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CONCEPT OF ENHANCEMENT IN WORKLOAD FOR STOCHASTIC ANALYSIS OF 3-UNIT STANDBY SYSTEM

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Abstract

This paper considers 3-unit cold standby system in which there is one main unit and two cold standby units. The system has a provision that whenever there is enhancement in workload of the system, both the cold standby systems also become operative with the main unit. This provision has been provided in the system in order to share the increased workload. There is a single repairman available in order to repair the breakdowns in the system. Various measures of system effectiveness such as Mean time to system failure (MTSF), Availability of the system, Busy period analysis and Profit evaluation has been conducted for the study using Semi-markov process and Regenerative point technique. Numerical study and Graphical interpretation has also been done for the present paper.

Keywords:

Standby systems;
Semi-markov process;
Regenerative point technique.

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1. Introduction:

The literature of Reliability holds a great importance for standby systems. Numerous reliability models have been developed by various researchers considering different working mechanism for the systems. Most of the studies deal with the system where the workload remains constant. But practically, there are situations where the system has increased workload. Such a situation can be seen in the Industrial power plant. Whenever more power is to be generated as per requirement of the system, there is enhancement in the workload. There is comparitively less work in the field of reliability for such conditional models where the workload is varying. Thus the study has been done to contribute into existing studies taking into account the problem of increased demand.

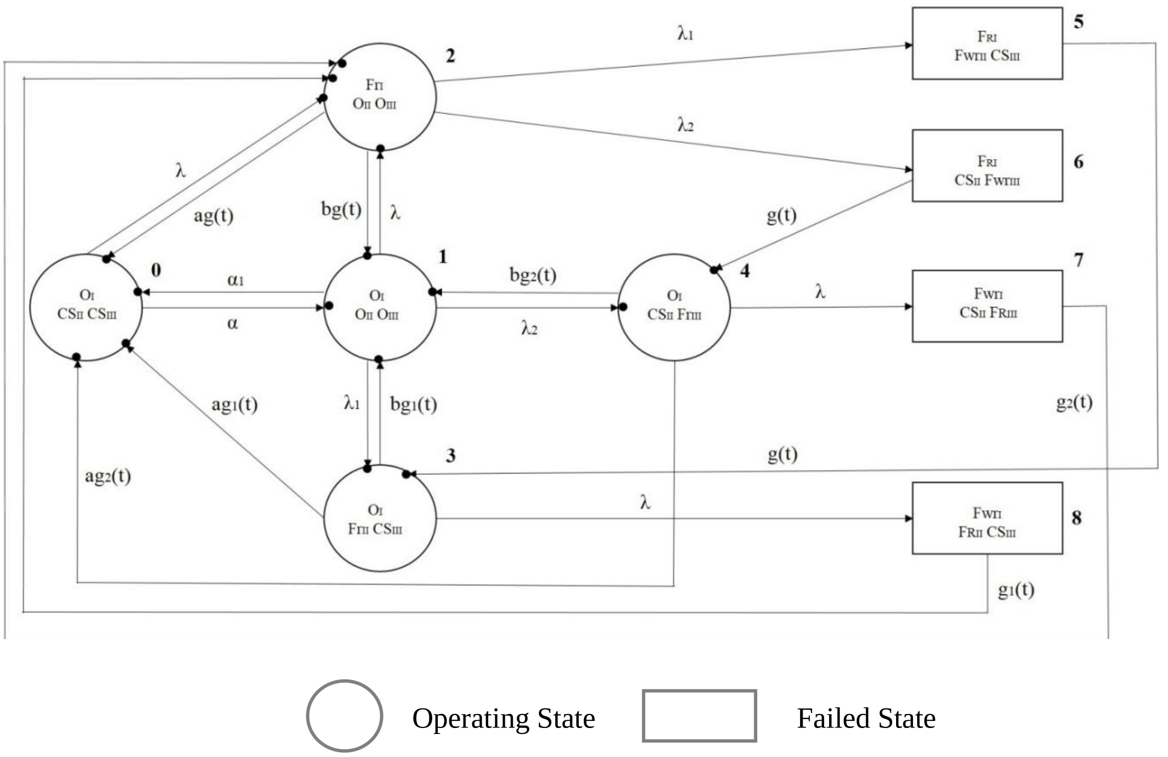
The system comprises of one main unit and two cold standby units. The work mechanism of the system is based upon the workload. Whenever there is more demand for generating power, both the cold standby units become operative with the main unit in order to meet the requirement. There is a single repairman facility available for repair of main as well as standby units. At a time, all standby units cannot fail simultaneously, i.e., failure cannot occur in any of the two among three cold standby units in a single state. On failure of one cold standby unit, the other standby unit go to standby state. Repair is done on FCFS basis.

2. Notations:

λ	Constant failure rate of main unit (Unit 1)
λ_1/ λ_2	Constant failure rate of cold standby units (Unit 2/3)
α	Constant rate of Unit 2 and 3 (both standby units) to become operative from standby state
α_1	Constant rate of Unit 2 and 3 (both standby units) to become standby from operative state
$g(t)$/ $G(t)$	pdf/ cdf of repair time of the main unit at failed state (Unit 1)
$g_1(t)$/ $G_1(t)$	pdf/ cdf of repair time of the standby unit at failed state (Unit 2)
$g_2(t)$/ $G_2(t)$	pdf/ cdf of repair time of the standby unit at failed state (Unit 3)
a	probability that after the repair of a unit, workload is only for one unit
b	probability that after the repair of a unit, workload is for all units (main and both standby units)
O_I/ O_{II}/ O_{III}	Unit 1/2/3 is in operative state
CS_{II}/ CS_{III}	Unit 2/3 is in cold standby state
F_{rI}/F_{rII}/F_{rII}	Unit 1/2/3 is under repair respectively
F_{wrI}/F_{wrII}/F_{wrIII}	Unit 1/2/3 is waiting for repair respectively
F_{RI}/F_{RII}/F_{RIII}	Unit 1/2/3 is under repair respectively from the previous state, i.e, Repair is continuing from previous state

3. Transition probabilities and mean sojourn times:

A state transition diagram in fig. 1 shows various transitions of the system. The epochs of entry into states 0,1,2,3 and 4 are regenerative points and thus these are regenerative states. The states 5, 6, 7 and 8 are failed states.



The non-zero elements p_{ij}, are obtained as

Fig. 1

$$p_{01} = \frac{\alpha}{\alpha + \lambda} \qquad p_{02} = \frac{\lambda}{\alpha + \lambda}$$

$$p_{10} = \frac{\alpha_1}{\lambda + \lambda_1 + \lambda_2 + \alpha_1} \qquad p_{12} = \frac{\lambda}{\lambda + \lambda_1 + \lambda_2 + \alpha_1}$$

$$p_{13} = \frac{\lambda_1}{\lambda + \lambda_1 + \lambda_2 + \alpha_1} \qquad p_{14} = \frac{\lambda_2}{\lambda + \lambda_1 + \lambda_2 + \alpha_1}$$

$$p_{20} = ag^*(\lambda_1 + \lambda_2) \qquad p_{21} = bg^*(\lambda_1 + \lambda_2)$$

$$p_{25} = \frac{\lambda_1[1 - g^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{23}^{(5)} \qquad p_{26} = \frac{\lambda_2[1 - g^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{24}^{(6)}$$

$$p_{30} = ag_1^*(\lambda) \qquad p_{31} = bg_1^*(\lambda)$$

$$p_{38} = 1 - g_1^*(\lambda) = p_{32}^{(8)} \qquad p_{40} = ag_2^*(\lambda)$$

$$p_{41} = bg_2^*(\lambda) \qquad p_{47} = 1 - g_2^*(\lambda) = p_{42}^{(7)}$$

$$p_{53} = g^*(0) = p_{64} \qquad p_{82} = g_1^*(0)$$

$$p_{72} = g_2^*(0)$$

By these transition probabilities, it can be verified that

$$p_{01} + p_{02} = 1 \qquad p_{10} + p_{12} + p_{13} + p_{14} = 1$$
$$p_{20} + p_{21} + p_{25} + p_{26} = 1 \qquad p_{20} + p_{21} + p_{23}^{(5)} + p_{24}^{(6)} = 1$$
$$p_{30} + p_{31} + p_{38} = 1 \qquad p_{30} + p_{31} + p_{32}^{(8)} = 1$$
$$p_{40} + p_{41} + p_{47} = 1 \qquad p_{40} + p_{41} + p_{42}^{(7)} = 1$$
$$p_{53} = 1 = p_{64} \qquad p_{82} = p_{72} = 1$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as –

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}^{*'}(0), Thus -$$
$$m_{01} + m_{02} = \mu_0 \qquad m_{10} + m_{12} + m_{13} + m_{14} = \mu_1$$
$$m_{20} + m_{21} + m_{25} + m_{26} = \mu_2 \qquad m_{20} + m_{21} + m_{23}^{(5)} + m_{24}^{(6)} = k$$
$$m_{30} + m_{31} + m_{38} = \mu_3 \qquad m_{30} + m_{31} + m_{32}^{(8)} = k_1$$
$$m_{40} + m_{41} + m_{47} = \mu_4 \qquad m_{40} + m_{41} + m_{42}^{(7)} = k_2$$

where,

$$k = \int_0^\infty \overline{G}(t)dt \qquad k_1 = \int_0^\infty \overline{G}_1(t)dt$$
$$k_2 = \int_0^\infty \overline{G}_2(t)dt$$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state, then we have -

$$\mu_0 = \frac{1}{\lambda + \alpha} \qquad \mu_1 = \frac{1}{\lambda + \lambda_1 + \lambda_2 + \alpha_1}$$
$$\mu_2 = \frac{1 - g^*(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2} \qquad \mu_3 = \frac{1 - g_1^*(\lambda)}{\lambda}$$
$$\mu_4 = \frac{1 - g_2^*(\lambda)}{\lambda} \qquad \mu_5 = -g^*(0) = \mu_6$$
$$\mu_8 = -g_1^*(0) \qquad \mu_7 = -g_2^*(0)$$

4. Mean time to system failure:

The mean time to system failure when the system starts from the state 0, is

$$T_0 = \frac{N}{D}$$

where

$$\begin{aligned} N = {} & \mu_0[1 - p_{12}p_{21} - p_{13}p_{31} - p_{14}p_{41}] + \mu_1[p_{01} + p_{02}p_{21}] \\ & + \mu_2[p_{02} + p_{01}p_{12} - p_{02}p_{13}p_{31} - p_{02}p_{14}p_{41}] \\ & + \mu_3[p_{01}p_{13} + p_{02}p_{13}p_{21}] + \mu_4[p_{01}p_{14} + p_{02}p_{14}p_{21}] \\ D = {} & 1 - p_{10}p_{10} - p_{02}p_{20} - p_{12}p_{21} - p_{13}p_{31} - p_{14}p_{41} - p_{02}p_{10}p_{21} - p_{01}p_{12}p_{20} - p_{01}p_{13}p_{30} \\ & - p_{01}p_{14}p_{40} - p_{02}p_{13}p_{21}p_{30} - p_{02}p_{14}p_{21}p_{40} + p_{02}p_{13}p_{31}p_{20} + p_{02}p_{14}p_{41}p_{20} \end{aligned}$$

5. Expected up-time of the system:

The steady state availability of the system is given by

$$A_0 = \frac{N_1}{D_1}$$

where

$$
\begin{aligned}
N_1 = \mu_0[1 &- p_{13}p_{31} - p_{14}p_{41} - p_{21}(p_{12} + p_{13}p_{32}^{(8)} + p_{14}p_{42}^{(7)}) \\
&- p_{23}^{(5)}\{p_{32}^{(8)}(1 - p_{14}p_{41}) + p_{31}(p_{12} + p_{14}p_{42}^{(7)})\} \\
&- p_{24}^{(6)}\{p_{42}^{(7)}(1 - p_{13}p_{31}) + p_{41}(p_{12} + p_{13}p_{32}^{(8)})\}] \\
&+ \mu_1[p_{01}(1 - p_{23}^{(5)}p_{32}^{(8)} - p_{24}^{(6)}p_{42}^{(7)}) + p_{02}(p_{21} + p_{31}p_{23}^{(5)} + p_{41}p_{24}^{(6)})] \\
&+ \mu_2[p_{01}(p_{12} + p_{13}p_{32}^{(8)} + p_{14}p_{42}^{(7)}) + p_{02}(1 - p_{13}p_{31} - p_{14}p_{41})] \\
&+ \mu_3[p_{01}\{p_{13}(1 - p_{24}^{(6)}p_{42}^{(7)}) + p_{23}^{(5)}(p_{12} + p_{14}p_{42}^{(7)})\} \\
&\quad + p_{02}\{p_{13}(p_{21} + p_{41}p_{24}^{(6)}) + p_{23}^{(5)}(1 - p_{14}p_{41})\}] \\
&+ \mu_4[p_{01}\{p_{14}(1 - p_{23}^{(5)}p_{32}^{(8)}) + p_{24}^{(6)}(p_{12} + p_{13}p_{32}^{(8)})\} \\
&\quad + p_{02}\{p_{14}(p_{21} + p_{31}p_{23}^{(5)}) + p_{24}^{(6)}(1 - p_{13}p_{31})\}]
\end{aligned}
$$

$$
\begin{aligned}
D_1 = \mu_0[&p_{10}(1 - p_{23}^{(5)}p_{32}^{(8)} - p_{24}^{(6)}p_{42}^{(7)}) + p_{12}(p_{20} + p_{30}p_{23}^{(5)} + p_{40}p_{24}^{(6)}) \\
&+ p_{13}\{p_{30} + p_{20}p_{32}^{(8)} - p_{24}^{(6)}(p_{30}p_{42}^{(7)} - p_{40}p_{32}^{(8)})\} \\
&+ p_{14}\{p_{40} + p_{20}p_{42}^{(7)} - p_{23}^{(5)}(p_{30}p_{42}^{(7)} - p_{40}p_{32}^{(8)})\} \\
&+ \mu_1[p_{01}(1 - p_{23}^{(5)}p_{32}^{(8)} - p_{24}^{(6)}p_{42}^{(7)}) + p_{02}(p_{21} + p_{31}p_{23}^{(5)} + p_{41}p_{24}^{(6)})] \\
&+ k[1 - p_{13}p_{31} - p_{14}p_{41} - p_{01}(p_{10} + p_{13}p_{30} + p_{14}p_{40})] \\
&+ k_1[p_{01}(p_{13} + p_{12}p_{23}^{(5)} - p_{13}p_{24}^{(6)}p_{42}^{(7)} + p_{14}p_{23}^{(5)}p_{42}^{(7)}) \\
&\quad + p_{02}(p_{23}^{(5)} + p_{13}p_{21} + p_{13}p_{41}p_{24}^{(6)} - p_{14}p_{41}p_{23}^{(5)})] \\
&+ k_2[p_{01}(p_{14} + p_{12}p_{24}^{(6)} + p_{13}p_{24}^{(6)}p_{32}^{(8)} - p_{14}p_{23}^{(5)}p_{32}^{(8)}) \\
&\quad + p_{02}(p_{24}^{(6)} + p_{14}p_{21} - p_{13}p_{31}p_{24}^{(6)} + p_{14}p_{31}p_{23}^{(5)})]
\end{aligned}
$$

6. Busy period of a repairman:

The steady state busy period of the system is given by:

$$B_R = \frac{N_2}{D_1}$$

where

$$
\begin{aligned}
N_2 = W_2[&p_{01}(p_{12} + p_{13}p_{32}^{(8)} + p_{14}p_{42}^{(7)}) + p_{02}(1 - p_{13}p_{31} - p_{14}p_{41})] \\
&+ W_3[p_{01}\{p_{13}(1 - p_{24}^{(6)}p_{42}^{(7)}) + p_{23}^{(5)}(p_{12} + p_{14}p_{42}^{(7)})\} \\
&\quad + p_{02}\{p_{13}(p_{21} + p_{41}p_{24}^{(6)}) + p_{23}^{(5)}(1 - p_{14}p_{41})\}] \\
&+ W_4[p_{01}\{p_{14}(1 - p_{23}^{(5)}p_{32}^{(8)}) + p_{24}^{(6)}(p_{12} + p_{13}p_{32}^{(8)})\} \\
&\quad + p_{02}\{p_{14}(p_{21} + p_{31}p_{23}^{(5)}) + p_{24}^{(6)}(1 - p_{13}p_{31})\}]
\end{aligned}
$$

and D_1 is already specified.

7. Expected no. of visits of repairman:

The steady state expected no. of visits of the repairman is given by:

$$V_R = \frac{N_3}{D_1}$$

where

$$\begin{aligned} N_3 = {} & [1 - p_{01} p_{10}][1 - p_{23}^{(5)} p_{32}^{(8)} - p_{24}^{(6)} p_{42}^{(7)}] \\ & + p_{02}[-p_{13} p_{31}(1 - p_{23}^{(5)} - p_{24}^{(6)} p_{42}^{(7)}) - p_{14} p_{41}(1 - p_{24}^{(6)} - p_{23}^{(5)} p_{32}^{(8)}) \\ & \quad + (1 - p_{32}^{(8)})(p_{21} p_{13} + p_{13} p_{41} p_{24}^{(6)}) \\ & \quad + (1 - p_{42}^{(7)})(p_{21} p_{14} + p_{14} p_{31} p_{23}^{(5)})] \end{aligned}$$

and D_1 is already specified.

8. Profit Analysis:

The expected profit incurred of the system is -

$$P = C_0 A_0 - C_1 B_R - C_2 V_R$$

C_0 = Revenue per unit up time of the system
C_1 = Cost per unit up time for which the repairman is busy in repair
C_2 = Cost per visit of the repairman

9. Graphical interpretation and conclusion:

For graphical analysis following particular cases are considered:

$$g(t) = \beta e^{-\beta t} \qquad g_1(t) = \beta_1 e^{-\beta_1 t}$$

$$g_2(t) = \beta_2 e^{-\beta_2 t}$$

Graphical study has been made for the MTSF and the profit with respect to failure rate of main unit (λ), revenue per unit uptime of the system (C_0) for different values of rate of failure rate of main unit (λ) , cost of repairman for busy in doing repair (C_1)for different values for different values of rate of failure rate of main unit (λ) and repair rate of main unit (β) for different values of rate of failure rate of main unit (λ).

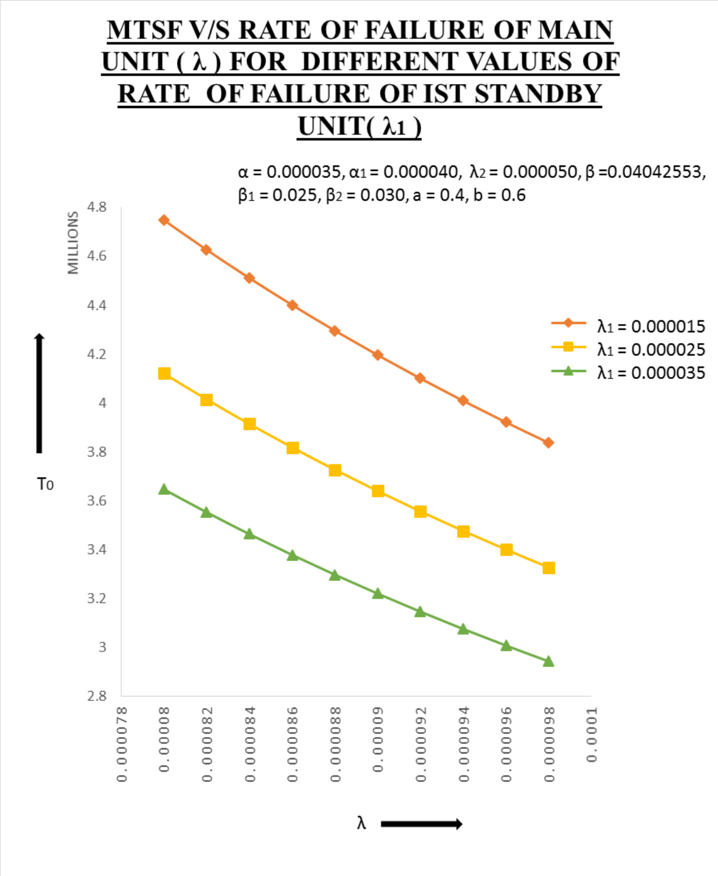


Fig. 2

In Fig. 2, the behaviour of MTSF w.r.t. failure rate of main unit (λ) for different values of rate of failure of Ist standby unit (λ_1) is shown. It is clear from the graph that MTSF gets decreased with the increase in the values of the failure rate of main unit (λ). Also, the MTSF decreases as failure rate of Ist standby unit (λ_1) increases.

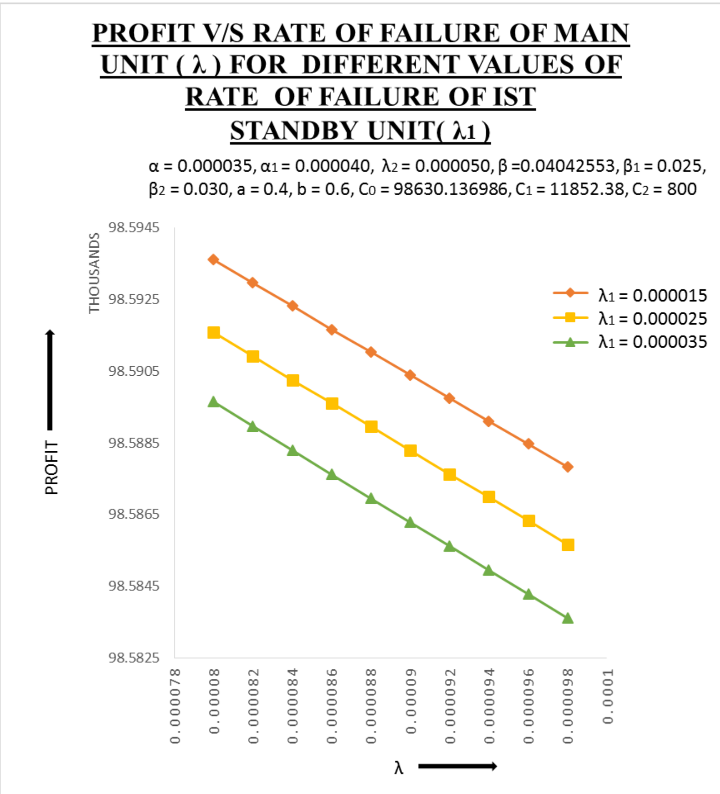


Fig. 3

Fig. 3 shows the behaviour of profit w.r.t. to failure rate of main unit (λ) for different values of failure rate of Ist standby unit (λ_1). As the values of failure rate of main unit (λ) increases, the profit decreases. Also, the profit decreases as failure rate of Ist standby unit (λ_1) increases.

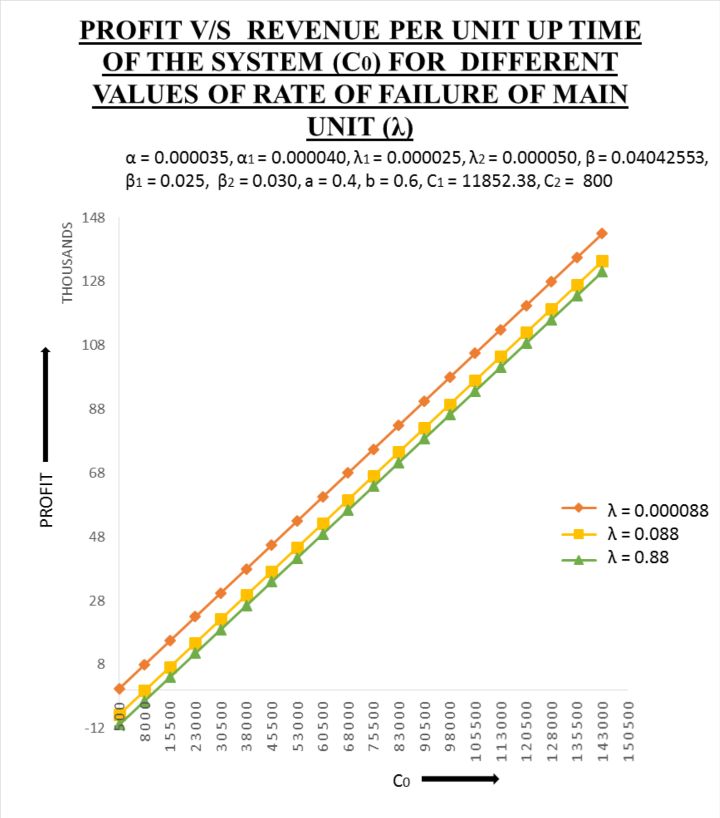


Fig. 4

Fig. 4 depicts the behaviour of the profit w.r.t. revenue per unit uptime of the system (C_0) for different values of rate of failure of main unit (λ). It can be interpreted that the profit increases with increase in the values of C_0. Following conclusions can be drawn from the graph:

1. For $\lambda = 0.000088$, profit is positive according as C_0 i.e. revenue per unit uptime of the system increases.
2. For $\lambda = 0.088$, profit is > or = or < according as C_0 > or = or < 8156.8 , i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 8156.8 to get positive profit.
3. For $\lambda = 0.88$, profit is > or = or < according as C_0 > or = or < 11365, i.e. , i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 11365 to get positive profit.

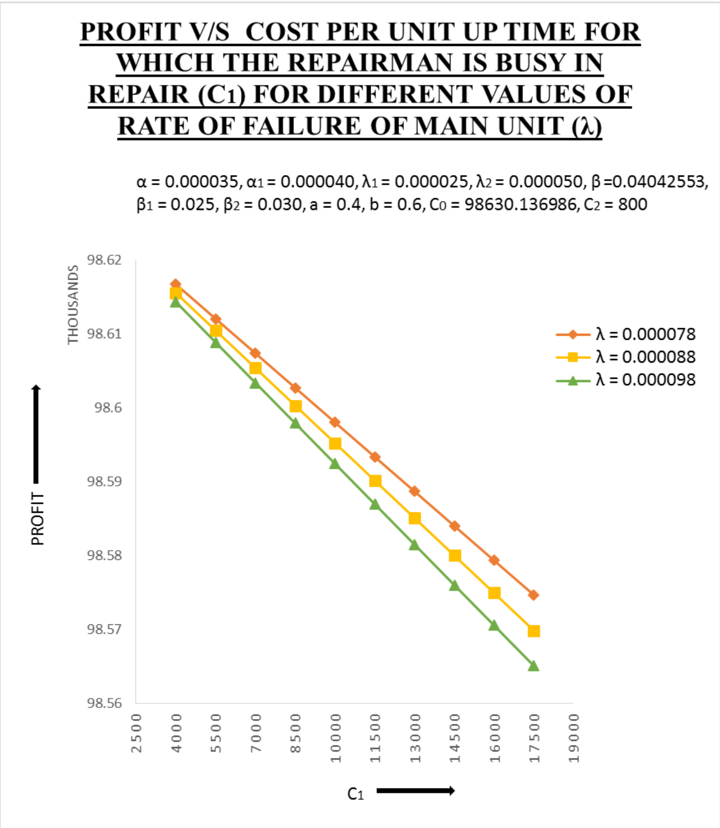


Fig. 5

Fig. 5 interprets the behaviour of profit w.r.t. to Cost per unit uptime for which the repairman is busy in Repair (C_1) for different values of rate of failure of main unit (λ). As the value of Cost per unit uptime for which the repairman is busy in Repair (C_1) increases, the profit decreases. Also, the profit decreases as failure of main unit (λ) increases.

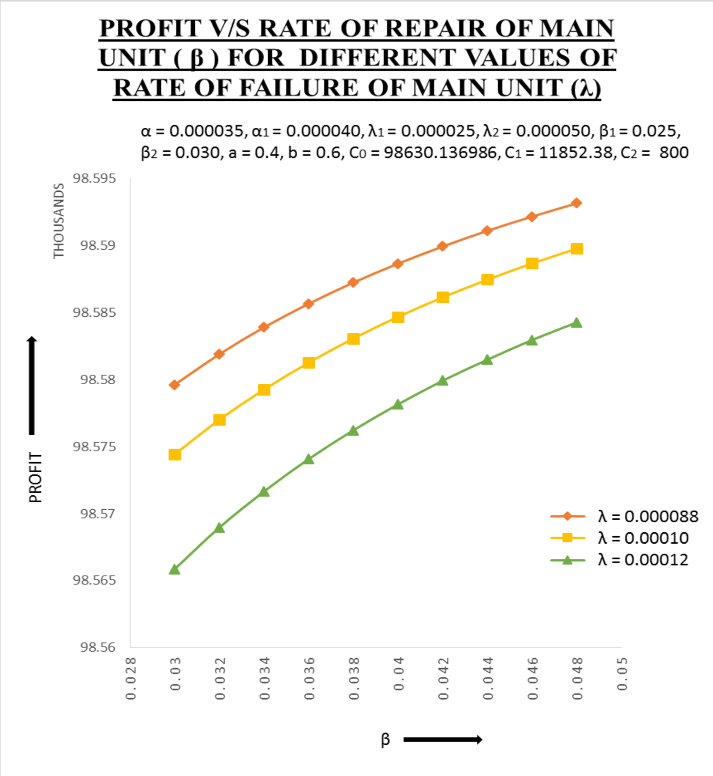


Fig. 6

Fig. 6 depicts the behaviour of profit w.r.t. to rate of repair of main unit (β) for different values of rate of failure of main unit (λ). As the values of rate of repair of main unit (β) increases, the profit increases. And, the profit decreases as failure rate of main unit (λ) increases.

10. Conclusion:

From the graphical interpretations given above, we can conclude that the cut off points for various rates/costs can be obtained which can assist in determining appropriate upper/lower acceptable values of rates/costs such that the system becomes profitable. It is clear from the study that the profit as well as MTSF decreases as failure rate increases. With the help of this research, one can obtain various measures for system effectiveness on the basis of which the company can build a proper model so that the system gives the positive profit. The upper/lower limits of various other rates/costs can also be obtained. Any company, industry or other user utilizing such systems can adopt exactly the same manner by taking the numerical values of various rates, costs, etc as existing there for such systems. Acquiring such values, numerous proposals can be given to the company using such systems.

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