

CERTAIN RULES REGARDING DIVISIBILITY BY PRIME NUMBERS BETWEEN 12 TO 100

Hoti Lal*

ABSTRACT

In Literature some rules of divisibility by prime numbers are available. In the present paper, we have formulated certain rules regarding the divisibility by prime numbers up to 100 (i.e. between 12 to 100). The paper is divided into two sections. Section 'A' describes rules for prime numbers with unity at unit place whereas Section 'B' discusses all other cases. Examples have been given to test the rules.

RULES OF THE TEST OF DIVISIBILITY BY ALL PRIME NUMBERS BETWEEN 12 TO 100 WITHOUT DIVISION PROCESS

The test of divisibility rules by all prime nos. between 12 to 100 can be done without division process. These rules are given by the author (**Hoti Lal**) himself and which are divided into two sections.

(A). Divisibility by prime nos. with unity at the unit place i.e. 31, 41, 61 and 71.

(B). Divisibility by other prime nos. i.e. 13, 17, 19, 23, 29, 37, 43, 47, 53, 59, 67, 73, 79, 83, 89 and 97.

SECTION 'A'

DETERMINATION OF RULE OF DIVISIBILITY BY PRIME NUMBERS

As every prime number in this class has digit 1 at unit place, hence we assume tenth place digit of these prime numbers as a standard number. For example, in prime nos. 31, 41, 61 and 71 the standard nos. are 3, 4, 6 and 7 respectively. Clearly these standard nos. are taken as follows:-

* **Science and Maths Teacher, Jr. High School, Pali Sadar, Achnera (Agra), (U.P.)**

In 31, 3 times of unit place no.
 In 41, 4 times of unit place no.
 In 61, 6 times of unit place no.
 and in 71, 7 times of unit place no.

Therefore in the test of divisibility by Prime number 31 we remove the unit place no. from given no. and subtract three times of removed no. (unit place no.) from remaining number, we get zero.

In the test of divisibility by 41 we remove unit place no. from given no. and subtract 4 times of removed no. (unit place number) from remaining no., we get zero.

In the test of divisibility by 61 we remove unit place no. from given no. and subtract 6 times of removing no. from remaining no. we get zero.

Similarly in the test of divisibility by 71, we remove unit place no. from given no. and subtract 7 times of unit place no. from remaining no. we get zero.

STEPS OF RULE TO TEST THE DIVISIBILITY BY PRIME NO. 31

- (i) First of all remove the unit place no. from the given no.
- (ii) Subtract three times of removed no. from remaining no.

If the difference is zero then given no. will be divisible by 31.

NOTE: If after first time's process the difference is not zero then repeat the process till a single digit no. is not obtained. If the last difference is zero, the given no. is certainly divisible by 31, otherwise not.

ILLUSTRATION: - Is the no. 4495, divisible by 31 (tell without division process).

CALCULATION

449(5)
 -15

 43(4)

ALGORITHMS

- (i) After removing the unit place no. (5) from the given no. 4495, the remaining no. is 449
- (ii) Now the difference between remaining no. 449 and 3 times of unit place no. (5) i.e. (5X3=15) is $449-15 = 434$

(iii) Again removing unit place no. (7) from 24637 , we get 2463

-49

 241(4) 2463-49=2414

(iv) Now difference between remaining no. 2463 & 7 times of removing no. (7) i.e. $7 \times 7 = 49$ is

(v) Again remaining no. after removing unit place no. (4) from 2414 is

-28

 21(3) 241

(vi) Difference between remaining no. 241 and 7 times of removing unit place no.(4) i.e. $(4 \times 7 = 28)$ is

241-28=213

(vii) Again remaining no. after removing the unit place no. (3) from 213 is

-21

 0 21

(viii) Now subtract 7 times of 3 (removing unit place no.) from remaining no. 21, the difference is

21-21=0

Verified:

Hence the no. 246725 is divisible by 71

$$246725/71=3475$$

SECTION 'B'

Section 'B' is divided into following two parts:-

I. The prime numbers who have '7' digit at the unit place (ie 17,37,47,67,97).**II.** Remaining other prime numbers (ie 13,19,23,29,43,53,59,73,79,83,89).

B (I). DETERMINATION OF THE RULE OF DIVISIBILITY BY PRIME NUMBERS

In this part of section 'B' we must be careful at the time of testing the rule of divisibility by prime numbers. If we want to check the divisibility of a number by a prime number first remove the unit place number of the prime number. Then write down the multiples of removing unit place number. Then subtract the tenth place number of the same prime number from every multiple of the removing unit place number. Now find out whose obtained number is the same prime number or any multiple of the prime number.

DETERMINATION OF RULE OF DIVISIBILITY BY PRIME NO. 17

In prime no. 17 the unit place no. is 7 and tenth place no. is 1. Therefore from 17, first remove unit place no. and write down its multiples. Then subtract tenth place no. (1) from every multiple. Now check the obtained numbers whose subtraction is equal to 17 or any multiple of 17.

For Example:-

The multiples of unit place no.(7) of 17	Multiple – Tenth place no.(1) of 17	whose subtraction is equal to 17 or equal to any multiple of 17
$7 \times 1 = 7$	$7 - 1 = 6$	Difference of 5 th multiple of 7 with tenth place no.(1) of 17 is 34 which is second multiple of 17.
$7 \times 2 = 14$	$14 - 1 = 13$	
$7 \times 3 = 21$	$21 - 1 = 20$	
$7 \times 4 = 28$	$28 - 1 = 27$	
$7 \times 5 = 35$	$35 - 1 = 34$	

It is clear from the above table, the difference of 5th multiple of unit place no. (7) of 17 with the tenth place no. (1) of the given prime no.(17) is 34 which is 2nd multiple of 17.

STEPS OF RULE TO TEST THE DIVISIBILITY BY PRIME NO. 17

- (i) First of all remove the unit place no. from the given no.
- (ii) Now subtract 5 times of removed no. from remaining no.

If obtained no. is zero or the simplest multiple of 17, then the given no. is divisible by 17.

NOTE:- If the obtained no. is neither equal to zero nor equal to simplest multiple of 17 after first time's process of both steps. Then repeat the process of both steps till at least two digits no. is not obtained. If thus obtained no. is a simple multiple of 17 or zero then the given no. is divisible by 17.

ILLUSTRATION: - How will you prove that the no. 41956 is divisible by 17 without division operation?

CALCULATION

4195(6)

-30

416(5)

-25

39(1)

-5

34 (2nd multiple of 17)

Verified :- $41956 / 17 = 2468$

ALGORITHMS

(i) The remaining no. after removing the unit place no.(6) from 41956 is

4195

(ii) Now difference between 4195 and 5 times of Unit place no. (6) is

$$4195 - 30 = 4165$$

(iii) Again removing the unit place no. (5) From 4165, the remaining no. is

416

(iv) Now difference between 416 and 5 times of removing no. (5) is

$$416 - 25 = 391$$

(v) Again the remaining no. after removing the unit place no. (1) from 391 is

39

(vi) Now subtract 5 times of removing no. (1) from 39 is

$$39 - 5 = 34 \dots\dots (2^{\text{nd}} \text{ multiple of } 17)$$

Thus the no. 41956 is divisible by 17.

DETERMINATION OF RULE OF DIVISIBILITY BY PRIME NO. 67.

In prime no 67, the unit place no. is 7 and tenth place no. is 6. Therefore, from 67, first remove unit place no. (7) and write down its multiples. Then subtract tenth place number (6) from every multiple. Now find out whose subtraction is equal to 67 or any multiple of 67.

The multiples of unit Place no. (7) of 67.	Multiple –tenth place no. (6) of 67.	Whose subtraction is equal to 67 or any multiple of 67.
$7 \times 1 = 7$	$7 - 6 = 1$	
$7 \times 2 = 14$	$14 - 6 = 8$	
$7 \times 3 = 21$	$21 - 6 = 15$	
.....	
.....	
$7 \times 11 = 77$	$77 - 6 = 71$	
$7 \times 12 = 84$	$84 - 6 = 78$	
$7 \times 13 = 91$	$91 - 6 = 85$	
.....	
.....	
$7 \times 18 = 126$	$126 - 6 = 120$	
$7 \times 19 = 133$	$133 - 6 = 127$	
$7 \times 20 = 140$	$140 - 6 = 134$	Difference of 20th multiple of 7 with tenth place number 6 of 67 is 134 which is 2nd multiple of 67.

It is clear from the above table that the difference between 20th multiple of unit place no. (7) of 67 with the tenth place no. 6 of the same prime number 67 is 134 which is the 2nd multiple of 67.

STEPS OF THE RULE TO TEST THE DIVISIBILITY BY THE PRIME NO. 67

- (i). First of all remove the unit place number from the given number.
- (ii). Subtract 20 times of removed number from remaining number.

If obtained number is either zero or simplest multiple of 67 then the given number is also divisible by 67.

NOTE: If the obtained number is neither equal to zero nor equal to simplest multiple of 67 after first time process of both above steps, repeat the process till last subtract is either zero or 1st or 2nd multiple of 67. If this is so then the given number is surely divisible by 67.

ILLUSTRATION: How can you prove that the number 454863 is divisible by 67 with out division process?

CALCULATION	ALGORITHMS
$\begin{array}{r} 45486(3) \\ -60 \\ \hline 4542(6) \end{array}$	(i). The remaining number after removing the unit place number (3) from 454863 is 45486 (ii). Then subtract 20 times of removed no. (3) from remaining no. 45486 $\text{Is } 45486 - (3 \times 20) = 45426$
$\begin{array}{r} 4542(6) \\ -120 \\ \hline 442(2) \end{array}$	(iii). Again removing the unit place no. (6) from 45426, the remaining number is 4542 (iv). Then subtract 20 times of removed no. (6) from 4542 is $4542 - (6 \times 20) = 4422$
$\begin{array}{r} 442(2) \\ -40 \\ \hline 40(2) \end{array}$	(v). Again removing the unit place no. (2) from 4422, the Remaining no. is 442. (vi). Then subtract 20 times of removed no. (2) from 442 is $442 - (2 \times 20) = 402$
$\begin{array}{r} 40(2) \\ -40 \\ \hline 0 \end{array}$	(vii). Again removing the unit place no. (2) from 402, the remaining no. is 40 (viii). Then subtract 20 times of removed no. (2) from remaining no. 40 is $40 - (2 \times 20) = 0$
Verified- $454863 \div 67$ $= 6789$	Thus the given no. 454863 is divisible by 67.

B(II). DETERMINATION OF THE RULE OF DIVISIBILITY BY PRIME NUMBERS.

In this part of section 'B' we must also careful at the time of testing the rule of divisibility by prime numbers. If we want to check the divisibility of a number by a prime number then according to it first remove the unit place number of the prime number. Then write down the multiples of the removing unit place number. Now add the tenth place number of same prime number with every multiple of the removing unit place number. Now find out whose obtained number is the same prime number.

DETERMINATION OF THE RULE OF DIVISIBILITY BY PRIME NO. 43

In prime no. 43, unit place no. is 3 & tenth place no. is 4. Therefore from 43 remove unit place no. 3 and write down its multiples. Now add tenth place no. (4) with every multiple. Now check the obtained numbers whose addition is equal to 43 or its any multiple.

For example

Multiples of unit place no.3 of 43	Multiple + tenth place no. (4) of 43	Whose addition is equal to 43 or its any multiple
$3 \times 1 = 3$	$3 + 4 = 7$	
$3 \times 2 = 6$	$6 + 4 = 10$	
$3 \times 3 = 9$	$9 + 4 = 13$	
$3 \times 4 = 12$	$12 + 4 = 16$	
$3 \times 5 = 15$	$15 + 4 = 19$	Addition of 13 th multiple of unit place no. (3) With 10 th place no. (4) of 43 is equal to 43 (the same prime no.)
$3 \times 6 = 18$	$18 + 4 = 22$	
$3 \times 7 = 21$	$21 + 4 = 25$	
$3 \times 8 = 24$	$24 + 4 = 28$	
$3 \times 9 = 27$	$27 + 4 = 31$	

$3 \times 10 = 30$	$30 + 4 = 34$	
$3 \times 11 = 33$	$33 + 4 = 37$	
$3 \times 12 = 36$	$36 + 4 = 40$	
$3 \times 13 = 39$	$39 + 4 = 43$	

From the above table it is clear that the addition of 13th multiple of unit place no. (3) With 10th place no.(4) of 43 is equal to the same prime no. 43.

STEPS OF RULE TO TEST THE DIVISIBILITY BY PRIME NO. 43

- (i) First of all remove the unit place no. from the given no.
- (ii) Then add thirteen times of the removed no. in the remaining no.

If the obtained no. is 43 or its simplest multiple then the given no. is divisible by 43

NOTE:- If the obtained no. is neither 43 nor simplest multiple of 43 after first two steps, repeat the above two steps process till maximum addition is near 200. If the last addition is 43 or its simplest multiple then given no. is divisible by 43.

ILLUSTRATION: - Without using division process how can you prove that the no. 31648 is divisible by 43.

CALCULATION

3164(8)

+104

ALGORITHMS

(i) The remaining no. after removing unit place no.(8) of 31648 is 3164

(ii) Now the no. after adding 13 times of the removing no. (8) with 3164 is

$$3164 + 8 \times 13 = 3268$$

326(8) (iii) Again the remaining no. after removing the unit place no. (8) from 3268 is 326

+104 (iv) Now the no. after adding 13 times of unit place no. (8) with 326 is

_____ 326+8x13 = 430

430
43 (Neglecting unit place (v) And now neglecting 0 (the unit place no.) No.(0) from 430 we get 43, the same prime no.

Verified : 31648/43

= 736 Therefore the no. 31648 is divisible by 43

NOTE: If after addition or subtraction, zero comes at unit place no. then neglect it and repeat the same process as need.

DETERMINATION OF RULE OF DIVISIBILITY BY PRIME NO. 59.

In prime no 59, the unit place no. is 9 and tenth place no. is 5. Therefore, from 59, remove unit place no. (9) and write down the multiples of removed no. 9. Then add these multiples with tenth place no. (5) of 59 and check the obtained nos. that whose addition no. is equal to 59 or any multiple of 59.

For example:-

The multiples of unit place no. (9) of the prime no. 59	Multiples + tenth place no. (5) of 59	Whose addition is equal to 59 or any multiple of 59

$9 \times 1 = 9$	$9 + 5 = 14$	Addition of 6 th multiple of 9 with tenth place no. (5) of 59 is himself the same prime no. (59)
$9 \times 2 = 18$	$18 + 5 = 23$	
$9 \times 3 = 27$	$27 + 5 = 32$	
$9 \times 4 = 36$	$36 + 5 = 41$	
$9 \times 5 = 45$	$45 + 5 = 50$	
$9 \times 6 = 54$	$54 + 5 = 59$	

From the above table it is clear that the addition of tenth place no. of 59 with 6 times of unit place no.(9) of 59 is himself the same prime no. Therefore

STEPS OF RULE TO TEST THE DIVISIBILITY BY PRIME NO. 59

- (i) First of all remove the unit place no. from the given no.
- (ii) Now add 6 times of removed no. with remaining no.

If thus obtained no. is 59 or simple multiple of 59, then the given no. is divisible by 59.

NOTE:- If the obtained no. is neither equal to 59 nor equal to simple multiple of 59 after first times process of both steps, then repeat the process till the maximum addition near 200 is not obtained . If the last addition is 59 or any simple multiple of 59, then the given no. is divisible by 59.

ILLUSTRATION:- Without division process how can you prove that the no.

5058188 is divisible by 59?

CALCULATION

505818(8)

ALGORITHMS

- (i) The remaining no. after removing the unit place no. (8) from 5058188 is

505818

___+48___
50586(6)

(ii) Now the no. after adding 6 times of unit place no. (8) with 505818 is

$$505818 + (8 \times 6) = 505866$$

(iii) Again the no. after removing the unit place no. (6) from the no. 505866 is

$$50586$$

+36

5062(2)

(iv) Now the no. after adding 6 times of unit place no. (6) with 50586 is

$$50586 + (6 \times 6) = 50622$$

(v) Again the no. After removing the unit place no. (2) from the no. 50622 is

$$5062$$

(vi) Now the no. after adding 6 times of unit place no. (2) with 5062 is

$$5062 + (2 \times 6) = 5074$$

+12

507(4)

(vii) Again the no. after removing the unit place no. (4) from the no. 5074 is

$$507$$

+24

53(1)

(viii) Now the no. after adding 6 times of unit place no.(4) with 507 is

$$507 + (4 \times 6) = 531$$

(ix) Again the no. after removing the unit place no. (1) from no. 531 is

$$53$$

+6

59... (The same Prime no.)

(x) Now the no. after adding 6 times of unit place no. (1) with 53 is

$$53 + (1 \times 6) = 59 \dots (\text{Same Prime no.})$$

Verified:- $5058188 / 59 = 85732$

Hence the no. 5058188 is divisible by 59.

In the same way the reader can also describes the rules of testing of divisibility by remaining prime nos. of the section “B” (Such as 17, 59 & 43 which are given above) However, the convenience of readers we are giving the table related to them

The remaining prime nos. of section “B”	For determination of Rules how many times of removed unit no. from prime no. should be added or subtracted with tenth place no.	
	Add	Subtract
13	4 times	
19	2 times	
23	7 times	
29	3 times	
37		11 times
47		14 times
53	16 times	
73	22 times	
79	8 times	
83	25 times	
89	9 times	
97		29 times