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STRONG AND BALANCED IRREGULAR INTERVAL-VALUED FUZZY GRAPHS

Selvanayaki. S*

Abstract

In this paper, the strong and balanced irregular interval-valued fuzzy graph concept is investigated and the size, order of the irregular interval-valued fuzzy graphs is derived and also the density of the graph is derived. Some basic propositions and theorem are also been presented.

Keywords: Balanced interval-valued fuzzy graphs, Interval-valued fuzzy graphs, Irregular interval-valued fuzzy graphs, Strong interval- valued fuzzy graphs.

* **Assistant Professor, Department of Mathematics, KG College of Arts and Science, Coimbatore, Tamilnadu.**

1 INTRODUCTION

The fuzzy graph theory as a generalization of Euler's graph theory was first introduced by Rosenfeld in 1975. The fuzzy relations between fuzzy sets were first considered by Rosenfeld, who developed the structure of fuzzy graphs obtaining analogous to several graph theoretical concepts. Later Bhattacharya gave some remarks on fuzzy graphs, and someoperations on fuzzy graphs were introduced by Modeson and Peng. Recently, Akrametalintroduced the concept of bipolar fuzzy graphs, interval-valued fuzzy graphs, Strong intuitionistic fuzzy graphs.

Talebi and Rashmanlou studied properties of isomorphism and complement on interval-valued fuzzy graphs. Rashmanlou and jun defined complete interval-valued fuzzy graphs. Talebi, Rashmanlou and Davvaz in investigated some properties of interval-valued fuzzy graphs such as regular interval-valued fuzzy graphs, totally regular interval-valued fuzzy graph and complement of interval-valued fuzzy graph. Talebi and Rashmanlou defined product bipolar fuzzy graphs and isomorphism and complement on bipolar fuzzy graphs. Recently Rashmanlou and Pal defined irregular interval-valued fuzzy graphs. More results on interval-valued fuzzy graphs, product interval-valued fuzzy graphs and their degrees, intuitionistic fuzzy graphs with categorical properties, some properties of highly irregular interval-valued fuzzy graphs. A study on bipolar fuzzy graphs and investigated several properties. They defined isometry on interval-valued fuzzy graphs. Samata and Pal introduced fuzzy tolerance graph,irregular bipolar fuzzy graphs, fuzzy k-competition graphs and p-competiotion fuzzy graphs, bipolar fuzzy hyper graphs. In 1975, Zadeh introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets in which the values of the membership degrees are intervals of numbers instead of the numbers. Interval-valued fuzzy sets provide more adequate description of uncertainty than traditional fuzzy sets.

2. PRELIMINARIES

In this section present some basic definitions. A fuzzy set A on X is characterized by a mapping $m: X \to [0,1]$, called the membership function. A fuzzy set is denoted as $A = (X, m)$. A fuzzy graph $\xi = (V, \sigma, \mu)$ is a non-empty set V together with a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that for all $u, v \in V, \mu(u, v) \leq \sigma(u)$^σ(v) (Here x^y denotes the minimum of x and y).

Definition 2.1. The interval-valued fuzzy set A in V is define by $A = \{(x[\mu_A^-(x),\mu_A^+(x)]):x\in V\}$ where $\mu_A^-(x)$ and $\mu_A^+(x)$ are fuzzy subsets of V, such that $\mu_A^-(x) \leq \mu_A^+(x)$ for all $x \in V$. For any two interval-valued set $A = [\mu_A^-(x),\mu_A^+(x)]$ and $B = [\mu_B^-(x),\mu_B^+(x)]$ in V ,we define

- $A\cup B=\{(x,\max(\mu_A^-(x),\mu_B^-(x)),$

$\max(\mu_A^+(x),\mu_B^+(x)): x\in V\}$

- $A\cap B=\{(x,\min(\mu_A^-(x),\mu_B^-(x)),$

$\min(\mu_A^+(x),\mu_B^+(x)): x\in V\}$

Definition 2.2. An interval-valued fuzzy graph G is called complete if $\mu_B(xy) = \min(\mu_A(x),\mu_A(x))$ and $\upsilon_B(xy) =\min(\upsilon_A(x),\upsilon_A(x))$, for each edge $xy \in E$.

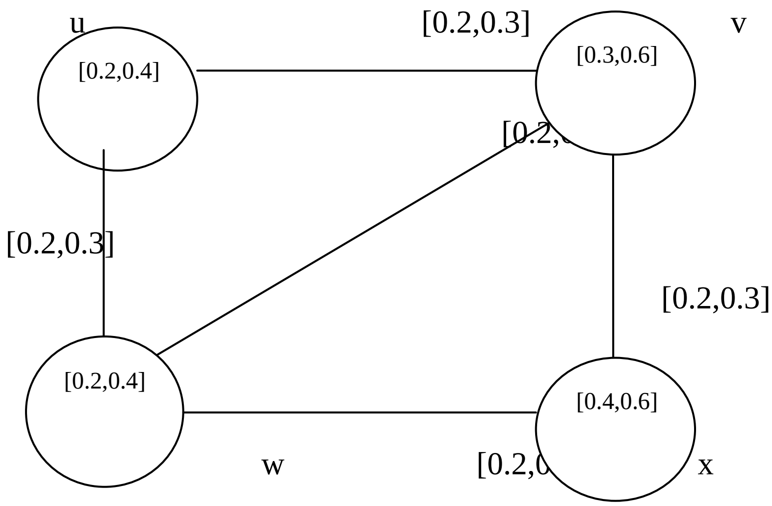
Definition 2.4. Let G be connected interval-valued fuzzy graph. The μ-distance, $\delta_\mu(v_i,v_j)$, is the smallest μ-length of any v_i-v_j path P in G, where $v_i,v_j \in V$. That is , $\delta_\mu(v_i,v_j) = \min\{L_\mu(p)\}$. The υ-distance , $\delta_\upsilon(v_i,v_j)$, is the largest υ-length of any v_i-v_j path P in G, where $v_i,v_j \in V$. That is, $\delta_\upsilon(v_i,v_j)=\max\{L_\upsilon(p)\}$. The distance, $\delta(v_i,v_j)$, is defined as $\delta(v_i,v_j) = [\delta_\mu(v_i,v_j), \delta_\upsilon(v_i,v_j)]$.

Definition 2.5. Let G be connected interval-valued fuzzy graph for each vi $\in V$, the μ-eccentricity of v_i, denoted by $e\mu(v_i)$, is defined as $e\mu(v_i) = \max \{ \delta_\mu(v_i,v_j)|v_i \in V, v_i \neq V_j\}$. For each $v_i \in V$, the υ- eccentricity of v_i, denoted by $e_\upsilon(v_i)$, is defined as $e_\upsilon(v_i) = \max \{ \delta_\upsilon(v_i,v_j)|v_i \in V, v_i \neq V_j\}$.For each $v_i \in V$, the eccentricity of v_i, denoted by $e(v_i)$, is defined as $e(v_i) = [e_\mu(v_i),e_\upsilon(v_i)]$.

Definition 2.6. Let G be connected interval-valued fuzzy graph. The μ-radius of G is denoted by $r_\mu(G)$ and is defined as $r_\mu(G) = \min\{e_\mu(v_i)|v_i \in V\}$. The υ -radius of G is denoted by $r_\upsilon(G)$ and is defined as $r_\upsilon(G) = \min\{e_\upsilon(v_i)|v_i\in V\}$.The radius of G is denoted by r(G) and is defined a $r(G) = [r_\mu(G),r_\upsilon(G)]$.

Definition 2.7. Let G be a connected interval-valued fuzzy graph. The μ-diameter of G is denoted by $d_\mu(G)$ and is defined as $d_\mu(G) = \max\{ e_\mu(v_i)|v_i \in V\}$. The υ-diameter of G is denoted by $d_\upsilon(G)$ and is defined as $d_\upsilon(G) = \max\{ e_\upsilon(v_i)|v_i\in V\}$.The diameter of G is denoted by d(G) and is defined as $d(G) =[d_\mu(G),d_\upsilon(G)]$.

Example 2.1 Consider a connected interval-valued fuzzy graph G such that V ={u,v,w,x}, E={(w,x),(w,v),(w,u),(x,v),(u,v)}



(i) Distance $\delta(v_iv_j)$ is $\delta(w,u)$ = [0.2,0.9], $\delta(w,x)$ =[0.2,0.6], $\delta(w,v)$ = [0.2,0.6], $\delta(v,x)$ = [0.2,0.9], $\delta(x,u)$ = [0.4,0.9], $\delta(v,u)$ = [0.2 ,0.9].

(ii) The eccentricities of the vertices are

e(w) = [0.2,0.9], e(x) = [0.4,0.9], e(v) = [0.2,0.9], e(u) = [0.4,0.9].

(iii) Radius of G is [0.2, 0.9], diameter of G is [0.4,0.9].

3. IRREGULAR INTERVAL- VALUED FUZZY GRAPHS

Irregular fuzzy graphs are important as regular interval- valued fuzzy graphs.

Definition 3.1. Let G = (A,B) be an interval-valued fuzzy graph. The degree of a vertex 'v' in G, denoted by d(v), is defined as $d(v) = [\sum_{u,v\in E} \mu_B^-(uv), \sum_{u,v\in E} \mu_B^+(uv)]$.

Definition 3.2. Let G= (A,B) be an interval valued fuzzy graph. An edge = uv is called effective if $\mu_B^-(uv) = \mu_A^-(u) \wedge \mu_A^-(v)$ and $\mu_B^+(uv) = \mu_A^+(u) \wedge \mu_A^+(v)$, $\forall$ u,v∈E. It is denoted by $\mu_{Be}(uv) = [\mu_{Be}^-(uv), \mu_{Be}^+(uv)]$. The effective degree of a vertex v in G, denoted by $d_e(v)$, is defined as

$$d_e(v) = [\sum_{u,v\in E} \mu_{Be}^-(uv) , \sum_{u,v\in E} \mu_{Be}^+(uv)].$$

Definition 3.3. Let G = (A,B) be an interval- valued fuzzy graph. The neighborhood degree of a vertex v is defined as the sum of the interval –valued membership value of the neighborhood vertices of v, and is denoted by $d_N(v)$.

Definition 3.4. Let G = (A,B) be an interval- valued fuzzy graph. The closed neighborhood degree of a vertex v is defined as the sum of the interval –valued membership value of the neighborhood vertices of v, and including the interval- valued membership value of v, and is denoted by $d_N[v]$.

Definition 3.5. Let G=(A,B) be an interval- valued fuzzy graph A= $[\mu_A^-, \mu_A^+]$ and B= $[\mu_B^-, \mu_B^+]$ be two interval valued fuzzy sets on a non-empty finite set V and $E \subseteq V\times V$ respectively. G is said to be irregular interval-valued fuzzy graph if there exists a vertex which is adjacent to vertex with distinct degrees.

Definition 3.6. Let G= (A,B) be an interval-valued fuzzy graph. Then the order of G is denoted by O(G) and is defined by $O(G) = [O^-(G), O^+(G)]$ where

$$O^-(G) = \sum_{u\in V} \mu_A^-(u) \text{ and } O^+(G) = \sum_{u\in V} \mu_A^+(u)$$

Definition 3.7. Let $G = (A, B)$ be an interval-valued fuzzy graph. Then the size of G is denoted by S(G) and is defined by $S(G) = [S^-(G), S^+(G)]$ where

$$S^-(G) = \sum_{\substack{u,v\in E\\ u\neq v}} \mu_B^-(uv) \;\&\; S^+(G) = \sum_{\substack{u,v\in E\\ u\neq v}} \mu_B^+(uv)$$

Definition 3.8. An interval-valued fuzzy graph $G = (A, B)$ is strong if $\mu_B^-(uv) = \mu_A^-(u) \wedge \mu_A^-(v)$ and $\mu_B^+(uv) = \mu_A^+(u) \wedge \mu_A^+(v)$, $\forall$ u,v∈E.

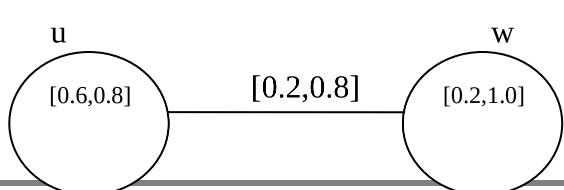
Definition 3.9. The density of an interval-valued fuzzy graphs G= (A,B) is $D(G) = [D^-(G), D^+(G)]$, where $D^-(G)$ and $D^+(G)$ is defined by

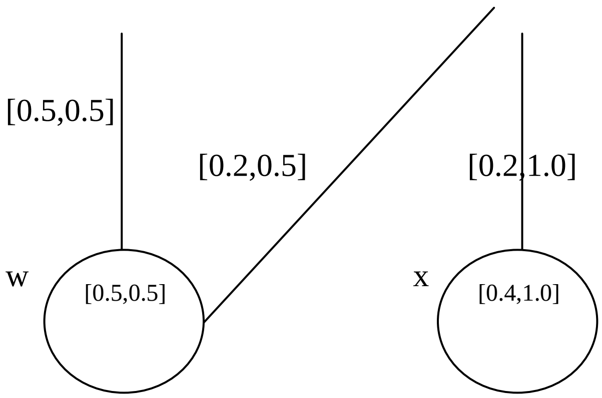
$$D^-(G) = \frac{2\sum_{u,v\in V} (\mu_B^-(uv))}{\sum_{(u,v)\in E} (\mu_A^-(u) \wedge \mu_A^-(v))} \qquad \text{for all } u,v\in E$$

$$D^+(G) = \frac{2\sum_{u,v\in V} (\mu_B^+(uv))}{\sum_{(u,v)\in E} (\mu_A^+(u) \wedge \mu_A^+(v))} \qquad \text{for all } u,v\in E$$

Definition 3.10. An interval-valued fuzzy graph G = (A,B) is balanced if $D(H) \leq D(G)$ that is $D^-(H) \leq D^-(G)$, $D^+(H) \leq D^+(G)$, for all sub graphs H of G.

Example 3.1. Let G = (A,B) be an interval valued fuzzy graph, where V={u,v,w,x} and E={uv,uw,vw,wx}. To find the O(G), S(G). Check the given graph is strong irregular interval valued fuzzy graph and also check the given graph is balanced or not.





Solution: The order of the graph G is

$O(G) = [O^{-}(G), O^{+}(G)] = O(G) = [1.7,3.3]$

The size of the graph G is $S(G) = [S^{-}(G), S^{+}(G)]$ $S(G) = [1.1, 2.8]$

The degree of the graph G is

$d(u) = [0.7,1.3]$, $d(v) = [0.7,1.0]$,

$d(w) = [0.4,2.3]$, $d(x) = [0.2,1.0]$

$d_e(u) = [0.7,1.3]$, $d_e(v) = [0.7,1.0]$,

$d_e(w) = [0.4,2.3]$, $d_e(x) = [0.2,1.0]$

$d_N(u) = [0.7,1.5]$, $d_N(v) = [0.8,1.8]$,

$d_N(w) = [1.5,2.3]$, $d_N(x) = [0.2,1.0]$,

$d_N[u] = [1.3,2.3]$, $d_N[v] = [1.3,2.3]$,

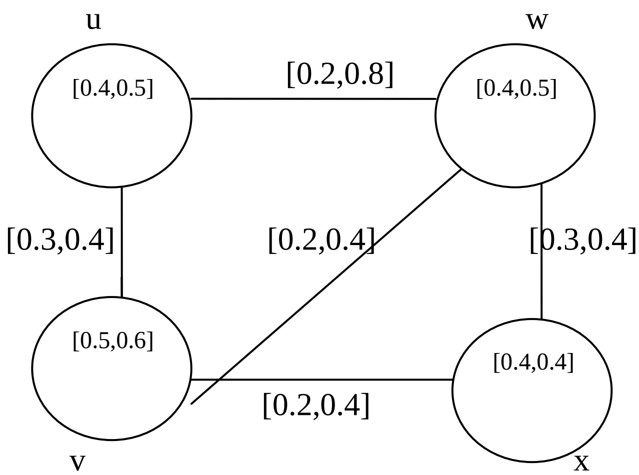
$d_N[w] = [1.7,3.3]$, $d_N[x] = [0.6,2.0]$,

The above graph contains different degree for each vertex. So it is irregular interval-valued fuzzy graph.

The graph is strong if $\mu_B^-(uv) = \mu_A^-(u) \wedge \mu_A^-(v) = 0.5$, $\mu_B^-(vw) = \mu_A^-(v) \wedge \mu_A^-(w) = 0.2$, $\mu_B^-(uw) = \mu_A^-(u) \wedge \mu_A^-(w) = 0.2$, $\mu_B^-(wx) = \mu_A^-(w) \wedge \mu_A^-(x) = 0.2$, $\mu_B^+(uv) = \mu_A^+(u) \wedge \mu_A^+(v) = 0.5$, $\mu_B^+(vw) = \mu_A^+(v) \wedge \mu_A^+(w) = 0.5$, $\mu_B^+(uw) = \mu_A^+(u) \wedge \mu_A^+(w) = 0.8$, $\mu_B^+(wx) = \mu_A^+(w) \wedge \mu_A^+(x) = 1.0$,

The density of G is denoted by is $D(G) = [D^-(G), D^+(G)]$, The graph is balanced if $D(H) \leq D(G)$ that is $D^-(H) \leq D^-(G)$, $D^+(H) \leq D^+(G)$, $D(G) = [2,2]$. Let $H_1 = \{u,v\}$, $H_2 = \{u,w\}$, $H_3 = \{u,x\}$, $H_4 = \{v,w\}$, $H_5 = \{v,x\}$, $H_6 = \{w,x\}$, $H_7 = \{u,v,w\}$, $H_8 = \{u,w,x\}$, $H_9 = \{u,v,x\}$, $H_{10} = \{v,w,x\}$, $H_{11} = \{u,v,w,x\}$ be a non-empty sub graphs of G. Densities of these subgraphs are $D[H_1] = [2,2]$, $D[H_2] = [2,2]$, $D[H_3] = [0,0]$, $D[H_4] = [2,2]$, $D[H_5] = [0,0]$, $D[H_6] = [2,1]$, $D[H_7] = [2,2]$, D[H_8] = [2,2], D[H_9] = [2,2], D[H_{10}] = [2,2], D[H_1] = [2,2]. The condition D(H) ≤ D(G) is satisfied for all subgraphs H of G. Hence G is a balanced interval valued fuzzy graphs.

Example 3.2. Let G=(A,B) be an interval valued fuzzy graph, where V={u,v,w,x} and E={uv,uw,vw,wx}.



d(u) = [0.3,0.4], d(v) = [0.8,1.2], d(w) = [0.5,0.8], d(x) = [0.5,0.8]. Here the interval valued fuzzy graph is highly irregular but not neighbourly irregular as d(w) = d(x) .

Corollary 3.1. In any interval-valued fuzzy graphs G = (A,B), the following conditions hold:
(i) $\sum_{v_i \in V} d(v_i) \geq \sum_{v_i \in V} d_e(v_i)$
(ii) $\sum_{v_i \in V} d_N(v_i) \leq \sum_{v_i \in V} d_N[v_i]$

Proof. Let G = (A,B) be an interval valued fuzzy graph. The degree of the graph if defined by d(v)=[$\sum_{u,v \in E} \mu_B^-(uv)$, $\sum_{u,v \in E} \mu_B^+(uv)$]. Consider the example 3.1. In that graph we found that

$\sum_{v_i \in V} d(v_i)$ = [2,5]. …(1)

The effective degree of a vertex v in G is defined as

$d_e(v)$ = [$\sum_{u,v \in E} \mu_{Be}^-(uv)$, $\sum_{u,v \in E} \mu_{Be}^+(uv)$].

In that example we found that

$\sum_{v_i \in V} d_e(v_i)$ = [2,5] …(2)

From (1) and (2) we get $\sum_{v_i \in V} d(v_i) = \sum_{v_i \in V} d_e(v_i)$

Therefore condition (i) holds.

Considering the example

$\sum_{v_i \in V} d_N(v_i)$ = [3.2, 6.6] …(3)

$\sum_{v_i \in V} d_N[v_i]$ = [4.9, 9.9] …(4)

From (3) and (4) we get

$$\sum_{v_i \in V} d_N(v_i) \leq \sum_{v_i \in V} d_N[v_i]$$

Therefore condition (ii) holds.

Corollary 3.2. Every strong interval –valued fuzzy graph is balanced.

Proof. The proof of this corollary is obvious from the example 3.1.An interval-valued fuzzy graph is said to be strong and balanced if it satisfies the conditions $S(G) = [S^{-}(G), S^{+}(G)]$ and $D(H) \leq D(G)$. The example 3.1 satisfies both this condition. So that Every strong interval–valued fuzzy graph is balanced.

Proposition 3.1. In any interval-valued fuzzy graph G = (A,B), the following inequalities hold:

(i) $O^{-}(G) \geq S^{-}(G)$ (ii) $O^{+}(G) \geq S^{+}(G)$

Proof. Let G = (A,B) be an interval valued fuzzy graph. Consider the example 3.1,

$O(G) = [O^{-}(G), O^{+}(G)] = [1.7, 3.3]$ …(5)

$O^{-}(G) = 1.7$, $O^{+}(G) = 3.3$

$S(G) = [S^{-}(G), S^{+}(G)] = [1.1, 2.8]$ …(6)

$S^{-}(G) = 1.1$, $S^{+}(G) = 2.8$

From (5) and (6), we get

$O^{-}(G) \geq S^{-}(G)$ and $O^{+}(G) \geq S^{+}(G)$.

Hence (i) and (ii) holds.

Proposition 3.2. In any interval-valued fuzzy graph G = (A,B), the following inequalities hold:

(i)$d(v_i) \geq d_e(v_i)$;(ii)$d_N(v_i) \geq d_N[v_i]$for any $v_i \in V$

Proof. Let G = (A,B) be an interval valued fuzzy graph. Consider the example 3.1, in that graph G contains four vertices {u,v,w,x}. Each vertex has degree that is equal to the effective degree of those vertices then the inequality (i) holds. In that graph the neighborhood degree of a vertex v is defined as $d_N(v_i)$ and closed neighborhood degree of a vertex v is defined as $d_N[v_i]$. In that example each $d_N(v_i)$ less than the $d_N[v_i]$. Then the (ii) inequality holds.

Proposition 3.3. In a strong interval-valued fuzzy graph G = (A,B), $d(v_i) = d_e(v_i)$ for all $v_i \in V$.

Proof. Let G = (A,B) be an interval valued fuzzy graph. Consider the example 3.1, in that graph G contains four vertices {u,v,w,x}. Each vertex has degree that is equal to the effective degree of those vertices. That is in the above graph we have d(u) = d_e(u), d(v) = d_e(v) , d(w) = d_e(w), d(x) = d_e(x). In a strong interval-valued fuzzy graph it should satisfies the condition $d(v_i) = d_e(v_i)$ for each vertices in graph.

Theorem 3.1. Let G= (A,B) be an interval- valued fuzzy graph. Then G is a highly irregular interval valued fuzzy graph and neighbourly irregular interval valued fuzzy graph if and only if the degrees of all vertices of G are distinct.

Proof. Let G = (A,B) be an interval-valued fuzzy graphs where A= $[\mu_A^-, \mu_A^+]$ and B= $[\mu_B^-, \mu_B^+]$ be two interval- valued fuzzy sets on a non-empty finite set V and V×V respectively. Let V = $\{v_1, v_2, v_3, \ldots.. v_n\}$. Assume that G is highly irregular and neighbourly irregular interval-valued fuzzy graphs. Let the adjacent vertices of $u_1$be $u_2,u_3,\ldots.,u_n$ with degrees $[k_2^-, k_2^+]$, $[k_3^-, k_3^+]$,…$[k_n^-, k_n^+]$ respectively. As G is highly irregular, $d(u_1) \neq d(u_2) \neq d(u_3) \ldots.. \neq d(u_n)$. So it is clear that all vertices are of distinct degrees.

Conversely, assume that the degrees of all vertices of G are distinct. That is every two adjacent vertices have distinct degrees and to every vertex the adjacent vertices have distinct degrees. Hence G is neighbourly irregular and highly irregular interval-valued fuzzy graphs.

4 CONCLUSION

In this paper described the order, size of the interval-valued fuzzy graphs. The interval- valued fuzzy graph model gives better accuracy compared to the classical fuzzy models. In this paper the concept of the strong interval-valued fuzzy graphs, irregular interval-valued fuzzy graphs and balanced interval-valued fuzzy graphs are discussed. Some relations about the defined graphs have been presented.

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