International Journal of Engineering& Scientific Research
Vol.5 Issue 5, May 2017,
ISSN: 2347-6532 Impact Factor: 6.660
JournalHomepage:http://www.ijmra.us,Email:editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

USE OF INFINITESIMAL TRANSFORMATIONS IN OBTAINING THE GENERATOR OF A HARMONIC FOURTH ORDER NON-LINEAR ORDINARY DIFFERENTIAL EQUATION

Rodgers K. Adenyah*

Titus J.O. Aminer**

Oduor M. Okoya**

Abstract

Lie group Theory is applied to differential equations occurring as mathematical models [4]. This paper seeks to obtain a generator T for a harmonic fourth order non-linear ordinary differential equation using the Lie Symmetry group invariant Method. This method makes use of infinitesimal transformations. The Generator T of infinitesimal transformation is then used to obtain the general solution to our harmonic differential equation. The solution to our differential equation is handy in the field of mechanics.

Mathematics Subject Classification: Primary 65N30; Secondary 65N22, 65N12, 65F10

Keywords—: Generator, Lie group, infinitesimal transformation, differential equations.

* **Department of Mathematics and Computer Science,Pwani University.**

** **Department of Pure and Applied Mathematics, Jaramogi Oginga Odinga University of Science and Technology.**

1. Introduction

Lie group theory provides very strong tools for solving ordinary and partial differential equations specifically those that are nonlinear [3]. The key point here is to obtain the Lie point symmetry of a given differential equation and then obtain its invariant solutions. This helps to reduce the order for the case of an ordinary differential equation [2]. However for a partial differential equation, this process reduces the number of independent variables [5]. In this paper we shall lay our emphasis on an ordinary differential equation which is fourth order, nonlinear and harmonic in nature. We seek to get its Generator of infinitesimal transformation.

2. Obtaining the Generator for our harmonic fourth order ordinary differential equation

The differential equation which is an identity in $x, y, y', y'' \, and \, y'''$ yielded by the harmonic fourth order differential equation $y^{(4)} = (y)^{-1} y'y'''$ is given by [1]

$$\lambda y^{-2}y'y''' - y^{-1}y'''\frac{\partial\lambda}{\partial x} - y^{-1}y'y'''\frac{\partial\lambda}{\partial y} + y^{-1}y'y'''\frac{\partial\xi}{\partial x} + y^{-1}y'^{2}y'''\frac{\partial\xi}{\partial y} - y^{-1}y'\frac{\partial^3\lambda}{\partial x^3}$$

$$-3y^{-1}y'^{2}\frac{\partial^3\lambda}{\partial x^2\partial y} - 3y^{-1}y'y''\frac{\partial^2\lambda}{\partial x\partial y} - y^{-1}y'y'''\frac{\partial\lambda}{\partial y} - 3y^{-1}y'^{3}\frac{\partial^3\lambda}{\partial x\partial y^2} - 3y^{-1}y'^{2}y''\frac{\partial^2\lambda}{\partial y^2}$$

$$-y^{-1}y'^{4}\frac{\partial^3\lambda}{\partial y^3} + 3y^{-1}y'y'''\frac{\partial\xi}{\partial x} + 3y^{-1}y'^{2}y'''\frac{\partial\xi}{\partial y} + 3y^{-1}y'^{2}y''\frac{\partial^2\xi}{\partial x^2} + 6y^{-1}y'^{2}y''\frac{\partial^2\xi}{\partial x\partial y}$$

$$+3y^{-1}y'^{3}y''\frac{\partial^2\xi}{\partial y^2} + 3y^{-1}y'y''^{2}\frac{\partial\xi}{\partial y} + y^{-1}y'^{2}\frac{\partial^3\xi}{\partial x^3} + 3y^{-1}y'^{3}\frac{\partial^3\xi}{\partial x^2\partial y} + 3y^{-1}y'^{2}y''\frac{\partial^2\xi}{\partial x\partial y}$$

$$+y^{-1}y'^{2}y'''\frac{\partial\xi}{\partial y} + 3y^{-1}y'^{4}\frac{\partial^3\xi}{\partial x\partial y^2} + 3y^{-1}y'^{3}y''\frac{\partial^2\xi}{\partial y^2} + y^{-1}y'^{5}\frac{\partial^3\xi}{\partial y^3} + \frac{\partial^4\lambda}{\partial x^4}$$

$$+4y'\frac{\partial^4\lambda}{\partial x^3\partial y} + 6y''\frac{\partial^3\lambda}{\partial x^2\partial y} + 4y'''\frac{\partial^2\lambda}{\partial x\partial y} + \left(y^{-1}y'y'''\right)\frac{\partial\lambda}{\partial y} + 3y'^{2}\frac{\partial^4\lambda}{\partial x^2\partial y^2} + 9y'y''\frac{\partial^3\lambda}{\partial x\partial y^2}$$

$$+4y'y'''\frac{\partial^2\lambda}{\partial y^2} + 3y''^{2}\frac{\partial^2\lambda}{\partial y^2} + 4y'^{3}\frac{\partial^4\lambda}{\partial x\partial y^3} + 6y'^{2}y''\frac{\partial^3\lambda}{\partial y^3} + 3y'^{2}\frac{\partial^4\lambda}{\partial x^2\partial y^2} + 3y'y''\frac{\partial^3\lambda}{\partial x\partial y^2}$$

$$y'^{4}\frac{\partial^4\lambda}{\partial y^4} - 4y^{-1}y'y'''\frac{\partial\xi}{\partial x} - 4y^{-1}y'^{2}y'''\frac{\partial\xi}{\partial y} - 6y'''\frac{\partial^2\xi}{\partial x^2} - 12y'y'''\frac{\partial^2\xi}{\partial x\partial y} - 6y'^{2}y'''\frac{\partial^2\xi}{\partial y^2}$$

$$-6y''y'''\frac{\partial\xi}{\partial y}-4y''\frac{\partial^3\xi}{\partial x^3}-12y'y''\frac{\partial^3\xi}{\partial x^2\partial y}-12y''^2\frac{\partial^2\xi}{\partial x\partial y}-4y''y'''\frac{\partial\xi}{\partial y}-12y'^2y''\frac{\partial^3\xi}{\partial x\partial y^2}$$

$$-12y'y''^2\frac{\partial^2\xi}{\partial y^2}-4y'^3y''\frac{\partial^2\xi}{\partial y^2}-y'\frac{\partial^4\xi}{\partial x^4}-4y'^2\frac{\partial^4\xi}{\partial x^3\partial y}-6y'y''\frac{\partial^3\xi}{\partial x^2\partial y}-4y'y'''\frac{\partial^2\xi}{\partial x\partial y}$$

$$-y'\left(y^{-1}y'y'''\right)\frac{\partial\xi}{\partial y}-3y'^3\frac{\partial^4\xi}{\partial x^2\partial y^2}-9y'^2y''\frac{\partial^3\xi}{\partial x\partial y^2}-4y'^2y'''\frac{\partial^2\xi}{\partial y^2}-3y'y''^2\frac{\partial^2\xi}{\partial y^2}-4y'^4\frac{\partial^4\xi}{\partial x\partial y^3}$$

$$-6y'^3y''\frac{\partial^3\xi}{\partial y^3}-3y'^3\frac{\partial^4\xi}{\partial x^2\partial y^2}-3y'^2y''\frac{\partial^3\xi}{\partial x\partial y^2}-y'^5\frac{\partial^4\xi}{\partial y^4}=0 \qquad (2.1)$$

Equation (2.1) simplifies to

$$\lambda y^{-2}y'y'''-y^{-1}y'''\frac{\partial\lambda}{\partial x}-y^{-1}y'y'''\frac{\partial\lambda}{\partial y}+y^{-1}y'y'''\frac{\partial\xi}{\partial x}+y^{-1}y'^2y'''\frac{\partial\xi}{\partial y}-y^{-1}y'\frac{\partial^3\lambda}{\partial x^3}$$

$$-3y^{-1}y'^2\frac{\partial^3\lambda}{\partial x^2\partial y}-3y^{-1}y'y''\frac{\partial^2\lambda}{\partial x\partial y}-y^{-1}y'y'''\frac{\partial\lambda}{\partial y}-3y^{-1}y'^3\frac{\partial^3\lambda}{\partial x\partial y^2}-3y^{-1}y'^2y''\frac{\partial^2\lambda}{\partial y^2}$$

$$-y^{-1}y'^4\frac{\partial^3\lambda}{\partial y^3}+3y^{-1}y'y'''\frac{\partial\xi}{\partial x}+3y^{-1}y'^2y'''\frac{\partial\xi}{\partial y}+3y^{-1}y'^2y''\frac{\partial^2\xi}{\partial x^2}+6y^{-1}y'^2y''\frac{\partial^2\xi}{\partial x\partial y}$$

$$+3y^{-1}y'^3y''\frac{\partial^2\xi}{\partial y^2}+3y^{-1}y'y''^2\frac{\partial\xi}{\partial y}+y^{-1}y'^2\frac{\partial^3\xi}{\partial x^3}+3y^{-1}y'^3\frac{\partial^3\xi}{\partial x^2\partial y}+3y^{-1}y'^2y''\frac{\partial^2\xi}{\partial x\partial y}$$

$$+y^{-1}y'^2y'''\frac{\partial\xi}{\partial y}+3y^{-1}y'^4\frac{\partial^3\xi}{\partial x\partial y^2}+3y^{-1}y'^3y''\frac{\partial^2\xi}{\partial y^2}+y^{-1}y'^5\frac{\partial^3\xi}{\partial y^3}+\frac{\partial^4\lambda}{\partial x^4}$$

$$+4y'\frac{\partial^4\lambda}{\partial x^3\partial y}+6y''\frac{\partial^3\lambda}{\partial x^2\partial y}+4y'''\frac{\partial^2\lambda}{\partial x\partial y}+y^{-1}y'y'''\frac{\partial\lambda}{\partial y}+3y'^2\frac{\partial^4\lambda}{\partial x^2\partial y^2}+9y'y''\frac{\partial^3\lambda}{\partial x\partial y^2}$$

$$+4y'y'''\frac{\partial^2\lambda}{\partial y^2}+3y''^2\frac{\partial^2\lambda}{\partial y^2}+4y'^3\frac{\partial^4\lambda}{\partial x\partial y^3}+6y'^2y''\frac{\partial^3\lambda}{\partial y^3}+3y'^2\frac{\partial^4\lambda}{\partial x^2\partial y^2}+3y'y''\frac{\partial^3\lambda}{\partial x\partial y^2}$$

$$y'^4\frac{\partial^4\lambda}{\partial y^4}-4y^{-1}y'y'''\frac{\partial\xi}{\partial x}-4y^{-1}y'^2y'''\frac{\partial\xi}{\partial y}-6y'''\frac{\partial^2\xi}{\partial x^2}-12y'y'''\frac{\partial^2\xi}{\partial x\partial y}-6y'^2y'''\frac{\partial^2\xi}{\partial y^2}$$

$$-6y''y'''\frac{\partial\xi}{\partial y}-4y''\frac{\partial^3\xi}{\partial x^3}-12y'y''\frac{\partial^3\xi}{\partial x^2\partial y}-12y''^2\frac{\partial^2\xi}{\partial x\partial y}-4y''y'''\frac{\partial\xi}{\partial y}-12y'^2y''\frac{\partial^3\xi}{\partial x\partial y^2}$$

$$-12y'y''^2\frac{\partial^2\xi}{\partial y^2}-4y'^3y''\frac{\partial^2\xi}{\partial y^2}-y'\frac{\partial^4\xi}{\partial x^4}-4y'^2\frac{\partial^4\xi}{\partial x^3\partial y}-6y'y''\frac{\partial^3\xi}{\partial x^2\partial y}-4y'y'''\frac{\partial^2\xi}{\partial x\partial y}$$

$$-y^{-1}\left(y'\right)^{2}y'''\frac{\partial \xi}{\partial y}-3y'^{3}\frac{\partial^{4}\xi}{\partial x^{2}\partial y^{2}}-9y'^{2}y''\frac{\partial^{3}\xi}{\partial x\partial y^{2}}-4y'^{2}y'''\frac{\partial^{2}\xi}{\partial y^{2}}-3y'y''^{2}\frac{\partial^{2}\xi}{\partial y^{2}}-4y'^{4}\frac{\partial^{4}\xi}{\partial x\partial y^{3}}$$

$$-6y'^{3}y''\frac{\partial^{3}\xi}{\partial y^{3}}-3y'^{3}\frac{\partial^{4}\xi}{\partial x^{2}\partial y^{2}}-3y'^{2}y''\frac{\partial^{3}\xi}{\partial x\partial y^{2}}-y'^{5}\frac{\partial^{4}\xi}{\partial y^{4}}=0 \quad (2.2)$$

Or better still

$$\lambda y^{-2}y'y'''-y^{-1}y'''\frac{\partial\lambda}{\partial x}-y^{-1}y'y'''\frac{\partial\lambda}{\partial y}-y^{-1}y'\frac{\partial^{3}\lambda}{\partial x^{3}}$$

$$-3y^{-1}y'^{2}\frac{\partial^{3}\lambda}{\partial x^{2}\partial y}-3y^{-1}y'y''\frac{\partial^{2}\lambda}{\partial x\partial y}-3y^{-1}y'^{3}\frac{\partial^{3}\lambda}{\partial x\partial y^{2}}-3y^{-1}y'^{2}y''\frac{\partial^{2}\lambda}{\partial y^{2}}-y^{-1}y'^{4}\frac{\partial^{3}\lambda}{\partial y^{3}}$$

$$+3y^{-1}y'y''\frac{\partial^{2}\xi}{\partial x^{2}}+9y^{-1}y'^{2}y''\frac{\partial^{2}\xi}{\partial x\partial y}+6y^{-1}y'^{3}y''\frac{\partial^{2}\xi}{\partial y^{2}}+3y^{-1}y'y''^{2}\frac{\partial\xi}{\partial y}$$

$$+y^{-1}y'^{2}\frac{\partial^{3}\xi}{\partial x^{3}}+3y^{-1}y'^{3}\frac{\partial^{3}\xi}{\partial x^{2}\partial y}+3y^{-1}y'^{4}\frac{\partial^{3}\xi}{\partial x\partial y^{2}}+y^{-1}y'^{5}\frac{\partial^{3}\xi}{\partial y^{3}}+\frac{\partial^{4}\lambda}{\partial x^{4}}$$

$$+4y'\frac{\partial^{4}\lambda}{\partial x^{3}\partial y}+6y''\frac{\partial^{3}\lambda}{\partial x^{2}\partial y}+4y'''\frac{\partial^{2}\lambda}{\partial x\partial y}+3y'^{2}\frac{\partial^{4}\lambda}{\partial x^{2}\partial y^{2}}+12y'y''\frac{\partial^{3}\lambda}{\partial x\partial y^{2}}$$

$$+4y'y'''\frac{\partial^{2}\lambda}{\partial y^{2}}+3y''^{2}\frac{\partial^{2}\lambda}{\partial y^{2}}+4y'^{3}\frac{\partial^{4}\lambda}{\partial x\partial y^{3}}+6y'^{2}y''\frac{\partial^{3}\lambda}{\partial y^{3}}+3y'^{2}\frac{\partial^{4}\lambda}{\partial x^{2}\partial y^{2}}$$

$$+y'^{4}\frac{\partial^{4}\lambda}{\partial y^{4}}-6y'''\frac{\partial^{2}\xi}{\partial x^{2}}-12y'y'''\frac{\partial^{2}\xi}{\partial x\partial y}-10y'^{2}y'''\frac{\partial^{2}\xi}{\partial y^{2}}-6y''y'''\frac{\partial\xi}{\partial y}$$

$$-4y''\frac{\partial^{3}\xi}{\partial x^{3}}-18y'y''\frac{\partial^{3}\xi}{\partial x^{2}\partial y}-12y''^{2}\frac{\partial^{2}\xi}{\partial x\partial y}-4y''y'''\frac{\partial\xi}{\partial y}-24y'^{2}y''\frac{\partial^{3}\xi}{\partial x\partial y^{2}}$$

$$-12y'y''^{2}\frac{\partial^{2}\xi}{\partial y^{2}}-10y'^{3}y''\frac{\partial^{2}\xi}{\partial y^{2}}-y'\frac{\partial^{4}\xi}{\partial x^{4}}-4y'^{2}\frac{\partial^{4}\xi}{\partial x^{3}\partial y}-4y'y'''\frac{\partial^{2}\xi}{\partial x\partial y}$$

$$-3y'^{3}\frac{\partial^{4}\xi}{\partial x^{2}\partial y^{2}}-3y'y''^{2}\frac{\partial^{2}\xi}{\partial y^{2}}-4y'^{4}\frac{\partial^{4}\xi}{\partial x\partial y^{3}}-3y'^{3}\frac{\partial^{4}\xi}{\partial x^{2}\partial y^{2}}-y'^{5}\frac{\partial^{4}\xi}{\partial y^{4}}=0 \quad (2.3)$$

Equation (2.3) can be further simplified to

$$\lambda y^{-2}y'y'''-y^{-1}y'''\frac{\partial\lambda}{\partial x}-y^{-1}y'y'''\frac{\partial\lambda}{\partial y}-y^{-1}y'\frac{\partial^{3}\lambda}{\partial x^{3}}-3y^{-1}y'^{2}\frac{\partial^{3}\lambda}{\partial x^{2}\partial y}$$

$$-3y^{-1}y'y''\frac{\partial^2\lambda}{\partial x\partial y}-3y^{-1}y'^3\frac{\partial^3\lambda}{\partial x\partial y^2}-3y^{-1}y'^2y''\frac{\partial^2\lambda}{\partial y^2}-y^{-1}y'^4\frac{\partial^3\lambda}{\partial y^3}$$

$$+3y^{-1}y'y''\frac{\partial^2\xi}{\partial x^2}+9y^{-1}y'^2y''\frac{\partial^2\xi}{\partial x\partial y}+6y^{-1}y'^3y''\frac{\partial^2\xi}{\partial y^2}+3y^{-1}y'y''^2\frac{\partial\xi}{\partial y}$$

$$+y^{-1}y'^2\frac{\partial^3\xi}{\partial x^3}+3y^{-1}y'^3\frac{\partial^3\xi}{\partial x^2\partial y}+3y^{-1}y'^4\frac{\partial^3\xi}{\partial x\partial y^2}+y^{-1}y'^5\frac{\partial^3\xi}{\partial y^3}+\frac{\partial^4\lambda}{\partial x^4}$$

$$+4y'\frac{\partial^4\lambda}{\partial x^3\partial y}+6y''\frac{\partial^3\lambda}{\partial x^2\partial y}+4y'''\frac{\partial^2\lambda}{\partial x\partial y}+6y'^2\frac{\partial^4\lambda}{\partial x^2\partial y^2}+12y'y''\frac{\partial^3\lambda}{\partial x\partial y^2}$$

$$+4y'y'''\frac{\partial^2\lambda}{\partial y^2}+3y''^2\frac{\partial^2\lambda}{\partial y^2}+4y'^3\frac{\partial^4\lambda}{\partial x\partial y^3}+6y'^2y''\frac{\partial^3\lambda}{\partial y^3}+y'^4\frac{\partial^4\lambda}{\partial y^4}$$

$$-6y'''\frac{\partial^2\xi}{\partial x^2}-16y'y'''\frac{\partial^2\xi}{\partial x\partial y}-10y'^2y'''\frac{\partial^2\xi}{\partial y^2}-10y''y'''\frac{\partial\xi}{\partial y}-4y''\frac{\partial^3\xi}{\partial x^3}$$

$$-18y'y''\frac{\partial^3\xi}{\partial x^2\partial y}-12y''^2\frac{\partial^2\xi}{\partial x\partial y}-24y'^2y''\frac{\partial^3\xi}{\partial x\partial y^2}-15y'y''^2\frac{\partial^2\xi}{\partial y^2}$$

$$-10y'^3y''\frac{\partial^2\xi}{\partial y^2}-y'\frac{\partial^4\xi}{\partial x^4}-4y'^2\frac{\partial^4\xi}{\partial x^3\partial y}-4y'y'''\frac{\partial^2\xi}{\partial x\partial y}-6y'^3\frac{\partial^4\xi}{\partial x^2\partial y^2}$$

$$-4y'^4\frac{\partial^4\xi}{\partial x\partial y^3}-y'^5\frac{\partial^4\xi}{\partial y^4}=0 \qquad (2.4)$$

From (2.4) we obtain the following coefficients

$$(y')^0\,y''':\ -y^{-1}\frac{\partial\lambda}{\partial x}+4\frac{\partial^2\lambda}{\partial x\partial y}-6\frac{\partial^2\xi}{\partial x^2}$$

$$(y')^1\,y''':\ \lambda y^{-2}-y^{-1}\frac{\partial\lambda}{\partial y}+4\frac{\partial^2\lambda}{\partial y^2}-16\frac{\partial^2\xi}{\partial x\partial y}$$

$$(y')^2\,y''':\ -10\frac{\partial^2\xi}{\partial y^2}$$

$$(y')^0\,y'':\ 6\frac{\partial^3\lambda}{\partial x^2\partial y}-4\frac{\partial^3\xi}{\partial x^3}$$

$$(y')^1\,y'':\ -3y^{-1}\frac{\partial^2\lambda}{\partial x\partial y}+3y^{-1}\frac{\partial^2\xi}{\partial x^2}+12\frac{\partial^3\lambda}{\partial x\partial y^2}-18\frac{\partial^3\xi}{\partial x^2\partial y}$$

$$(y')^2\,y'':\ -3y^{-1}\frac{\partial^2\lambda}{\partial x^2}+9y^{-1}\frac{\partial^2\xi}{\partial x\partial y}+6\frac{\partial^3\lambda}{\partial y^3}-24\frac{\partial^3\xi}{\partial x\partial y^2}$$

$$(y')^3 y'' : \; 6y^{-1}\frac{\partial^2\xi}{\partial y^2} - 10\frac{\partial^3\xi}{\partial y^3}$$

$$(y'')^0 y''' : \; -y^{-1}\frac{\partial\lambda}{\partial x} + 4\frac{\partial^2\lambda}{\partial x\partial y} - 6\frac{\partial^2\xi}{\partial x^2}$$

$$y''y''' \quad : -6\frac{\partial\xi}{\partial y} - 4\frac{\partial\xi}{\partial y}$$

$$: -10\frac{\partial\xi}{\partial y}$$

$$(y')^0 (y'')^2 : \; 3\frac{\partial^2\lambda}{\partial y^2} - 12\frac{\partial^2\xi}{\partial x\partial y}$$

$$(y')^1 (y'')^2 : \; 3y^{-1}\frac{\partial\xi}{\partial y} - 15\frac{\partial^2\xi}{\partial y^2}$$

$$(y')^0 : \; \frac{\partial^4\lambda}{\partial x^4}$$

$$(y')^1 : \; -y^{-1}\frac{\partial^3\lambda}{\partial x^3} + 4\frac{\partial^4\xi}{\partial x^3\partial y} - \frac{\partial^4\xi}{\partial y^4}$$

$$(y')^2 : \; -3y^{-1}\frac{\partial^3\lambda}{\partial x^2\partial y} + 3\frac{\partial^4\lambda}{\partial x^2\partial y^2} + 3\frac{\partial^4\lambda}{\partial x^2\partial y^2} - 4\frac{\partial^4\xi}{\partial x^3\partial y} + y^{-1}\frac{\partial^3\xi}{\partial x^3}$$

$$: -3y^{-1}\frac{\partial^3\lambda}{\partial x^2\partial y} + 6\frac{\partial^4\lambda}{\partial x^2\partial y^2} - 4\frac{\partial^4\xi}{\partial x^3\partial y} + y^{-1}\frac{\partial^3\xi}{\partial x^3}$$

$$(y')^3 : \; -3y^{-1}\frac{\partial^3\lambda}{\partial x\partial y^2} + 3y^{-1}\frac{\partial^3\xi}{\partial x^2\partial y} + 4\frac{\partial^4\lambda}{\partial x\partial y^3} - 3\frac{\partial^4\xi}{\partial x^2\partial y^2} - 3\frac{\partial^4\xi}{\partial x^2\partial y^2}$$

$$: -3y^{-1}\frac{\partial^3\lambda}{\partial x\partial y^2} + 3y^{-1}\frac{\partial^3\xi}{\partial x^2\partial y} + 4\frac{\partial^4\lambda}{\partial x\partial y^3} - 6\frac{\partial^4\xi}{\partial x^2\partial y^2}$$

$$(y')^4 : \; -y^{-1}\frac{\partial^3\lambda}{\partial y^3} + 3y^{-1}\frac{\partial^3\xi}{\partial x\partial y^2} + \frac{\partial^4\lambda}{\partial y^4} - 4\frac{\partial^4\xi}{\partial x\partial y^3}$$

$$(y')^5 : \; y^{-1}\frac{\partial^3\xi}{\partial y^3} - \frac{\partial^4\xi}{\partial y^4}$$

From which we obtain the following determining equations

$$-10\frac{\partial^2\xi}{\partial y^2} = 0 \tag{2.5}$$

$$3\frac{\partial^2\lambda}{\partial y^2} - 12\frac{\partial^2\xi}{\partial x\partial y} = 0 \tag{2.6}$$

$$-y^{-1}\frac{\partial\lambda}{\partial x}+4\frac{\partial^2\lambda}{\partial x\partial y}-6\frac{\partial^2\xi}{\partial y^2}=0 \quad (2.7)$$

$$-3y^{-1}\frac{\partial^2\lambda}{\partial x\partial y}+3y^{-1}\frac{\partial^2\xi}{\partial y^2}+12\frac{\partial^3\lambda}{\partial x\partial y^2}-18\frac{\partial^3\xi}{\partial x^2\partial y}=0 \quad (2.8)$$

From (2.5) we have

$$\xi=\phi_1 y+\phi_2 \quad (2.9)$$

Where ϕ_1 and ϕ_2 are functions of x.

From (2.6) we have

$$\lambda=2\phi_1' y^2+\phi_3 y+\phi_4 \quad (2.10)$$

From equation (2.7) we get

$$8\phi_1'' y+3\phi_3'-6\phi_2''-\phi_4' y^{-1}=0 \quad (2.11)$$

We equate the coefficients of powers of y of (2.11) to zero

$$y^{-1} \quad :\phi_4'=0 \quad (2.12)$$

$$y^{0} \quad :3\phi_3'-6\phi_2''=0 \quad (2.13)$$

$$y^{1} \quad :8\phi_1''=0 \quad (2.14)$$

Solving the differential equations (2.12), (2.13) and (2.14) we get

$$\phi_1=R_1x+R_2 \quad (2.15)$$

$$\phi_4=R_3 \quad (2.16)$$

From equation (2.8) we have

$$-3y^{-1}\frac{\partial^2\lambda}{\partial x\partial y}+3y^{-1}\frac{\partial^2\xi}{\partial y^2}+12\frac{\partial^3\lambda}{\partial x\partial y^2}-18\frac{\partial^3\xi}{\partial x^2\partial y}=0$$

$$-3y^{-1}\frac{\partial}{\partial x}\left(\frac{\partial\lambda}{\partial y}\right)+3y^{-1}\left(\frac{\partial^2\xi}{\partial y^2}\right)+12\frac{\partial}{\partial x}\left(\frac{\partial^2\lambda}{\partial y^2}\right)-18\frac{\partial}{\partial x}\left(\frac{\partial^2\xi}{\partial x\partial y}\right)=0$$

$$-3y^{-1}\frac{\partial}{\partial x}\left(\frac{\partial\lambda}{\partial y}\right)+3y^{-1}\left(\frac{\partial^2\xi}{\partial y^2}\right)+12\frac{\partial}{\partial x}\left(\frac{\partial^2\lambda}{\partial y^2}\right)-18\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial\xi}{\partial y}\right)\right)=0$$

$$-3y^{-1}\frac{\partial}{\partial x}\left(4\phi_1'y+\phi_3\right)+3y^{-1}(0)+12\frac{\partial}{\partial x}\left(4\phi_1'\right)-18\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(\phi_1)\right)=0$$

$$-3y^{-1}\frac{\partial}{\partial x}\left(4\phi_1''y+\phi_3'\right)+12\left(4\phi_1''\right)-18\frac{\partial}{\partial x}\left(\phi_1'\right)=0$$

$$-3y^{-1}\left(4\phi_1''y+\phi_3'\right)+48\phi_1''-18\left(\phi_1''\right)=0$$

$$-12\phi_1''-3\phi_3'y^{-1}+48\phi_1''-18\phi_1''=0$$

$$18\phi_1''-3\phi_3'y^{-1}=0 \qquad (2.17)$$

Equating the coefficients of powers of y of (2.17) to zero we obtain

$$y^{-1} \quad :\phi_3'=0 \qquad (2.18)$$

$$y^{0} \quad :\phi_1''=0 \qquad (2.19)$$

Solving (2.18) we obtain

$$\phi_3=R_4 \qquad (2.20)$$

From (2.13) we have

$$2\phi_2''=\phi_3' \qquad (2.21)$$

Substituting (2.20) into (2.21) we get

$$\phi_2''=0$$

$$\phi_2'=R_5$$

$$\phi_2=R_5x+R_6 \qquad (2.22)$$

Substituting values of ϕ_1,ϕ_2,ϕ_3 and ϕ_4 into (2.9) and (2.10) we get

$$\xi=\phi_1y+\phi_2$$

$$\xi=\left(R_1x+R_2\right)y+R_5x+R_6 \qquad (2.23)$$

We also have

$$\lambda=2\phi_1'y^2+\phi_3y+\phi_4$$

$$\lambda = 2\left(R_1 x + R_2\right)' y^2 + R_4 y + R_3$$

$$\lambda = 2R_1 y^2 + R_3 + R_4 y \qquad (2.24)$$

The generator T of infinitesimal transformation is given by

$$T = \xi \frac{\partial}{\partial x} + \lambda \frac{\partial}{\partial y} \qquad (2.25)$$

Values of ξ and λ are defined in equations (2.23) and (2.24). Equation (2.25) becomes

$$T = \left(\left(R_1 x + R_2\right) y + R_5 x + R_6\right) \frac{\partial}{\partial x} + \left(2R_1 y^2 + R_3 + R_4 y\right) \frac{\partial}{\partial y}$$

$$T = R_1 xy \frac{\partial}{\partial x} + R_2 y \frac{\partial}{\partial x} + R_5 x \frac{\partial}{\partial x} + R_6 \frac{\partial}{\partial x} + 2R_1 y^2 \frac{\partial}{\partial y} + R_3 \frac{\partial}{\partial y} + R_4 y \frac{\partial}{\partial y}$$

$$T = R_1\left(xy \frac{\partial}{\partial x} + 2y^2 \frac{\partial}{\partial y}\right) + R_2\left(y \frac{\partial}{\partial x}\right) + R_3\left(\frac{\partial}{\partial y}\right) + R_4\left(y \frac{\partial}{\partial y}\right) + R_5\left(x \frac{\partial}{\partial x}\right) + R_6\left(\frac{\partial}{\partial x}\right) \qquad (2.26)$$

Where R_1, R_2, R_3, R_4, R_5 and R_6 are arbitrary constants of integration.

Equation (2.26) which is a six parameter symmetry is the required Generator T for our nonlinear harmonic fourth order differential equation.

Conclusion

In this paper we used infinitesimal transformations to obtain the Generator T to a fourth order nonlinear differential equation which is harmonic in nature. The Generator forms an n-parameter symmetry which may be split into n one-parameter symmetries [6].

References

1. **Adenyah, R., Aminer, T., Oduor, M. (2016**). *Lie Symmetry analysis: Fourth order non-linear ordinary differential equation as a case study.* IJESR, Vol 4, Issue 6, ISSN 2347-6532.

2. **Aminer, T.J.O, (2014).** *Lie symmetry Solution of Fourth Order Non-linear Ordinary Differential Equations, PhD thesis,* Jaramogi Oginga Odinga University of Science and Technology, Bondo, Kenya.

3. **Bluman, G.W., Anco, S.C. (2002).** *Symmetry and Integration Methods for Differential Equations;* Springer: New York, NY, USA.

4. **Bluman, G.W., Kumei, S. (1989).** *Symmetries and differential Equations;* Springer-Verlag: New York, NY, USA.

5. **Dresner, L. (1999).** *Applications of Lie's Theory of Ordinary and Partial Differential Equations;* London, Institute of Physics.

6. **H.Stephani (1989)**. *Differential Equations: Their solutions using symmetries,* Cambridge University Press, Cambridge 1989.