International Journal of Marketing and Technology
Vol. 7 Issue 4, April 2017,
ISSN: 2249-1058 Impact Factor: 6.559
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

PRIME LABELING OF HEAWOOD GRAPHS

S.Chandrakala*

Abstract: A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers 1,2,…|V| such that for edge xy the labels assigned to x and y are relatively prime . A graph which admits prime labeling is called a prime graph . In this paper I investigate prime labeling for some special graph namely Heawood graph . I also discuss prime labeling in the context of graph operations namely duplication and switching in Heawood graph .

Keywords : Prime labeling , Heawood graph , Duplication,Switching.

I.INTRODUCTION

In this paper, I consider only finite simple undirected graph . The graph G has vertex set V =V(G) and edge set E =E(G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [6].Two integers a and b are said to be relatively prime if their greatest common divisor is 1.Many researchers have studied prime graph. Fu.H[3] has proved that the path P_n on n vertices is a prime graph.Deretsky et al [2] have proved that the cycle C_n on n vertices is a prime graph .Around 1980 Roger Entringer conjectured that all trees have prime labeling which is not settled today.

In [7] S.K. Vaidhya and K.K. Kanmani have proved that the graphs obtained by identifying any two vertices duplicating arbitrary vertex and switching of any vertex in cycle C_n admit prime

* **Assistant professor of Mathematics,T.D.M.N.S College ,T.Kallikulam,Tamilnadu,India.**

labeling. In [5] Meena and Vaithilingam have proved the Prime labeling for some helm related graphs.

The following definitions are necessary for the present study.

Definition 1.1

Let G = (V(G),E(G)) be a graph with p vertices . A bijection f :V(G)→ {1,2,…p} is called a prime labeling if for each edge e $=uv$, gcd$\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.2

Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that N$(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' $in\ G'$.

Definition 1.3

Duplication of a vertex v_k by a new edge e = $v_k' v_k''$ in a graph G produces a new graph G' such that N$(v_k') = \{v_{k,} v_k''\} and\ N(v_k'') = \{v_{k,} v_k'\}$.

Definition 1.4

A vertex switching G_v of a graph G is obtained by taking a vertex v of G ,removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.5

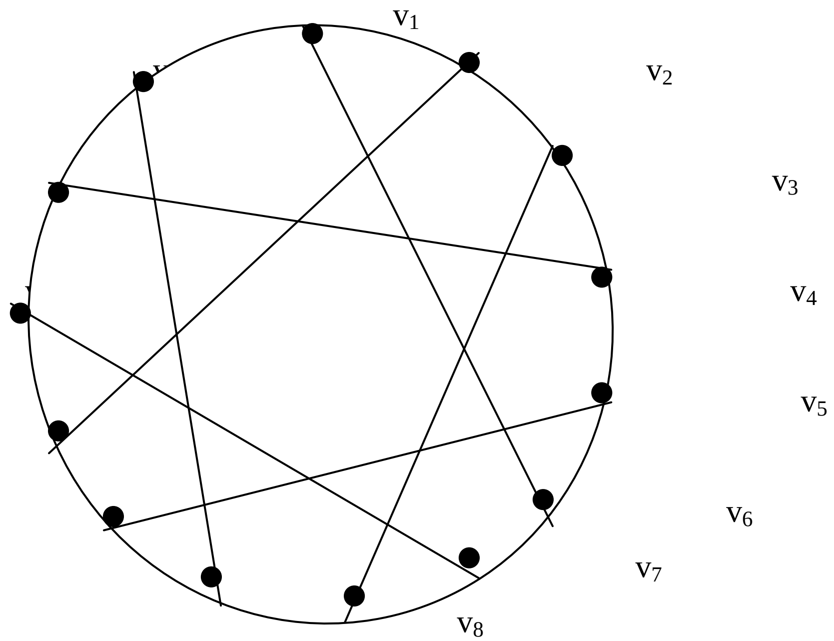
A regular graph is a graph where every vertex has the same degree. A regular graph with vertices of degree k is called a k- regular graph.

III. HEAWOOD GRAPH

Definition 1.6

The heawood graph is an undirected graph with 14 vertices and 21 edges .Heawood graph is a 3-regular graph.

Example



IV. MAIN RESULTS

Theorem 3.1

Heawood graph is a prime graph .

Proof :

Let G be the heawood graph with 14 vertices and 21 edges .

Then |V(G)|= 14 and |E(G)| =21.

The edge set E(G) ={ $v_i v_{i+1}$ / $1 \le i \le 13$ } $\cup \{v_1 v_{14}\} \cup \{v_i v_{i+5} \,/\, i = 1,3,5,7,9\}$

$\cup \{v_2 v_{11}\} \cup \{v_4 v_{13}\}$.

Define a labeling $f.V(G) \to \{1,2, \dots \dots \dots \dots ..14\}$

$by\ f(v_i) = i\ ,\ i = 2,3,4,6,7,8,9,10,11,12,13,14.$

$f(v_1) = 5$

$f(v_5) = 1$

Then $g.c.d\ \{f(v_i), f(v_{i+1})\} = 1, \quad 1 \le i \le 13$

$g.c.d\ \{f(v_i), f(v_{i+5})\} = 1, \quad i = 1,3,5,7,9.$

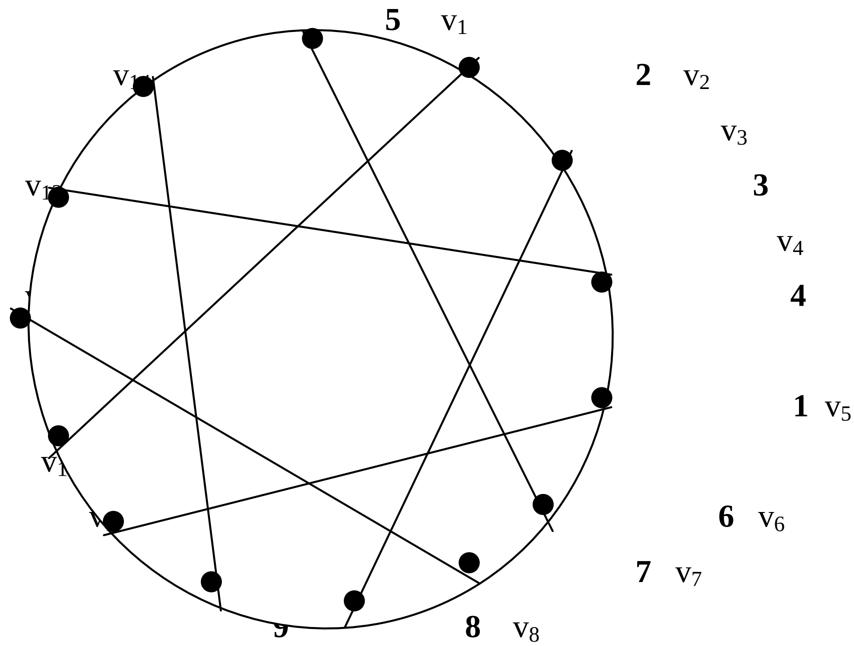
$g.c.d\ \{f(v_1), f(v_{14})\} = 1$

$g.c.d\ \{f(v_2), f(v_{11})\} = 1$

$g.c.d\ \{f(v_4), f(v_{13})\} = 1$

Then f admits a prime labeling. Hence G is a prime graph .

Illustration of Theorem 3.1



Prime labeling for Heawood graph

Theorem 3.2

Duplication of a vertex of Heawood graph is a prime graph .

Proof:

Let G be the Heawood graph .

Then $|V(G)| = 14 \; and \; |E(G)| = 21$.

Let G^* be the graph obtained by duplication of a vertex v_3 of Heawood graph .Let the new vertex be v_3' .

Then $|V(G^*)| = 15$ and $|E(G^*)| = 24$

The edge set $E(G^*) = \{v_i v_{i+1} / 1 \leq i \leq 13\} \cup \{v_1 v_{14}\} \cup \{v_i v_{i+5} / i = 1,3,5,7,9\} \cup \{v_2 v_{11}\} \cup \{v_4 v_{13}\} \cup \{v_2 v_3'\} \cup \{v_4 v_3'\} \cup \{v_8 v_3'\}$.

Define a labeling $f: V(G^*) \rightarrow \{1,2, \dots \dots \dots .15\}$ by

$$f(v_i) = i \, , \; i = 2,3,4,6,7,8,9,10,11,12,13,14.$$

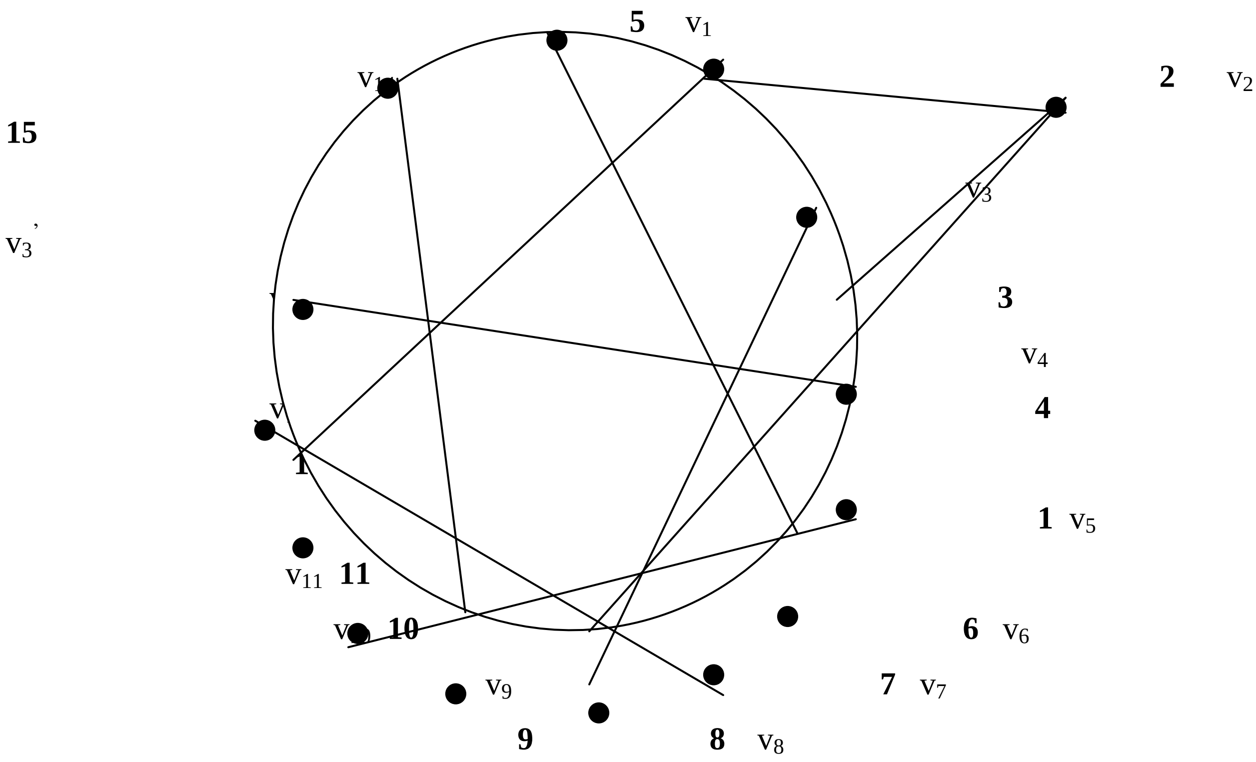
$$f(v_1) = 5$$

$f(v_5) = 1$

$f(v_3') = 15$

Then f admits Prime labeling .Hence G^* is a prime graph.

Illustration of Theorem 3.2



Prime labeling for duplication of a vertex v_3 in Heawood graph

Theorem 3.3

Duplication of an edge by a vertex of Heawood graph is a prime graph .

Proof:

Let G be the Heawood graph .Then $|V(G)| = 14$ *and* $|E(G)| = 21$.

Let G^* be the graph obtained by duplication of an edge by a new vertex w of Heawood graph .

Then $|V(G^*)| = 15$ and $|E(G^*)| = 23$

The edge E(G) = $\{v_i v_{i+1}/1 \le i \le 13\} \cup \{v_1 v_{14}\} \cup \{v_i v_{i+5}/i = 1,3,5,7,9\} \cup \{v_2 v_{11}\} \cup \{v_4 v_{13}\} \cup \{v_i v_w \ : i = 4,5\}$

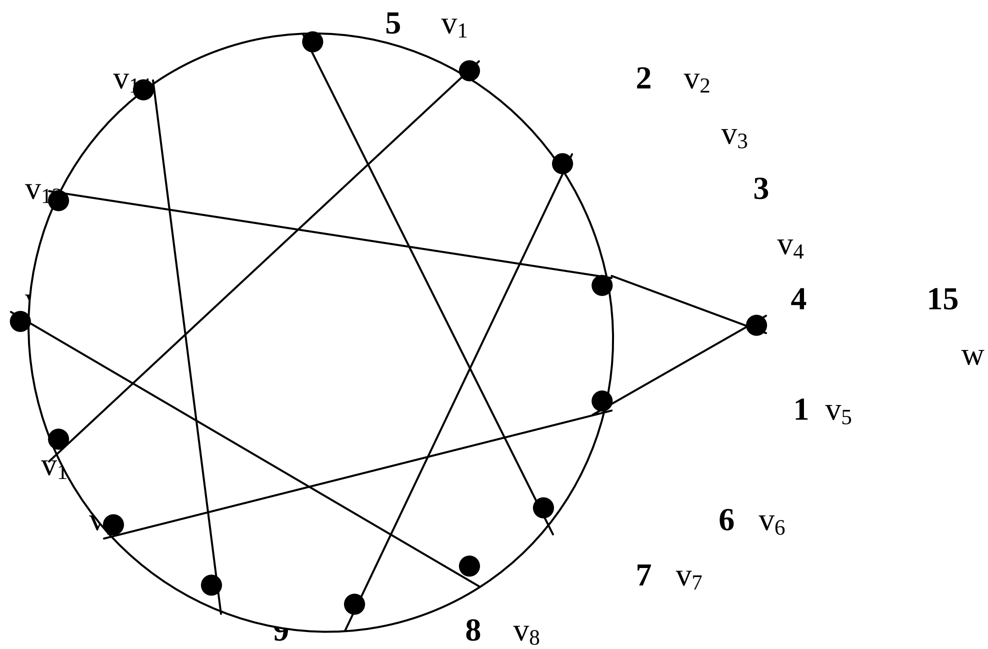
Define a labeling $f: V(G^*) \to \{1,2, \dots \dots \dots .15\}$

by $f(v_i) = i \,, i = 2,3,4,6 \dots \dots \dots \dots \dots 14$

$f(v_1) = 5$; $f(v_5) = 1$; $f(\omega) = 15.$

Then f admits prime labeling .Hence G* is a prime graph .

Illustration of theorem 3.3



Prime labeling for duplication of an edge v_4v_5 by a vertex w in Heawood graph

Theorem 3.4

Let G^* be the graph obtained by switching a vertex in the Heawood graph .Then G^* is a Prime graph.

Proof:

Let G be the Heawood graph .

Then $|V(G)| = 14\ and\ |E(G)| = 21$.

Let G* be the graph obtained by switching a vertex v_5 of Heawood graph .

Then $|V(G^*)| = 14$ and $|E(G^*)| = 28$

The edge $E(G^*) = \{v_iv_{i+1}\ : i = 1,2,3,6,7,8,9,10,11,12,13\} \cup \{v_1v_{14}\} \cup \{v_iv_{i+5}/i = 1,3,7,9\} \cup \{v_2v_{11}\} \cup \{v_4v_{13}\} \cup \{v_5v_i\ : i = 1,2,3,7,8,9,10,11,12,13,14\}$

Define a labeling $f: V(G^*) \to \{1,2, \dots \dots \dots .14\}$

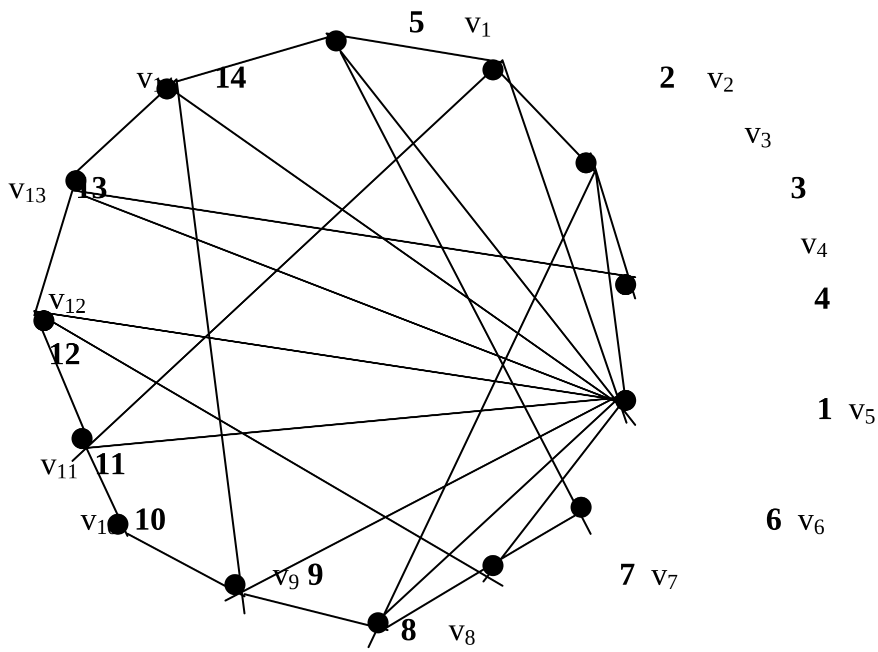
by $f(v_i) = i\,, i = 2,3,4,6,7 \dots \dots \dots \dots \dots 14$

$f(v_1) = 5$

$f(v_5) = 1$

Then f admits prime labeling .Hence G* is a prime graph .

Illustration of theorem 3.4



Prime labeling for switching of a vertex v_5 in Heawood graph

Theorem 3.5

Let G^* be the graph obtained by adjoining a vertex to each vertex in the Heawood graph.Then G^* is a prime graph.

proof:

Let G be the Heawood graph .Then $|V(G)| = 14\ and\ |E(G)| = 21$.

Let G* be the graph obtained by adjoining a vertex to each vertex in the Heawood graph .

Let $w_{1,}w_2,\ldots,w_{14}$ be the vertices adjoining to each vertex in the Heawood graph .

Then $|V(G^*)| = 28\ and\ |E(G^*)| = 35$.

The edge set E(G^*) ={ $v_i v_{i+1}$ / $1 \le i \le 13$ } $\cup \{v_1 v_{14}\} \cup \{v_i v_{i+5}$ / $i = 1,3,5,7,9\}$

$\cup \{v_2 v_{11}\} \cup \{v_4 v_{13}\} \cup \{v_{i+1}\, w_i\ : i = 1 \le i \le 13\}$.

Define a labeling $f.V(G) \to \{1,2, \ldots \ldots \ldots \ldots .28\}$

$by\ f(v_i) = i\ ,\ i = 2,3,4,6,7,8,9,10,11,12,13,14.$

$f(v_1) = 5$,

$f(v_5) = 1$,

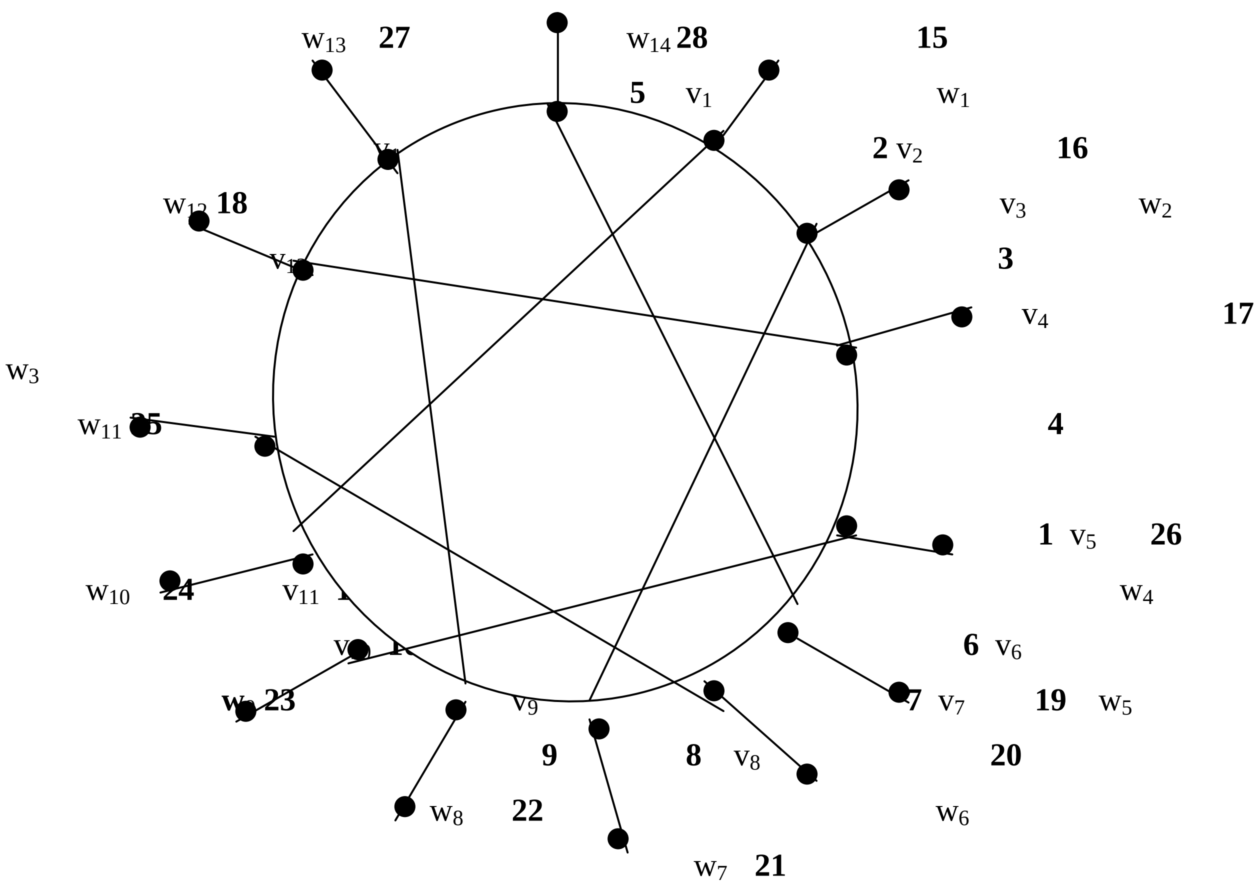
$f(w_i) = 14 + i\ : i = 1,2,3,5,6,7,8,9,10,11,13,14$.

$f(w_4) = 26$,

$f(w_{12}) = 18$.

Then f admits a Prime labeling .Then G^* is a Prime Graph.

Illustration of Theorem 3.5



prime labeling for adjoining of a vertex to each vertex in the Heawood graph

V.CONCLUSION

In the present work I investigate prime labeling of the special graph namely Heawood graph .To investigate similar result for other graph families and in the context of different graph operations is an open area of ressarch.

VI. ACKNOWLEDGEMENTS

The author is thankful to the refree for the valuable comments which improves the standard of the paper

REFERENCES

[1] Bondy .J.A and Murthy U.S.R, "Graph Theory and Application" (North Holland). New York (1976).

[2] Deretsky .T,Lee.S.M and Mitchem .J, "On Vertex Prime Labeling of Graphs in Graph Theory,Combinatorics and Applications", Vol.I (Ed.Alavi .J, Chartrand . G,Oellerman.O,and Schwenk . A) Proceedings of the 6th international conference Theory and Applications of Graphs, Wiley ,New York,(1991) 359-369.

[3] Fu, H.C and Huany .K.C. , " On prime Labelings Discrete Mathematics", 127(1994), 181-186.

[4] Gallian J.A , "A Dynamic survey of Graph labeling" ,The Electronic Journal of Combinatorics ,18(2011), 147,#DS6.

[5] Meena.S and Vaithilingam.K "Prime Labeling for some Helm Related Graphs", International Journal of innovative research in science ,Engineering and Technology ,Vol.2, 4 April 2009.

[6] Tout.A,Dabboucy .A.N and Howalla . K, "Prime Labeling of Graphs" ,Nat.Acad .Sci letters 11 (1982) 365-368

[7] Vaidya S.K and Kanmani K.K "Prime Labeling for some cycle Related Graphs",Journal of Mathematics Research vol.2. No.2.pp 98-104,May 2010.