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CERTAIN STRUCTURES OF FUZZY SOFT G-MODULES

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Abstract:

In this paper, we study soft set theory initiated by Molodtsov [1]. we introduce the notions of fuzzy version of soft G-module and obtain basic properties of such soft G-modules using Molodtsov's definition of the soft sets. We study the concept fuzzy soft G-modules over a commutative ring with respect to t- norm. Some Properties of fuzzy soft G-modules are investigated. In Particular, we consider properties of intersection and direct product for fuzzy soft G-modules. Moreover we examine irreducibility, reducibility and complete reducibility of fuzzy soft G-modules.

Keywords: Fuzzy soft Set, , Soft Module, G-module, fuzzy Soft G Module , Direct product, T-norm.

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1. Introduction The concept of soft set theory is introduced by Molodtsov to overcome uncertainties which cannot be dealt with by classical methods in many areas such as engineering, economics, medical science and social science. At present, work on the soft set theory is progressing rapidly. P.K.Maji at all [2] defined basic properties of soft set theory. Aktaş and Çağman [3] compared to soft sets to the related concepts of fuzzy sets and rough sets and introduced soft group and derived their basic properties. Sujoy Das and S.K. Samanta [4-6] studied soft real sets, soft real numbers, soft complex sets, soft complex numbers and soft metric. Soft linear spaces and soft norm on soft linear spaces are given and some of their properties are studied by Samanta, Das ve P. Majumdar [7]. In [8] Q. Sun, Z. Zang and J. Liu, introduced the definition of soft modules and constructed some basic properties of soft modules.

Module theoretic approach is better suited to deal with deeper results in representation theory. Moreover, module theoretic approach gives more elegance to the theory. In particular, the G-module structure has been extensively used for the study of representations of finite groups. Group theory is now factored into two parts. First, there is a study of the structure of abstract groups. Second is the companion question: given a group G, how can we describe all the ways in which G may be embedded in a linear group GL(V)? This is the subject matter of representation theory [9,10,11]. Soon after the introduction of fuzzy set theory by L.A. Zadeh [12] in 1965, Rosenfield [13] initiated the fuzzification of algebraic structures. Recently, some researchers studied G-modules on fuzzy sets. As a continuation of these works S. Fernandez [14] introduced fuzzy parallels of the notions of G-modules, group representations, reducibility, irreducibility and completely reducibility and observe, some of their basic properties. In [15] A.K.Sinho and K. Dewangan studied isomorphism theorems for fuzzy submodules of G-modules. Recently, many authors have studied some algebraic structures of soft set theory. [16,17,18,19,20] Some interesting results in the theory of soft modules are still being explored currently. However the theory of soft modules has not yet been studied. The main purpose of this paper is to introduce soft parallels of the notions of G-modules, reducibility, irreducibility and completely reducibility and observe, some of their basic properties.

2.Preliminaries: In this section as a beginning, the concepts of G-module[36] soft sets introduced by Molodtsov [29] and the notions of fuzzy soft set introduced by Maji et al. [26] have been presented.

2.1 Definition [2] For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if

i) $A \subset B$ and,

ii) $\forall\ \mathcal{E} \in A$, $F(\mathcal{E})$ and $G(\mathcal{E})$ are identical approximations.We write $(F,A) \tilde{\subseteq} (G,B)$.

2.2 Definition [2] The union of two soft sets (F,A) and (G,B) over the common universe U is the soft set (H,C), where $C = A \cup B$ and for all $x \in C$,

$$H(x) = \begin{cases} F(x), & if\ x \epsilon A - B \\ G(x), & if\ x \epsilon B - A \\ F(x) \cup G(x), & if\ x \epsilon A \cap B \end{cases}$$

We write $(F,A) \tilde{\cup} (G,B) = (H,C)$.

2.3 Definition [2] The intersection of two soft sets (F,A) and (G,B) over the common universe U is the soft set (H,C) , where $C = A \cap B$ and for all $x \in C$, $H(x) = F(x)$ or $G(x)$,(as both are same set). We write $(F,A) \tilde{\cap} (G,B) = (H,C)$.

2.1.Theorem [13]: Let F_H and T_K be soft sets over U, F^r_H, T^r_K be their relative soft sets, respectively and ψ be a function from H to K. then, i) $\psi^{-1}(T^r_K) = (\psi^{-1}(T_K))^r$,

ii) $\psi(F^r_H) = (\psi^\star(F_H))^r$ and $\psi^\star(F^r_H) = (\psi(F_H))^r$.

2.4 Definition [3] Let G be a group and (F,A) be a soft set over G. Then (F,A) is said to be a soft group over G if $F(a)$ is a subgroup of G, for each $a \in A$.

Let V be a vector space over a field K and let A be a parameter set. A soft set (F, A) where $F : A \to \wp(V)$ will be denoted by F only.

2.5 Definition [7] Let V be a vector space over a field K and let A be a parameter set. Let G be a soft set over (V,A). Now G is said to be a soft vector space or soft linear space of V over K if $G(x)$ is a vector subspace of V, $\forall x \in A$.

2.1 Example [7] Consider the Euclidian n-dimensional space $\mathcal{R}^n$ over $\mathcal{R}$. Let $A = \{1, 2, 3, \ldots, n\}$ be the set of parameters. Let $G: A \to \wp(\mathcal{R}^n)$ be defined as follows:

$G(i) = \{t \in R^n ;\ \ i-\text{th coordinate of } t \text{ is } 0\}$, $i = 1, 2, \ldots, n$.

Then G is a soft vector space or soft linear space of $\mathcal{R}^n$ over $\mathcal{R}$.

2.6 Definition[7] Let F be a soft vector space of V over K. Let $G : A \to \wp(V)$ be a soft set over (V,A). Then G is said to be a soft vector subspace of F if

i) For each $x \in A$, $G(x)$ is a vector subspace of V over K and

ii) $F(x) \supseteq G(x)$, $\forall x \in A$.

2.7 Definition [9] Let G be a finite group. A vector space M over a field K is called a *G-module* if for every $g \in G$, $m \in M$, there exists a product (called the action of G on M) $m.g \in M$ satisfying the following axioms.

i) $m.1_G = m$, $\forall\, m \in M$ (1_G being the identity element in G)

ii) $m.(g.h) = (m.g).h$, $\forall\, m \in M$; $g, h \in G$

iii) $(k_1 m_1 + k_2 m_2).g = m_1(k_1.g) + k_2(m_2.g)$, $\forall\, k_1, k_2 \in K$; $m_1, m_2 \in M$; $g \in G$.

2.1 Remark [9] The operation $(m, g) \to m.g$ defined above may be called a right action of G on M and M may be said to a right *G-module*. We shall consider all *G-modules* as right *G-modules*.

2.2 Example [9] Let $G = \{1, -1, i, -i\}$ and $M = \mathbb{C}^n$ ($n \geq 1$). Then M is a vector space over $\mathbb{C}$ and under the usual addition and multiplication of complex numbers, we can show that M is a *G-module*.

2.3 Example [9] For any prime p, we have $M = (Z_p, +, .)$ is a field. Let G=M–{0}. Then under the field operations of M, it is a *G-module*.

2.4 Example [9] Let $G = \{1, -1\}$ and $\mathbb{Q}(\sqrt{2}) = \{x + y\sqrt{2}: \quad x, y \in \mathbb{Q}\}$. Then $\mathbb{Q}(\sqrt{2})$ is a vector space over $\mathbb{Q}$ and $\mathbb{Q}(\sqrt{2})$ is a *G-module*.

2.8 Definition [9] Let M be a *G-module*. A vector subspace N of M is a *G-sub module* if N is also a *G-module* under the same action of G.

2.9 Definition [9]Let M and M^* be *G-modules*. A mapping $\varphi : M \to M^*$ is a *G-module homomorphism* if

i) $\varphi(k_1.m_1 + k_2.m_2) = k_1.\varphi(m_1) + k_2.\varphi(m_2)$ and,

ii) $\varphi(m.g) = \varphi(m).g, \quad \forall\ k_1, k_2 \in K ; m, m_1, m_2 \in M ; g \in G$

Further if, φ is 1-1, then φ is an isomorphism. The *G-modules* M and M* are said to be isomorphic if there exists an isomorphism φ of M onto M*. Then we write M $\cong M^*$.

2.10 Definition [9] Let M be a nonzero *G-module*. Then M is irreducible if the only *G-sub modules* of M are M and {0}.Otherwise M is reducible**.**

2.11 Definition [10] Let $\boldsymbol{M_1}, \boldsymbol{M_2}, \boldsymbol{M_3}, \dots, M_n$ be vector spaces over a field K. Then the set $\{ m_1 + m_2 + \dots + m_n \ ; \ m_i \in M_i \}$ becomes a vector space over K under the operations

$$(m_1 + m_2 + \dots + m_n) + (m_1' + m_2' + \dots + m_n') = (m_1 + m_1') + (m_2 + m_2') + \dots + (m_n + m_n')$$

and

$$\alpha(m_1 + m_2 + \dots + m_n) = \alpha\, m_1 + \alpha\, m_2 + \dots + \alpha\, m_n \ ; \ \alpha \in K, \ m_n' \in M_i$$

It is the called direct sum of the vector spaces $\boldsymbol{M_1}, \boldsymbol{M_2}, \boldsymbol{M_3}, \dots, M_n$ and is denoted by $_{i=1}^{n}\bigoplus M_i$.

2.2 Remark [10] The direct sum $M = {}_{i=1}^{n}\bigoplus M_i$ of vector spaces M_i has the following properties.

i) Each element $m \in M$ has a unique expression as the sum of elements of M_i.

ii) The vector subspaces $\boldsymbol{M_1}, \boldsymbol{M_2}, \boldsymbol{M_3}, \dots, M_n$ of M are independent.

iii) For each $1 \leq i \leq n$, $M_j \cap (M_1 + M_2 + \ldots + M_{j-1} + M_{j+1} + \ldots + M_n) = \{0\}$.

iv)

2.12 Definition [9] A nonzero G-module M is completely reducible if for every G-sub module N of M there exists a G-sub module N* of M such that $M = N \oplus N^*$

2.1 Proposition[10] Let V be a finite dimensional vector space over a field F and let W_1 be any subspace of V. Then there exist a subspace W_2 of V such that $V = W_1 \oplus W_2$.

2.1 Corollary[9] All finite dimensional G- modules are completely reducible.

2.2 Proposition[9] A G-sub module of a completely reducible G-module is completely reducible.

3. Characterization of Fuzzy Soft G-Modules

3.1Definition: Let $\mu_{\tilde{A}} : U \rightarrow [0,1]$ be any function and A be a crisp set in the universe 'U'. Then the ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x) / x \in U\}$ is called a fuzzy set and $\mu_{\tilde{A}}$ is called a membership function.

3.2 Definition: By a t- norm 'T' , we mean a function $T: [0,1]\times [0,1]\rightarrow [0,1]$ satisfying the following conditions ;

(T1) $T(0,x) = 0$,(T2) $T(x,y) \leq T(x,z)$ if $y \leq z$

(T3) $T(x,y) = T(y,x)$,(T4) $T(x, T(y,z)) = T(T(x,y),z)$, for all $x,y,z \in [0,1]$.

3.1 Proposition : For a t-norm , then the following statement holds $T(x,y) \leq \min\{x,y\}$, for all $x,y \in [0,1]$.

3.3 Definition : Define

$T_n((x_1,x_2, \ldots, x_n),q) = T(x_i, T_{n-1}((x_1,x_2,\ldots x_{i-1}, x_{i+1},\ldots, x_n),q))$ for all $1 \leq i \leq n$, $n \geq 2$, $T_1 = T$. Also define $T_\infty(x_1,x_2,\ldots, q) = \lim T_n((x_1,x_2,\ldots,x_n),q)$ as $n \rightarrow \infty$.

3.4 Definition : By the intersection of fuzzy soft subsets A_1 and A_2 in a set X with respect to an t-norm T, we mean the fuzzy soft subset $A = A_1 \cap A_2$ in the set X such that for any x ε X $A(x,q) = (A_1 \cap A_2)(x,q) = T(A_1(x,q), A_2(x,q))$. By the intersection of a collection of fuzzy soft subsets $\{A_1, A_2, \dots\}$ in a set X with respect to a t- norm T, we mean the fuzzy soft subset $\cap A_i$ such that for any x ε A, $(\cap A_i)(x,q) = T_\infty(A_1(x,q), A_2(x,q), \dots)$.

3.5Definition : By the direct product of fuzzy soft sets $\{A_1, A_2, \dots\dots\}$ with respect to t- norm T we mean the fuzzy soft subset $A = \prod A_i$ such that

$A(x_1, x_2, \dots\dots x_n), q) = (\prod A_i)((x_1, x_2, \dots\dots x_n), q) = T_n(A_1(x_1, q), A_2(x_2, q), \dots\dots, A_n(x_n, q))$.

3.6.Definition: Let G be a group. Let M be a G-module of V and A_M be a fuzzy soft set over V.

Then A_M is called Intersection Fuzzy Soft G-module of V (IFSG-m), denoted by $A_M \mathrel{\tilde{<}_i} V$ if the following properties are satisfied

(FSG-m_1) $A(ax + by) \geq A(x) \cap A(y)$

(FSG-m_2) $A(\alpha x) \geq A(x)$, for all x, y∈ M, a,b, $\alpha \in$ F.

3.1Example: Let G = {1,-1} , M = R^4 over R. Then M is a G-module. Define A on M by,

$$A(x) = \begin{cases} 1, & \text{if } x_i = 0 \ \forall i. \\ 0.5, & \text{if atleast } x_i \neq 0. \end{cases}$$

Where $x = \{x_1, x_2, x_3, x_4\}$; $x_i \in$ R. Then A is a fuzzy soft G-Module.

3.2Example. Suppose that $G = A = \{1, -1\}$ and that we define the set valued function $F: G \to G$, $F(x) = \{y \in G : x = y^n,\ n \in \mathbb{N}\}$. Then the soft group (F,A) is a parametrized family $\{F(x) : x \in A\}$ of subsets, which gives us a collection of subgroups of G.

Consider the Euclidian n-dimensional space $M = \mathcal{R}^n$ over $\mathcal{R}$. Then M is a G-module. Let $B = \{1, 2, 3, \dots, n\}$ be the set of parameters. Let $V: B \to \wp(\mathcal{R}^n)$ be defined as follows:

$V(i) = \{t \in R^n$; i–th coordinate of t is 0} , $i = 1, 2, \dots, n$.

Then (V,B) is a soft vector space or soft linear space of $\mathcal{R}^n$ over $\mathcal{R}$.

Therefore (V,B) is a soft G-module (the action of (F,A) on (V,B)).

3.3 Example . Suppose that $G = A = \{1, -1, i, -i\}$ and that we define the set valued function $F{:}A \rightarrow G$, $F(x) = \{y \in G : x = y^n\ ,\ n \in \mathbb{N}\}$. Then (F,A) is a soft group on G.

Consider the n-dimensional space $M = \mathbb{C}^n$ over $\mathbb{C}$. Then M is a G-module. Let B= {1, 2, 3, . . . , n} be the set of parameters and let S = $\{e_1,\ e_2, \ldots, e_n\}$ be a base for $\mathbb{C}^n$. Let V: $B \rightarrow \mathcal{P}(\mathbb{C}^n)$ be defined as $V(i) = \text{Span}\{e_i\}$, $\forall\, i \in B$. Then (V,B) is a soft vector space of $\mathbb{C}^n$ over $\mathbb{C}$.

Therefore (V,B) is a soft G-module (the action of (F,A) on (V,B)).

3.1Corollary . Let M be a fuzzy *G–module* (G is a finite group and M is a vector space over a field K). Let A and B be parameter sets, (F,A) be a fuzzy soft group over G and (V,B) be soft vector space over M. Then (V,B) can be define on a soft subgroup of (F,A) as a fuzzy soft G-module.

3.4 Example . Suppose that $G = A = \{1, -1\}$ and that we define the set valued function $F{:}A \rightarrow G$, $F(x) = \{y \in G : x = y^n\ ,\ n \in \mathbb{N}\}$. Then (F,A) is a fuzzy soft group on G.

Consider the n-dimensional space $M = \mathbb{C}^n$ over $\mathbb{C}$. Then M is a G-module. Let B= {1, 2, 3, . . . , n} be the set of parameters and let S = $\{e_1,\ e_2, \ldots, e_n\}$ be a base for $\mathbb{C}^n$. Let V: $B \rightarrow \mathcal{P}(\mathbb{C}^n)$ be defined as $V(i) = \text{Span}\{e_i\}$, $\forall\, i \in B$. Then (V,B) is a soft vector space of $\mathbb{C}^n$ over $\mathbb{C}$.

Therefore (V,B) is a fuzzy soft G-module (the action of (F,A) on (V,B)).

3.7 Definition . Let (F,A) and (H,B) be two fuzzy soft G-modules over M. Then (H,B) is soft G-sub module of (F,A) if

i) $B \subseteq A$,

ii) $H(x) < F(x)$, $\forall\, x \in B$,

This is denoted by $(H,B) \precsim (F,A)$

3.8 Definition. Let (V,A) is any fuzzy soft G-module over M and let $(V_0$,B) is a soft G-submodule over {0} subspace of (V,A). Then two fuzzy soft G-sub modules (V,A) and (V_0,B) of (V,A) called trivial fuzzy soft G-sub modules.

3.5 Example . Suppose that $G = A = \{1, -1\}$ and that we define the set valued function $F{:}A \rightarrow G$, $F(x) = \{y \in G : x = y^n\ ,\ n \in \mathbb{N}\}$. Then (F,A) is a fuzzy soft group on G.

Consider the Euclidian n-dimensional space $M = \mathcal{R}^n$ over $\mathcal{R}$

Consider the n-dimensional space $N = \mathbb{C}^n$ over $\mathbb{C}$. Then *M* and *N* are *G*-module. Let *B*= {1, 2, 3, . . . , *n*} be the set of parameters and let S = {e_1, e_2, ..., e_n } be a base for $\mathcal{R}^n$ and let S' = {e'_1, e'_2, ..., e'_n } be a base for $\mathbb{C}^n$.Let *V* : B → $\mathcal{P}(\mathcal{R}^n)$ and *W*: *B* → $\mathcal{P}(\mathbb{C}^n)$ be defined as *V*(i) = Span{e_i}, *W*(*i*) = Span{e'_i} , ∀ i ϵ B with respectively. Then (*V*,*B*) is a fuzzy soft *G*-sub module of (*W*,*B*).

3.8 Definition . Let (*F*,*A*) and (*H*,*B*) be two fuzzy soft *G*-modules over *M* and *N* respectively. Let *f*:*M*→*N* and *g*:*A*→*B* be two functions. Then we say that (*f*,*g*) is a fuzzy soft *G*-module homomorphism is the following conditions are satisfied:

i) *f: M* →*N* is a *G*-module homomorphism.

ii) *g* :A→B is a mapping;

iii) *f*(*F*(*x*)) = *H*(*g*(*x*)); ∀ *x* ϵ *A*

In this definition, if *f* is an isomorphism from *M* to *N* and *g* is a one-to-one mapping from *A* onto *B*, then we say that (*f*,*g*) is a fuzzy soft G-module isomorphism and that (*F*,*A*) is isomorphic to (*H*,*B*), this is denoted by (*F*,*A*) ≅ (*H*,*B*).

3.6 Example . Consider (*V*,*B*) and (*W*,*B*). are two fuzzy soft *G*-module over $M = \mathcal{R}^n$ and $N = \mathbb{C}^n$ respectively in Example 3.8.

i) *f*: $\mathbb{C}^n \to \mathcal{R}^n$, *f*(Span{$e'_i$}) =Span {$e_i$}, a *G*-module homomorphism

ii) *A* = *B* = {1, 2, 3, … ,n} *g: A* → *B* , *g*(*x*) = *x*, identity mapping

iii) *f*(*W*(i)) = *V*(*g*(i)) ∀ i ϵ A .

Hence (*f*,*g*) is a fuzzy soft *G*-module homomorphism. In addition (*V*,*A*) ≅ (*W*,*B*)..

3.2 Proposition. Let (F,A) and (*H*,*B*) be two fuzzy soft *G*-modules over *M*. Then (*F*,*A*)$\tilde{\cap}$ (*H*,*B*) is a fuzzy soft *G*-module over *M*.

Proof: From Definition 2.3 we know that (*F*,*A*)$\tilde{\cap}$ (*H*,*B*)=(K, C) is a fuzzy soft set over *M*, where *C*=*A* ∩ *B* and *K*(i) = *H*(i) or *K*(i) = *F*(i) for all i ϵ *C*= *A* ∩ *B*. (G,B)$\tilde{\cap}$(F,A) is a fuzzy soft *G*-module over *M* since (*G*,*B*) and (*F*,*A*) are fuzzy soft *G*-module over *M*.

3.3 Proposition . Let (F,A) and (H,B) be two fuzzy soft modules over M. Then $(F,A)\ \tilde{\cup}\ (H,B)$ is a fuzzy soft G-module over M if $A \cap B = \emptyset$.

Proof: From definition 2.4 we know that $(H,B)\tilde{\cup}(F,A) = (K,C)$ is a fuzzy soft set, where $C = A \cup B$ and

$$K(x) = \begin{cases} F(x), & x\epsilon A - B \\ H(x), & x\epsilon B - A \\ F(x) \cup H(x), & x\epsilon A \cap B \end{cases}$$

$x\epsilon A - B$ or $x\epsilon B - A$ As $A \cap B = \emptyset$, thus (K,C) is a fuzzy soft G-module over M.

3.9 Definition. Let $\{(F_i, B_i)$: i = 1, 2, 3, …, n$\}$ be a nonempty family of fuzzy soft G-modules M_i over K. Then the set $\{ F_1(x_{i_1}) + F_2(x_{i_2}) + \ldots + F_n(x_{i_n}):\ x_{i_k} \in B_i \}$ becomes a soft vector space over K under the operations

$$[F_1(x_{i_1}) + F_2(x_{i_2}) + \ldots + F_n(x_{i_n})] + [F_1(x'_{i_1}) + F_2(x'_{i_2}) + \ldots + F_n(x'_{i_n})] =$$

$$[F_1(x_{i_1}) + F_1(x'_{i_1})] + [F_2(x_{i_2}) + F_2(x'_{i_2})] + \ldots + [F_n(x_{i_n}) + F_2(x'_{i_n})] \quad \text{and}$$

$$\alpha[F_1(x_{i_1}) + F_2(x_{i_2}) + \ldots + F_n(x_{i_n})] = \alpha F_1(x_{i_1}) + \alpha F_2(x_{i_2}) + \ldots + \alpha F_n(x_{i_n})\ ;\ \alpha \in K,\ x'_{i_k}, x_{i_k} \in B_i$$

It is called the direct of the soft vector spaces $\{(F_i, B_i)$: i = 1, 2, 3, …, n$\}$ and it is denoted by $\widetilde{\bigoplus}_{i=1}^{n}(F_i, B_i)$.

The notion of direct sum extends to fuzzy soft G-modules since fuzzy soft G-modules are fuzzy soft vector spaces

3.4 Remark. The direct sum $(F,B) = \widetilde{\bigoplus}_{i=1}^{n}(F_i, B_i)$ of soft vector spaces $\{(F_i, B_i)$: i= 1, 2, 3, …, n$\}$ has the following properties.

i) Each element F(i)ϵ (F,B) has a unique expression as the sum of element of (F_i, B_i), where i ϵ B

ii) The soft vector spaces $\{(F_i, B_i)$: i = 1, 2, 3, …, n$\}$ are independent.

iii) $B = \cup_{i=1}^{n} B_i$

iv) For each $1 \le j \le n$, $1 \le k \le n$, $1 \le i \le n$

$B_j \cap (B_1 + B_2 + \ldots + B_{j-1} + B_{j+1} + \ldots + B_n) = \emptyset$ and

For each $i_k \epsilon B_i$ $F_j(i_j) \cap \{(F_1(i_1) + F_2(i_2) + \ldots + F_{j-1}(i_{j-1}) + F_{j+1}(i_{j+1}) + \ldots + F_n(i_n)\} = \{0\}$

3.3 Corollary. Let (*F*, *B*) is a fuzzy soft *G*-module over *M*, and $\{(F_i, B_i)\colon i \epsilon I\}$ be a nonempty family of fuzzy soft G-sub modules of (*F*,*B*). Then

i) $\tilde{\cap}_{i=1}^{n}(F_i, B_i)$ is a fuzzy soft *G*-sub module of (*F*, *B*).

ii) $\tilde{\cup}_{i=1}^{n}(F_i, B_i)$ is a fuzzy soft *G*-sub module of (*F*,*B*), if $B_i \cap B_j = \emptyset$ for all ∀ i,j ϵ I.

3.3 Proposition : If A is fuzzy soft G- sub module of M with respect to a t- norm T, then M_1 = { x / x ε M, A(x) = 0} is a sub module of the module M and A is fuzzy soft G- sub module of M_1 , with respect to the t- norm T.

Proof: Let x,y ε M_1 and α ε R. Then, according to condition (FSG-m1), A (ax+by) ≥ T(A(x), A(y)) = T(1,1) = 1. Thus, A (ax+by) =1. Hence x+y ε M_1. According to condition (FSG-m2), A(αx) ≥ A(x) = 1. Thus, we have A(αx) = 1. From here it follows that αx ε M_1. Thus M_1 is a sub module of module M.

3.4 Proposition: Let 'T' be a t-norm. Then every sensible is fuzzy soft G- sub module 'A' of R is fuzzy soft G- sub module of R.

Proof: Assume that 'A' is a sensible is fuzzy soft G- sub module of R, then we have (FSG-m1) A (ax+by) ≥ T (A(x), A(y)) and (FSG-m2) A(αx) ≥ A(x) for all x,y ε R.

Since 'A' is sensible, we have

$$\begin{aligned} \text{Min}\{A(x), A(y)\} &= T(\min\{A(x), A(y)\}, \min\{A(x), A(y)\}) \\ &\geq T(A(x), A(y)) \\ &\geq \min\{A(x), A(y)\} \end{aligned}$$

And so T (A(x), A(y)) = min { A(x), A(y) }. It follows that

A(ax+by) ≥ T(A(x), A(y) } = min { A(x), A(y) } for all x,y in R.

clearly A(αx) ≥ A(x) for all x in R. so 'A' is is fuzzy soft G- sub module of R.

3.5 Proposition : If A is is fuzzy soft G- sub module of M with respect to the t-norm min, then for any θ ε [0,1], M_θ = { x / x ε M, A(x) ≥ θ } is a sub module of the module M and A is fuzzy soft G- sub module of M_θ with respect to min.

Proof; Let x,y ε M_1, and αε R, Then A(ax+by) ≥ min { A(x), A(y)} = min {θ,θ} = θ. Thus A(ax+by)≥ θ. Hence x+y ε M_θ. Further, we have A(αx) ≥ A(x) ≥ θ. From here we conclude that αx ε M_θ. Finally, from A(0) = 1 ≥ θ it follows that 0 ε M_θ.

3.6 Proposition: Let $A : B \rightarrow [0,1]$ be the characteristic function of a subset B is contained in M and M be an G-module. Then A is is fuzzy soft G- sub module of M with respect to t-norm T if and only if B is a sub module of the module M.

Proof: Let A be is fuzzy soft G- sub module of M with respect to T. Then, according to (FSG-m2), $A(\alpha x,q) \geq A(x,q) = 1$. Hence $\alpha x \in B$. Finally, according to condition .Thus, B is a sub module of the module M.

Conversely, Let B be a sub module of the module M. Then for any $x,y \in M$,

$$A(ax+by) \geq T(A(x), A(y)\}.$$

Indeed, for any $x,y \in B$, $A(ax+by) = 1 \geq 1 = T(1,1) = T(A(x), A(y))$

For any $x \in B$, and y is not in B, $T(A(x), A(y)) = T(1.0) = 0 \geq A(ax+by)$

For any x is not in B, and $y \in B$, $T(A(x), A(y)) = T(0,1) = 0 \geq A(ax+by)$

Finally, for any x,y does not belong to B, $T(A(x), A(y)) = T(0,0) = 0 \geq A(ax+by)$

Further for all $x \in M$, and $\alpha \in R$, we have $A(\alpha x) \geq A(x)$. Indeed, for all $x \in B$ we have $\alpha x \in B$, hence $A(\alpha x) = 1 \geq A(x)$, and for all x does not belong to B we have $A(x) = 0 \leq A(\alpha x)$.Therefore, A is is fuzzy soft G- sub module of M with respect to T.

3.7 Proposition; The intersection of any collection of is fuzzy soft G- sub module of an G-module M is fuzzy soft G- sub module of this module.

Proof: For all $x,y \in M$, and any $\alpha \in R$, we have

$$\cap A_i (x+y) = T_\infty(A_1(x+y), A_2(x+y), \ldots) \geq T_\infty(T_\infty(A_1(x), A_1(y)), T_\infty(A_2(x), A_2(y)), \ldots)$$

$$= T_\infty (T_\infty(A_1(x), A_2(x), , \ldots), T_\infty(A_1(y), A_2(y), \ldots))$$

$$= T_\infty ((\cap A_i)(x) , (\cap A_i)(y))$$

$$(\cap A_i)(\alpha x) = T_\infty (A_1(\alpha x), A_2(\alpha x), \ldots) \geq T_\infty(A_1(x), A_2(x),\ldots) = (\cap A_i) (x)$$

Proposition is proved.

3.8 Proposition: Let $\{ M_1,M_2,\ldots,M_n\}$ be a collection of G-modules and $M = \prod A_i$ be its direct product. Let $\{A_1,A_2,\ldots,A_n\}$ be is fuzzy soft G- sub module of the R-modules$\{ M_1,M_2,\ldots, M_n\}$ with respect to a t-norm T.Then $A = \prod A_i$ is is fuzzy soft G- sub module of the R- module M with respect to the t-norm T.

Proof: Let $x,y \in M$, $x = (x_1,x_2,\ldots x_n)$, and $y = (y_1,y_2,\ldots y_n)$. Also let $\alpha \in R$.Then

$$A(x+y) = A ((x_1+y_1, x_2+y_2, \ldots, x_n+y_n))$$

$= T_n(A_1((x_1+y_1)), A_2((x_2+y_2)), \ldots, A_n((x_n+y_n)))$

$\geq T_n(T (A_1(x_1), A_1(y_1)), T(A_2(x_2), A_2(y_2)), \ldots, T(A_n(x_n), A_n(y_n)))$

$= T (T_n(A_1(x_1), A_2(x_2),\ldots,A_n(x_n)), T_n(A_1(y_1), A_2(y_2), \ldots, A(y_n)))$

$= T (A(x), A(y))$

$A(\alpha x) = A((\alpha x_1,\alpha x_2,\ldots,\alpha x_n))$

$= T_n(A_1(\alpha x_1), A_2(\alpha x_2), \ldots, A_n(\alpha x_n))$

$\geq T_n(A_1(x_1), A_2(x_2) ,\ldots ,A_n(x_n))$

Therefore, A is is fuzzy soft G- sub module of the module M with respect to T.

4. Various Reducibility of fuzzy Soft *G*-modules.

In this section we define irreducibility, reducibility and complete reducibility of fuzzy soft *G*-modules. We will give some examples related to this subject.

4.1Definition: Let (*V*,*A*) be a nonzero *G*-module of *M* over *K*. Then (*V*,*A*) is irreducible if the only fuzzy soft *G*-sub modules of (*V*,*A*) are (*V*,*A*) and (V_0 ,A), otherwise (*V*,*A*) is reducible.

4.1 Example: For any prime p, we have *M* = (Z_p, +, .) is a field. Let *G*=*M*–{0}. Then under the field operations of *M*, it is a *G-module*.

Suppose that *A* = Z_p and that we define the set valued function *F*: $Z_p \rightarrow Z_p$,

F(x)={yϵZ_p:y=x^n;n$\epsilon\mathbb{N}$}.Then (*F*,*A*) is a fuzzy soft group on Z_p.

B = Z_p and that we define the set valued function *V*: $Z_p \rightarrow Z_p$, *V*(x) = {yϵZ_p : y = x^n; nϵ $\mathbb{N}$}.

Then (*V*,*B*) is a fuzzy soft *G-module* on *M* and is irreducible.

4.2 Example: Let **V**: *B* → ($\mathcal{R}^n$) on $\mathcal{R}^n$ and let W: *B* → ($\mathbb{C}^n$) on $\mathbb{C}^n$. Then (*V*,*B*) is a fuzzy soft *G*-sub module of (*W*,*B*). Therefore (*W*,*B*) is a reducible *G*-module.

4.2 Definition: Let (*V*,*A*) be a nonzero *G*-module of *M* over *K*. Then (*V*,*A*) is called completely reducible if for every fuzzy soft *G*-sub modules (*W*,*B*) of (*V*,*A*) there exists a fuzzy soft *G*-sub module (W^*, C) of (*V*,*A*) such that (*V*,*A*)=(*W*,*B*)$\bigoplus(W^*, C)$, where $B \cap C = \emptyset$ and $B \cup C = A$.

4.3 Example: Let *G* = {1, -1}, M=$\mathbb{Q}(\sqrt{2})$ = {x + y$\sqrt{2}$: x, y ϵ $\mathbb{Q}$}, M_0 ={0}, M_1= $\mathbb{Q}$, M_2=$\sqrt{2}$ and $\mathbb{Q}$ = {$\sqrt{2}b$: b $\epsilon\mathbb{Q}$ }. Then *M* is a *G*-module on $\mathbb{Q}$.

Let A = G and F:$A \rightarrow G$, $F(x) = \{y \in G : y = x^n\ ,\ n \in \mathbb{N}\}$. Then ($F$, A) is a fuzzy soft group on G.

Let B = $\mathbb{Z}$, V: $\mathbb{Z} \rightarrow \mathcal{P}(\mathbb{Q}(\sqrt{2}))$; V(k) = {k.x + k.y$\sqrt{2} : x, y \in \mathbb{Q}$}. Then ($V$,$B$) is a soft G-module of $\mathbb{Q}(\sqrt{2})$ on $\mathbb{Q}$.

Let B = $\mathbb{Z}$, V_0: $\mathbb{Z} \rightarrow (\{0\})$; ($V$,$B$) and ($V_0$,B) are trivial G-sub modules of (V,B) of $\mathbb{Q}(\sqrt{2})$ on $\mathbb{Q}$.

Let B = $\mathbb{Z}$, V_1: $\mathbb{Z} \rightarrow \mathcal{P}(\mathbb{Q})$; V(k) = {k.x: $x \in \mathbb{Q}$}. Then (V_1,B) is a soft G-module on $\mathbb{Q}$.

Let B = $\mathbb{Z}$, V_2: $\mathbb{Z} \rightarrow \mathcal{P}(\sqrt{2}\mathbb{Q})$; V(k) = {k.x + k.y$\sqrt{2} : x, y \in \mathbb{Q}$}. Then ($V_2$, B) is a fuzzy soft G-module of on $\mathbb{Q}$.

Therefore the only decompositions (V,B) = (V,B)$\oplus$ (V_0,B) and (V,B) = (V_1,B)$\oplus$ (V_2,B) and hence (V,B) is completely reducible G-module.

4.1Proposition: Let (F,A) be a soft vector space of finite dimensional V over K and let (W_1, B) be any soft subspace of (F,A). If $C \cap B = \emptyset$ and B$\cup C = A$, there exists a soft subspace (W_2, C) of (F,A) such that (F,A)=$(W_1, B) \oplus (W_2$, C).

Proof: Let V be a finite dimensional vector space over a field F and let W_1 be any subspace of V. Then there exist a subspace W_2 of V such that $V = W_1 \oplus W_2$ by Proposition 2.21.
We obtain desired result from the definition of direct sum and $C \cap B = \emptyset$, $B \cup C = A$

4.1 Corollary: Let G be a finite soft group and M be a G-module. Then all finite dimensional fuzzy soft G-modules over M are completely reducible.

Proof: From Proposition 4.1, for any finite dimensional (F,A) fuzzy soft G-module and (W_1, B) fuzzy soft G-sub module of (F,A) there exist a fuzzy soft G-sub module (W_2, C) such that (F,A)=$(W_1, B) \oplus (W_2, C)$. Therefore (F,A) is a completely reducible fuzzy soft G-module.

4.1 Theorem : A fuzzy soft G-sub module of a completely reducible fuzzy soft G-module over M is completely reducible.

Let (V,A) be a completely reducible fuzzy soft G-module of M over K. Assume that (W,A_1) is a fuzzy soft G-sub module of (V,A) and (N,A_2) is a fuzzy soft G-sub module of (W,A_1). Then

(N,A_2) is a fuzzy soft *G*-sub module of (*V*,*A*). There exist a fuzzy soft *G*-sub module (T,A_3) such that (*V*,*A*)=$(N, A_2)\bigoplus(T,A_3)$ since (*V*,*A*) is a completely reducible fuzzy soft *G*-module.

We have $(N',A_4) = (T, A_3)\cap (W, A_1)$ for $A_2 \cap A_4 = \emptyset$. From definition of direct sum $N(i_2)\cap N'(i_4) \subset N(i_2)\cap T(i_3) = \{0\}$ $\forall\, i_k \epsilon A_k$ $1 \leq k \leq 4$.

If $W(i)\epsilon\ (W,A_1)$ for $i\neq 0$ $i\ \epsilon\ A_1$ then $W(i)\epsilon\ (V,A)$ and $W(i) = N(i)+T(i)$. Here $N(i)\epsilon\ (N, A_2)$ and $T(i)\epsilon(T, A_3)$. Therefore we obtain $T(i)\epsilon(W,A_1)$. Hence $T(i)\epsilon\ (T,A_3)\cap(W, A_1) = (N', A_4)$ and $(W, A_1) = (N, A_2)\bigoplus(N', A_4)$.

4.2 Proposition : Let (*V*,*A*) be a completely reducible fuzzy soft *G*-module over *M*. Then there is a irreducible fuzzy soft *G*- sub module of (*V*,*A*).

Proof: Let (*V*,*A*) be a completely reducible fuzzy soft *G*-module over *M* and let (N,A_1) be a fuzzy soft *G*-sub module of (*V*,*A*). $N(i_1)\epsilon\ (N,A_1)$ for $i_1\ \epsilon\ A_1$ Consider the collection sets of submodules of (N,A_1) such that (N,A_1) does not contain $N(i_1)$. This set is not empty. Because there is at least (M_0, A) fuzzy soft *G*-sub module of (*V*,*A*). This collection sets has maximal element (N_0, A_2) from Zorn's Lemma.

(N, A_1) is a completely reducible soft *G*-module and there exist a fuzzy soft *G*-sub

module $(N_1, A_3)\ \tilde{\lesssim}(N,A_1)$ such that $(N, A_1) = (N_0, A_2)\bigoplus (N_1, A_3)$ by Theorem 4.1.

Now we must show that (N_1, A_3) is irreducible fuzzy soft *G*-sub module of (*V*,*A*).

Assume that (N_1, A_3) is reducible fuzzy soft *G*-sub module of (*V*,*A*). Then there exist a fuzzy soft *G*-sub modules (N_2,A_4), $(N_3,A_5)\ \epsilon\ (N_1, A_3)$ such that $(N_1, A_3) = (N_2,A_4)\bigoplus (N_3,A_5)$ by Theorem 4.1. Hence we obtain $(N, B) = (N_0, A_2)\bigoplus (N_2,A_4)\bigoplus (N_3,A_5)$. From definition of direct sum we have $\{N_0(i_2)+ N_2(i_4)\} \cap\{N_0(i_2)+N_3(i_5)\} = \{0\}$ $\forall\ i_k \epsilon\ A_k$, $1\leq k\leq 5$.Then $N(i_1) \notin \{(N_0, A_2)+ (N_2,A_4)\}$ or $N(i_1) \notin\{(N_0, A_2)+(N_3,A_5)\}$. This is a contradiction. Because (N_0, A_2) is an maximal element of the collection sets. Therefore (N_1, A_3) is irreducible fuzzy soft *G*-sub module of (*V*,*A*).

4.3 Proposition: Let (*V*,*A*) be completely reducible fuzzy soft *G*-module over *M*. Then (*V*,*A*) is a direct sum of irreducible *G*- sub modules of (*V*,*A*).

Proof: Let $\{(V_{\mathcal{B}},A_{\mathcal{B}}) : \mathcal{B}\epsilon\mathrm{I}\ \}$ be a family of fuzzy soft G-sub modules of (V,A) and let (N,B) = $\bigoplus(V_{\mathcal{B}},A_{\mathcal{B}})$. Assume that (V,A) ≠(N,B). Then (N,B)$\widetilde{<}$(V,A).

There exist a fuzzy soft G-sub modules (N',C)$\widetilde{<}$(V,A) such that (V,A) = (N,B)$\bigoplus(N',C)$ since (V,A) is completely reducible soft G-module. From Theorem 4.1. (N',C) is completely reducible fuzzy soft G-sub module and (N',C) has an irreducible fuzzy soft G-sub module by Proposition 4.2. Hence $N(i)\cap(N'(i) \neq \{0\}\ \forall\ i\ \epsilon\ A$. This is a contradiction with (V,A) = (N,B)$\bigoplus$(N',C). Therefore (V,A) = $\bigoplus(V_{\mathcal{B}},A_{\mathcal{B}})$.

Conclusion: Module theoretic approach is better suited to deal with deeper results in representation theory. Moreover, module theoretic approach gives more elegance to the theory. In particular, the G-module structure has been extensively used for the study of representations of finite groups. In this paper, we have studied notions of fuzzy version of soft G-module and obtain basic properties of such soft G-modules, the concept fuzzy soft G-modules over a commutative ring with respect to t- norm. Moreover we examine irreducibility, reducibility and complete reducibility of fuzzy soft G-modules.

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