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DECOMPOSITION OF α-CONTINUITY , α*G-CONTINUITY AND G#CLOSED SETS IN TOPOLOGY

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ABSTRACT

In this paper , α*g closed ,Dη* , Dη**,$\hat{\eta}$ sets are defined and a schematic representation is obtained relating various open sets. Using these sets , α*g , Dη* , Dη**, $\hat{\eta}$ continuities are defined and finally decompositions ofα-continuity in terms of D*η*-continuity and α*g continuity ,α*g-continuous in terms of D*η*continuous and $\hat{\eta}$ continuous and a decomposition of continuity using $g^{\#}$ continuous and $g^{\#}$ lc* continuous are obtained.

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INTRODUCTION :

In 1961 Levine gave a decomposition of continuity as a mapping f : X → Y is continuous if and only if it is weakly continuous and weak* continuous. After Levine's decomposition of continuity mathematician gave in several papers different and interesting new decomposition of continuous as well as generalized continuous functions. Noiri and Sayed [10] introduced the notions of η-sets and obtained some decomposition of continuity ,Veera Kumar [15] introduced the notions of α*g-closed sets. Recently , Palaniappan [11] introduced and studied the notions of $\hat{n}$-closed sets. In this paper I investigate the properties of α-closed, Dη*-set, Dη**-set and α*g-closed.and a decompositions of α-continuity , α∗g-continuity and$g^{\#}$ closed sets are studied.

1.DECOMPOSITION OF α-CONTINUITY AND α∗g-CONTINUITY

Definition :1.1

A subset A of a space X is called

(1) an **η-set** [10] if A = V⋂T where V is open and T is an α-closed set.

(2) an **α*g-closed** [1,15] if αcl(A) ⊂ U whenever A ⊂ U and U is $\hat{g}$-open in X. The complement of α*g- closed

set is called α*g open.

(3) an **$\hat{n}$-closed** [61] if pcl(A) ⊂ U whenever A ⊂ U and U is $\hat{g}$-open in X.

The complement of $\hat{n}$-closed set is called $\hat{n}$-open . The collection of all α*g-open (resp. $\hat{n}$-open) sets in X will be denoted by α*g O(X) (resp. $\hat{n}$O(X)).

Definition :1.2

A subset A of a space X is said to be

(1) an **Dη*-set** if A = U ⋂T where U is $\hat{g}$-open and T is α-closed in X.

(2) an **Dη**-set** if A = U ⋂T where U is α * g-open and T

is a t-set in X.

The collection of all Dη*-sets (resp. Dη**-sets) in X will be denoted by Dη*(X) (resp. η**(X)).

Theorem :1.1

For a subset A of a space X, the following are equivalent

(1) A is an $D\eta^*(X)$.

(2) A=U ⋂cl(A) for some ĝ-open set U.

Proof:

(1)⇒(2)

Since A is an $D\eta^*(X)$, then A = U ⋂T, where U is ĝ -open and T is αclosed.

So A⊂ U and A ⊂ T.

Hence α cl(A) ⊂ α cl(T).

Therefore A ⊂ U ⋂αcl(A)

⊂ U⋂αcl(T)

= U ⋂ T

= A.

Thus A = U ⋂ α cl(A).

(2)⇒(1)

It is obvious because αcl(A) is α-closed.

Remark :1.1

In a space X, the intersection of two $D\eta^{**}$-sets is an $D\eta^{**}$-sets.

Remark :1.2

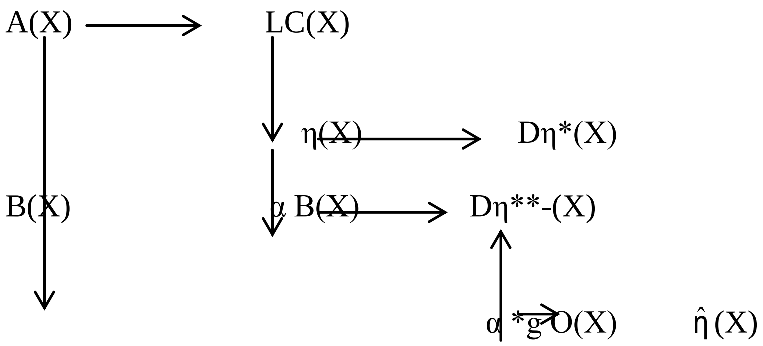
Since the union of t-sets need not be a t-set , the union of two $D\eta^{**}$-sets need not be an $D\eta^{**}$-sets as seen from the following examples.

Example :1.1

Let X = {a, b, c} ,τ = {X , ϕ, {b, c}}.The sets {a} and {b} are $D\eta^{**}$ -sets in (X, τ) but their union {a, b} is not an $D\eta^{**}$ -set in (X, τ).

Remark :1.3

We have the following implications



Where none of these implications is reversible as shown in the following examples.

Example :1.2

(i) Let X = {a, b, c} and τ = {X, ϕ, {b, c}}. Then {b} is an Dη*- set but not an η-set in (X, τ).

(ii) Let X = {a, b, c} and τ = {X, ϕ, {b, c}}. Then set {a} is an Dη**-set but not an α*g -open set in (X, τ).

(iii) Let X = {a, b, c} and τ = {X, ϕ, {b, c}}.Then {b} is an Dη** set but an αB-set in (X, τ).

Remark :1.4

(1) The notions of Dη*-sets and α*g-closed sets are independent.

(2) The notions of Dη**-sets and n̂ -open sets are independent.

Example :1.3

Let X = {a, b, c} and τ = {X ,ϕ, {b, c}}. Then {a, b} is α*g closed but not an Dη*-set and the set {b} is an Dη*-set but not an α*g-closed in (X, τ).

Example :1.4

Let X = {a, b, c} and τ= {X, ϕ, {b, c}}.Then the set {a} is an Dη**-set but not an n̂ -open set and and also that {a, b} is an n̂ -open set but not an Dη**-set (X, τ).

Theorem :1.2

For a subset A of a space X, the following are equivalent

(1) A is α-closed

(2) A is an Dη*-set and α*g-closed.

Proof :

The proof of (1) ⇒(2) follows from the definitions.

(2) ⇒(1)

Since A is an Dη*-set, by Definition 1.2 , A = U ∩ αcl(A) where U is ĝ-open in X. So A ⊂ U and since A is α*g-closed, then αcl(A) ⊂ U .

Therefore, αcl(A) ⊂U ∩ αcl(A) = A.

Hence A is α-closed.

Proposition :1.1

Let A and B be subsets of a space X. If B is an α*-set, then αint (A ∩B) = αint(A) ∩ int(B).

Proof :

B is an α*-set ⇒int B = int cl int B.

Now A∩ B ⊆ B. Therefore α int (A ∩ B) = α int A ∩ α int B. But αint B is α-open and so (α int B) ⊆int cl int B = int B and in general int B ⊆ α int B.

Therefore α int B = int B.

Hence α int (A ∩ B) = αint A ∩int B.

Theorem :1.3

For a subset S of a space X, the following are equivalent:

(1) S is α*g-open.

(2) S is a Dη**-set and n̂-open.

Proof:

Necessity:

The proof follows from the definitions.

Sufficiency:

Assume that S is n̂-open and an Dη**-set in X. Then S = A ∩ B where A is α*g-open and B is a t-set in X.

Let F ⊂S, where F is ĝ-closed in X. Since S is n̂-open in X,

$F \subset pint(S) \quad = \quad S \cap int(cl(S))$

$= (A \cap B) \cap int[cl(A \cap B)]$

$\subset A \cap B \cap int(cl(A)) \cap int(cl(B))$

$= A \cap B \cap int(cl(A)) \cap int(B),$

since B is a t-set.

This implies, $F \subset int(B)$. As A is α*g-open and that $F \subset A$. We get $F \subset \alpha\text{-}int(A)$. Therefore, $F \subset \alpha\text{-}int(A) \cap int(B) = \alpha\ int(S)$ by Proposition 1.1.

Hence S is α*g-open.

2.DECOMPOSITION OF CONTINUITY USING $g^{\#}$ CLOSED SETS

Definition :2.1

A function $f :(X, \tau) \rightarrow (Y, \sigma)$ is said to be **$g^{\#}$- continuous** if for each closed set V of Y, $f^{-1}(V)$ is $g^{\#}$-closed in X.

Proposition :2.1[14]

Every closed set is $g^{\#}$-closed but not conversely.

Proposition :2.2[14]

Every continuous map is $g^{\#}$–continuous but not conversely.

Definition :2.2

A subset A of a space (X, τ) is called **$g^{\#}$lc* -set** if $A = M \cap N$, where M is αg-open and N is closed in(X, τ).

Example :2.1

Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{b\}, X\}$. Then $\{a\}$ is $g^{\#}$lc* -set in (X, τ).

Remark : 2.1

Every closed set is $g^{\#}$lc*-set but not conversely.

Example :2.2

Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{c\}, X\}$. Then $\{b, c\}$ is $g^{\#}$lc* -set but not closed in (X, τ).

Remark :2.2

$g^{\#}$-closed and $g^{\#}lc^*$-sets are independent of each other.

Example :2.3

Let X = {a, b, c} with τ = {ϕ, {b, c}, X}.Then {a, b} is an $g^{\#}$-closed set but not $g^{\#}lc^*$ -set in (X, τ).

Example :2.4

Let X = {a, b, c} with τ = {ϕ, {b}, X}.Then {a, b} is an $g^{\#}lc^*$ -set but not$g^{\#}$–closed set in (X, τ).

Proposition :2.3

Let (X, τ) be a topological space. Then a subset A of (X, τ) is closed if and only if it is both $g^{\#}$-closed and $g^{\#}lc^*$-set.

Proof:

Necessity is trivial.

To prove the sufficiency,

Assume that A is both $g^{\#}$-closed and $g^{\#}lc^*$-set. Then A = M ⋂ N, where M is αg-open and N is closed in (X, τ). Therefore, A ⊆ M and A ⊆ N and so by hypothesis, cl(A) ⊆ M and cl(A) ⊆ N. Thus cl(A) ⊆ M ⋂ N = A.

Hence cl(A) = A.

(i.e.) A is closed in (X, τ).

Definition :2.3

A mapping f : (X, τ) →(Y, σ) is said to be **$g^{\#}lc^*$-continuous** if for each closed set V of (Y, σ), $f^{-1}(V)$ is a $g^{\#}lc^*$-set in (X, τ).

Example :2.5

Let X = Y = {a, b, c} with τ = {ϕ, {a}, X} and σ = {ϕ, {a}, {b, c}, Y}.

Let f : (X, τ) →(Y, σ) be the identity map. Then f is $g^{\#}lc^*$- continuous map.

Remark :2.3

Every continuous map is $g^{\#}lc^*$-continuous but not conversely.

Example :2.6

Let X = Y = {a, b, c} with τ = {ϕ, {b}, X} and σ = {ϕ, {b}, {a, c}, Y}.

Let f : (X, τ)→(Y, σ) be the identity map. Then f is $g^{\#}lc^*$ -continuous map. Since for the closed set {b} in (Y, σ),f^{-1}(V) = {b}, which is not closed in (X, τ), f is not continuous.

Remark :2.4

$g^{\#}$-continuity and $g^{\#}lc^*$ -continuity are independent of each other.

Example :2.7

Let X = Y = {a, b, c} with τ = {ϕ, {a, b}, X} and σ = {ϕ, {a}, Y}.

Let f: (X, τ) → (Y, σ) be the identity map. Then f is $g^{\#}$-continuous but not$g^{\#}lc^*$-continuous.

Example : 2.8

Let X = Y = {a, b, c} with τ = {ϕ, {b}, X} and σ = {ϕ, {b, c}, Y}. Let f : (X, τ) →(Y,σ) be the identity map. Then f is $g^{\#}lc^*$-continuous but not $g^{\#}$-continuous.

The decomposition of continuity is given using a generalized continuity is obtained in the following theorem.

DECOMPOSITION OF CONTINUITY

Theorem :2.1

A mapping f : (X, τ) → (Y, σ) is continuous if and only if it is both $g^{\#}$-continuous and $g^{\#}lc^*$-continuous.

Proof:

Assume that f is continuous.

Then by Proposition 2.2 and Remark 2.3,

f is both $g^{\#}$-continuous and $g^{\#}lc^*$-continuous.

Conversely, assume that f is both $g^{\#}$- continuous and $g^{\#}lc^*$-continuous. Let V be a closed subset of (Y, σ). Then f^{-1}(V) is both $g^{\#}$-closed and $g^{\#}lc^*$-set.

As in Proposition 2.3, we prove that $f^{-1}(V)$ is a closed set in (X, τ) and so f is continuous.

CONCLUSION :

The main aim of this paper is to discuss the decompositions of continuity in topological spaces. The definitions and results which are needed for the course of this paper is also discussed.Finally the decomposition of continuity in α-continuity, α∗g-continuity and $g^{\#}$ closed sets are obtained.

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