table , the satisfaction level regarding the first topic is represented by a fuzzy set F(T_1) which is

F(T_1) ={(E,0),(VG,0.7),(G,0.8),(MG,0.5),(F,1.0),(MB,0),(B,0),(VB,0),(EB,0)}

= {(VG,0.7),(G,0.8),(MG,0.5),(F,1.0)}

It indicates that the satisfaction level of the student's learning quality with respect to the first topic is defined as 70% excellent, 80% good, 50% more or less good and 100% fair. The proposed algorithm is explained by the following steps as given below:

Topic no.	Satisfaction level									Degree of satisfaction
	E	VG	G	MG	F	MB	B	VB	EB	
T_1	0	0.7	0.8	0.5	1.0	0	0	0	0	
…	…	…	…	…	…	…	…	…	…	…
T_i	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	…
….	…	…	…	…	…	…	…	…	…	…
										Total marks=

Table 3: Fuzzy marks of topic T_1 in a fuzzy performance sheet

Here $y_i \in [0,1]$ which is the membership values awarded to each level of performance and $1 \leq i \leq 9$. F(y_i) is the respective maximum degree of satisfaction. Now the degree of satisfaction D(T_1) of the topic T_1 of the student's performance can be evaluated by the function

$$\text{D as } D(T_i) = \frac{\sum T_i(y_i) \times F(y_i)}{\sum T_i(y_i)}$$

$$= \frac{y_1 \times H(E) + y_2 \times H(VG) + \cdots . + y_9 \times H(EB)}{y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9} \text{ where } D(T_i) \in [0,1].$$

Step 2: Assume that student's performance marks to a subject consists of 100 marks. Let that there are n number of topics to be covered.

Total marks = 100

T_1 carries C M_1 marks, T_2 carries C M_2 marks, …… T_n carries C M_n marks where $\sum_{i=1}^{n} CM_1 = 100$.

Let, that the evaluated degree of satisfaction of the topic $T_1, T_2, \ldots$ and T_i are D(T_1), D(T_2)…. and D(T_n) respectively, then the total marks (TM) of the student can be evaluated as follows:

$$\text{TM} = \sum_{i=1}^{n} CM(T_i) \times D(T_i)$$

$$= CM_1 \times D(T_1) + CM_2 \times D(T_2) + \cdots . + CM_n \times D(T_n)$$

Here, $CM(T_i)$ is the marks allocated for each topic by the evaluator and D(T_i) is the calculated degrees of satisfaction for T_i. Shows this total marks in the appropriate box at the bottom of the fuzzy performance sheet.

Step 3: Construct a generalized fuzzy evaluation method. Assume that the evaluator evaluated the student's performance score using the following criteria as given below:

l_1: prior knowledge, l_2: memory power, l_3: problem solving skill, l_4: cognition power,

l_5 : independent idea generation, l_6 :presence of mind, l_7 : concentration power, l_8 : time management, l_9: language skill to understand the problem, l_{10}: concept understanding etc.

Criteria based parameters are related with different disciplines and related papers within it.

Topic no.	Learning skill parameter	Satisfaction level									Degree of satisfaction for criteria	Degree of satisfaction for topic
		E	VG	G	MG	F	MB	B	VB	EB		
T_1	l_1										D(l_{11})	P(l_1)
	l_2										D(l_{12})	
	l_3										D(l_{13})	
	…	..	..	..	..	..	..	..	..	..	…	
	l_m										D(l_{1m})	
T_2	l_1										D(l_{21})	P(l_2)
	l_2										D(l_{22})	
	l_3										D(l_{23})	
	…	..	..	..	..	..	..	..	..	..	…	
	l_m										D(l_{2m})	
	…	..	..	..	..	..	..	..	..	..	…	…
T_n	l_1										D(l_{n1})	P(l_n)
	l_2										D(l_{n2})	
	l_3										D(l_{n3})	
	…	..	..	..	..	..	..	..	..	..	…	
	l_m										D(l_{nm})	
Total marks = $CM_1 \times P(l_1) + CM_2 \times P(l_2)$+…..+$CM_n \times P(l_n)$												

Table :4 Generalized fuzzy performance sheet

Assume that the weights of the given learning skill parameter l_1, l_2 , l_3 … l_m are $w_1, w_2, w_3 \ldots w_m$ respectively, where $w_i \in [0,1] and\ 1 \leq i \leq m$. Also assume that an evaluator can evaluate each topic of a student's performance score sheet using the above learning skill parameter. It evaluates the student's performance used the proposed the method as shown in above the table where the degrees of satisfaction of topic T_i of a student performance score in

respect to the parameter l_1, l_2 , l_3 … l_m evaluated by the method are D(l_{i1}), D(l_{i2}), D(l_{i3}), … D(l_{im}) respectively where 0≤ $D(l_{im}) \leq 1$.

The degree of satisfaction P(l_i) of the topic T_i of the student's performance sheet can be evaluated as:

$P(T_i) = \frac{w_1 \times D(l_{i1}) + w_2 \times D(l_{i2}) + \cdots . + w_m \times D(l_{im})}{w_1 + w_2 + \cdots . + w_m}$ where P(T_i) ∈ [0,1] and $0 \leq i \leq n$. The total marks of the student can be evaluated and is equal to

$CM_1 \times P(T_1) + CM_2 \times P(T_2) + \ldots . + CM_n \times P(T_n)$. Apply this formula to calculate the total marks in the appropriate box at the bottom of the fuzzy performance sheet.

IV. Case Study

Consider a student's performance sheet which consists of 100 marks. Let that there are 4 topics to be covered with total marks 100. We analyze the mathematical learning skill of students of R.A. college of women, Thiruvarur and to be covered the following topics:

T_1 stands for abstract algebra which carries 30 marks.

T_2 stands for real and complex analysis which carries 30 marks.

T_3 stands for ordinary & partial differential equations which carries 20 marks.

T_4 stands for calculus which carries 20 marks.

There are many tests are conducted based on the parameters l_1, l_2, l_3 and l_4 where l_1- prior knowledge, l_2- memory power, l_3- problem solving skill and l_4- cognition power respectively. The faculty evaluates a student's performance by generalized fuzzy performance sheet as in shown below table.

Topic No.	Given learning skill parameter	Satisfaction level									Degree of satisfaction for criteria	Degree of satisfaction for topic n
		E	VG	G	MG	F	MB	B	VB	EB		
T_1	l_1	0	0	0	0	0.3	0.6	0.7	0.5	0	0.4190	0.4705
	l_2	0	0	0.3	0.8	0.6	0	0	0	0	0.6824	
	l_3	0	0	0	0	0.5	0.4	0.6	0.3	0	0.4511	
	l_4	0	0	0	0	0	0.4	0.3	0.7	0	0.3488	
T_2	l_1	0	0	0	0.2	0.5	0.7	0	0	0	0.5643	0.5524

	l_2	0	0	0.5	0.9	0.3	0.2	0	0	0	0.6895	
	l_3	0	0	0	0.2	0.5	0.6	0.1	0	0	0.5571	
	l_4	0	0	0	0	0.2	0.3	0.6	0.5	0	0.3938	
T_3	l_1	0	0	0.6	0.4	0.3	0.5	0	0	0	0.6611	0.7722
	l_2	0	0.7	0.8	0.5	0.6	0	0	0	0	0.7615	
	l_3	0.5	0.9	0.8	0	0	0	0	0	0	0.8864	
	l_4	0	0	0.6	0.9	0.7	0.4	0	0	0	0.6654	
T_4	l_1	0	0	0.8	0.5	0.6	0.2	0	0	0	0.6905	0.7866
	l_2	0	0.7	0.6	0.5	0.5	0	0	0	0	0.7652	
	l_3	0.6	0.9	0.7	0	0	0	0	0	0	0.8955	
	l_4	0	0	0.5	0.9	0.8	0	0	0	0	0.6864	

Table :5 An example of fuzzy performance sheet

Assume that the weights (w_1, w_2, w_3, w_4) of given learning skill parameter l_1, l_2, l_3, l_4 are respectively 0.2, 0.2, 0.4, 0.2.

$$D(l_{11}) = \frac{0.3\times H(F)+0.6\times H(MB)+0.7\times H(B)+0.5\times H(VB)}{0.3+0.6+0.7+0.5}$$

$$= \frac{0.3\times 0.6+0.6\times 0.5+0.7\times 0.4+0.5\times 0.24}{2.1} = 0.4190.$$

Similarly, the other degrees of satisfaction D(l_{12}), D(l_{13})……D(l_{44}) are shown in table 5.

Degree of satisfaction for topic is calculated as given below:

$$P(T_1) = \frac{w_1\times D(l_{11})+w_2\times D(l_{12})+w_3\times D(l_{13})+w_4\times D(l_{14})}{w_1+w_2+w_3+w_4}$$

$$= \frac{0.2\times 0.4190+0.2\times 0.6824+0.4\times 0.4511+0.2\times 0.3488}{1} = 0.4705$$

Similarly, $P(T_2) = 0.5524$, $P(T_3) = 0.7722$, $P(T_4) = 0.7866$.

Total marks = $CM_1 \times P(T_1) + CM_2 \times P(T_2) + CM_3 \times P(T_3) + CM_4 \times P(T_4)$

$= 30 \times 0.4705 + 30 \times 0.5524 + 20 \times 0.7722 + 20 \times 0.7866.$

$= 14.115 + 16.572 + 15.444 + 15.732 = 61.863 \cong 62.$

V . Comparative Anatysis With Non-Fuzzy Approach

The result got from fuzzy evaluation can be compared with non-fuzzy approach. In the non-fuzzy approach marks in each topic T_i is calculated as given below in table 6.

Topic No.	Learning skill parameter	Maximum membership value	Satisfaction level	Degree of satisfaction	Non-fuzzy marks (100 grade scale)
T_1	l_1	0.7	B	25-40	32
	l_2	0.8	MG	61-70	65
	l_3	0.6	B	25-40	32
	l_4	0.7	VB	10-24	17
T_2	l_1	0.7	MB	41-50	45
	l_2	0.9	MG	61-70	65
	l_3	0.6	MB	41-50	45
	l_4	0.6	B	25-40	32
T_3	l_1	0.6	G	71-80	75
	l_2	0.8	G	71-80	75
	l_3	0.9	VG	81-90	85
	l_4	0.9	MG	61-70	65
T_4	l_1	0.8	G	71-80	75
	l_2	0.7	VG	81-90	85
	l_3	0.9	VG	81-90	85
	l_4	0.9	MG	61-70	65

Table :6 Non-fuzzy performance sheet

Assume that the non fuzzy mark in each learning skill parameter is the average of the range of the degree of satisfaction in each learning skill parameter.

Non-fuzzy marks in $T_1 = (32 + 65 + 32 + 17)/4 = 36.5$ in 100 grade scale and its equivalent marks according to total marks to allotted to T_1 is 36.5x30/100 = 10.95. Similarly the other non-fuzzy marks are $T_2 = 14.025, T_3 = 15, T_4 = 15.5$

Total marks = 10.95+14.025+15+15.5 = 55.475≅ 55.

The figure 2 shows the graphical representation of fuzzy and non fuzzy marks of each topic.

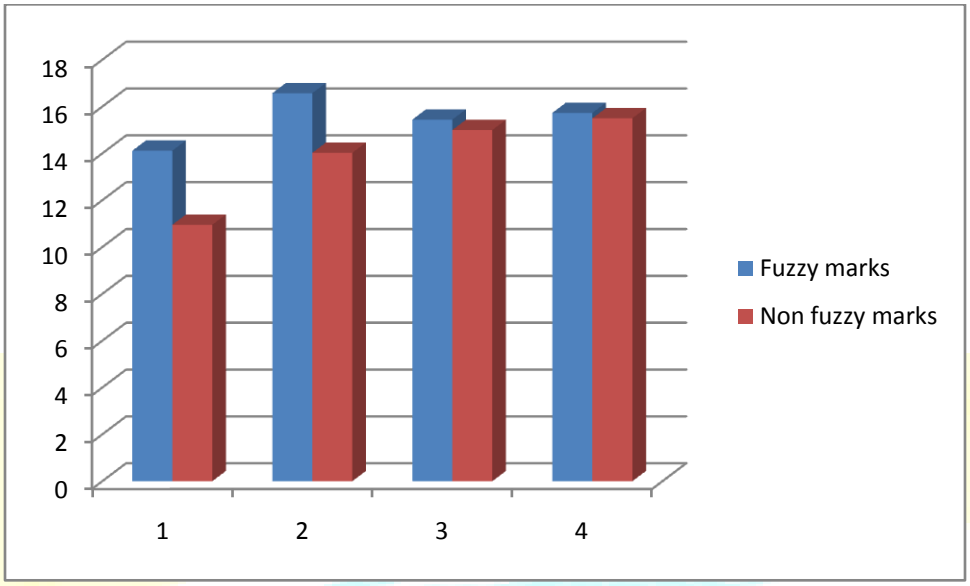

Figure :2 results of student marks in each topic.

The figure 3 shows the graphical representation of fuzzy and non fuzzy marks in the subject. Fuzzy marks obtained are more than the non-fuzzy marks. The satisfaction level differs in both the approach where the fuzzy result shows MG(more or less good) and non-fuzzy result shows F(fair) ranking. The fuzzy result is more accurate for performance evaluation than the non fuzzy method.

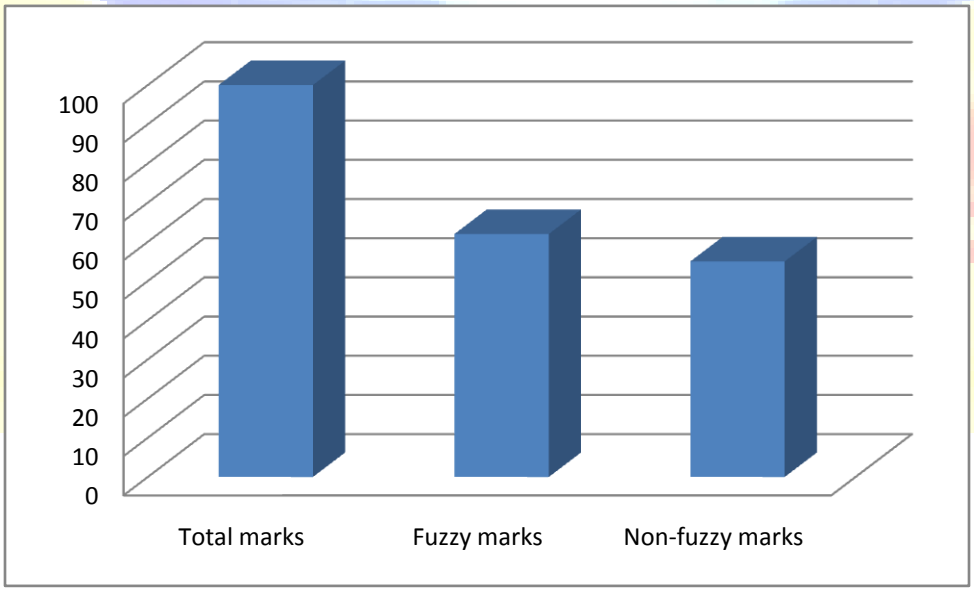

Figure 3: comparative results of fuzzy and non fuzzy marks in the mathematical skill

VI. Conclusion

In this article, a model for evaluating learning mathematical skill of the students has been discussed. The above method is applied to analyze the skill of mathematical subject based on some learning parameter. The model is explained with the help of proper case study. From that the most of the students need more concentration on problem solving skill for theory papers. The comparative performance analysis is presented and the fuzzy performance result is better than the non-fuzzy result. The model can be used to analyze the various skill of the students. From that, the faculty members train the each student in suitable way.

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