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STUDENTS' ERROR ANALYSIS ON A GIVEN GEOMETRIC PROOFS AND SOLUTIONS: ITS EFFECT ON THEIR ACHIEVEMENT AND CONCEPTUAL UNDERSTANDING

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Abstract

Proof and reasoning are essential mathematical skills in Geometry subject. Engaging in this level of higher order thinking skills, students' need to develop and learn basic concepts competencies in proving especially in geometry. Students must have a good conceptual understanding of mathematical ideas and theories to have a proper understanding of mathematical proof and proving techniques. This study investigated the effect of students' error analysis of geometric proofs on their achievement and conceptual understanding. It employed quasi experimental non-equivalent control group design and was conducted at Prosperidad National High School, Prosperidad, Agusan del Surfrom August to October 2016. The participants were the two intact classes of Grade 10 students assigned as experimental and control

Keywords:

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group. Both groups were given pre-test of the teacher made two tiered achievement test and conceptual understanding test before the actual experiment. To develop students' capacity to detect error in a given geometric proofs, a worktext was developed. The control group was exposed to traditional mathematics instruction with group work answering, activity sheets, journal writing and short quizzes. The experimental group was taught using error analysis with group work, find an expert activity, journal writing and short quizzes which emphasized on detecting errors in a given geometric proofs and after experimental period both groups were given the post-test. The analysis using one way ANCOVA was used at 0.05 level of significance. Results revealed that students taught with error analysis have better performance in the achievement and conceptual understanding tests than the students taught using traditional method. Moreover, experimental group students had a better conceptual understanding of geometric concepts and improve their proof skills compared to the control group.

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1. Introduction

Proving is one of the most interesting but difficult competency in mathematics. Teaching students' how to write mathematical proof is essential because this skill develops students reasoning and critical thinking ability. [1] pointed out that all students must learn to argue mathematical ideas, give reasons on the existence of mathematical objects and prove mathematical arguments which are vital in science and technology, engineering and mathematics (STEM) courses. However, despite the importance of proving skills, little attention had been given to this field of mathematics. This may be due to teachers' lack of knowledge in writing mathematical proof and their educational experience did not require to do so. [2] believed that teachers' play a critical role in classroom learning hence it is important to investigate what influence them from doing away of teaching mathematical proof to their students. Letting students experienced how to write mathematical proof at the early years even though it is difficult at the start will enhance their proving skill later. Proof writing is an essential aspect of mathematics teaching and students must not be deprive of experiencing it [3]. [4] observed that it takes students several years to master the competency in proof writing. This indicates that learning experience is significant and writing proof is a vehicle that best enhance students' mathematical assimilation and understanding [5].

The results of the National Career Assessment Examination (NCAE) and the Mathematical Society of the Philippines (MSP) competitions where logical reasoning and critical thinking were given more emphasis showed that most students had difficulty in proving mathematical arguments [6]. To address this, the Department of Education (DepEd) introduce geometric proof in Grade 9 and Grade 10 and mandated teachers to teach mathematics with varied teaching techniques and strategies to develop students' higher order thinking skills [7],[8]. Exposing students to mathematical conjectures, proving theorems, justify processes and generate their own proof make them develop a deeper understanding of the abstract entities of mathematics that will eventually result to satisfactory level of conceptual understanding. To improve students' proof writing skills, they need to understand the structure of proving mathematical arguments and its theoretical construct to develop fluency. In this regard, students may commit errors of writing geometric proofs. Hence, it is important to analyze these errors since these are fundamental on the growth of their mathematical abstraction and learning.

The goal of error analysis in writing geometric proofs is to gauge students' mathematical concept acquisition and learning progress. [9] said that error analysis is integral in mathematics instruction and integrating it in writing geometric proof is indispensable for effective learning to happen. Since proving ability is not natural to all students, teacher must design pedagogy that will enable his students to be inquisitive to discover his own errors in writing geometric proofs. [10] further said that since geometric proofs are complex in nature, teachers should organize interventions and learning activities that focus on identifying students' errors in order to be successful in fostering proof writing competence. In order to construct geometric proofs elimination of cognitive constraints such as misconceptions of mathematical ideas and theorems. This may result to students' development of critical thinking and problem solving skills since students will become the backbone of the country's incremental growing economy there is a need of human resources who are problem solvers and critical thinkers that are free of errors.

Thus, this study was undertaken to investigate the effect of students' error analysis on geometric proofs and solutions to their achievement and conceptual understanding.

2. Research Method

This study employed quasi-experimental non-equivalent control group design. The participants of the study were the 96 Grade 10 students of the two intact classes randomly assigned as experimental and control group. They were chosen from among the six sections of Grade 10 students of Prosperidad National High School, Prosperidad, Agusandel Surduring the School Year 2016 – 2017. Before the study started, a pretest was administered to both groups using teacher made two tiered test and conceptual understanding test. The first tier is a 22 item multiple test that assessed students' cognitive knowledge in Geometry and the second tier ask the students to give justifications of their answers. The conceptual understanding test determine students' interpretation, explanation and application of mathematical concepts to the 5 open ended questions. The test included the topics in chords, arcs and central angle, inscribed angle, sector and segments of a circle, tangents and secants to circle, tangent and secant segments of a circle, rectangular coordinates, distance formula, midpoint formula and equation of a circle. To develop students' ability to detect error patterns in a given geometric proof and solutions, a develop worktext was used in the class.

After the teacher equipped both group of students through lecture discussion, control group students were exposed to the traditional mathematics instruction of group work answering, activity sheets and journal writing. However, in the experimental group students underwent activities like group discussion, find an expert activity and journal writing that focused on error analysis of writing geometric proof and solutions. Quizzes were given after every lesson. At the end of the experimental period, both groups of students were given posttest on achievement and conceptual understanding test. Students proficiency level was determine using the K-12 proficiency level while students level of conceptual understanding were assessed based on Wiggins and McTighes (1998) facets of understanding. The data gathered were analyzed using Analysis of Covariance (ANCOVA).

2.1 Developing Students' Error Analysis and Proving ability through Group Discussion

The experimental group were first equipped with mathematical concept and skills through lecture-discussion method. During the discussion, they used the worktext on error analysis and proved different theorems in Geometry. In group work activity, the teacher presented a theorem or mathematical argument and solutions where students wrote their answers in a manila paper. After the groups had finished answering the task, they were asked to pass their work to another group in a circular manner. The task of the group who received the other group's work was to identify the errors committed in the geometric proof. When the group identified a possible error on the statement or reason of the proof and solutions, they needed to investigate and discussed the possible causes of the error which determine what mathematical concepts were used incorrectly. After identification of this error, they wrote the correct statement and reason of the geometric proof in the same manila paper without deleting the proof of the group they received. The group who did error analysis may provide alternative statements and reason to the proofs of the other groups of which they thought was correct. The process continued until all the manila papers or proofs of each group returned to them. The students and the teacher agreed the time allotment for the activity which is 5 minutes. When the groups' work was returned to them, each group discussed their proof and solutions to the class. After all the groups have presented their work, the teacher summarized the activity.

2.2 Developing Students' Error Analysis and Proving ability through Find an Expert Activity

In this activity, after the teacher have discussed the lesson, he posted a problem on the board and instructed students to solve it using activity notebooks and showed their answers to the teacher. For those students who were not able to write the correct geometric proof, they asked help from the list of expert students provided by the teacher prior to the discussion of the lesson. These expert students were selected based from their academic rank from previous years. The expert student was instructed not to show his proof to anyone but to help find errors in his answer by asking questions to led to the mistake. The student who failed to write the correct proof on his activity notebook, together with the expert analyzed the errors on the statement and reason of the proof. After identifying the error on his proof, the student wrote the correct geometric proof on his activity notebook.

2.3 Developing Students' Error Analysis and Proof technique through Journal writing

To further enhance proving skills, students were tasked to make a reflection through journal. Entries in the journal are from the students common errors committed while working collaboratively in the error analysis activities. Students were ask to give their insights on how the different errors they committed in writing geometric proof enabled them to understand the theorem and mathematical argument discussed. This was given at the end of every lesson. The teacher also gave a geometric proofs that contained incorrect statements and reasons. The tasked of the students was to correct the statements and correct reasons of a geometric proof which are wrongly explained the causes of errors committed in the proof.

3. Results and Analysis

The result of the analysis is presented in the following tables.

Table 1. Means, standard deviation and descriptive level of students' mathematical achievement score

	Experimental group (n=48)		Control group (n=48)	
	Pretest	Posttest	Pretest	Posttest
Mean Score	7.28	62.33	6.66	36.52
SD	2.41	10.73	2.46	12.29
Descriptive level	Beginning	Approaching Proficiency	Beginning	Developing

Table 2. One way ANCOVA summary for students' achievement score

Source	Df	Adjusted SS	Adjusted MS	F	P
Treatment within	1	12621.9	12621.9	89.58	0.0001*
Error	94	12543.8	140.9		
Total	95	25165.7			

*Significant at 0.05 level

Table 1 shows experimental and control group mean scores in their achievement pretest which are 7.28 and 6.66 respectively both describe at beginning level. The students' scores are low since the total points is 110. In posttest, it shows a significant increase of both groups means scores. The experimental group proficiency level had improved to approaching proficiency with score more than 50% of the total points while the control group attained developing level where score is less than 50%.

The results of experimental group and control group standard deviations both in pretest and posttest are comparable an indication that their scores increase proportionally. The smaller standard deviations of both groups in achievement pretests suggest that students have common acquisition of geometric concepts during their earlier years in schools. However, after the instruction their scores tend to vary from each other an indication that the treatment received by the participants affected their achievement score in Geometry.

To determine if there is significant difference in students' mean scores, Analysis of Covariance (ANCOVA) was used.

Table 2 shows the result of the analysis of covariance on students' scores on the effect of teaching method used which resulted to a computed F ratio of 89.55 with a probability value of 0.0001 lesser than 0.05 level of significance. This led to the researcher's none acceptance of the null hypothesis which implies that there is significant difference in the experimental group students' posttest mean scores which is 62.33 in the achievement test compare to the control group which is 36.52.

The results further imply that students who are exposed in error analysis have a better performance in the achievement test. The students proving skills in writing geometric proof had also improve as exhibited during the different activities using error analysis. Doing error analysis in the different geometric proofs enable students to enhance their logical reasoning which is essential in writing proof. They combine logic and imagination to visualize their proofs which is clearly evident when students' engage in the different activities. Participants' analysis of the statements and reasons using two column proof become better after the instruction as shown in their posttest mean scores. This was observable when students' find hard time in proving theorem using two column proof for the first time since they were not exposed before in writing mathematical proofs. However, when lessons progresses, their proof skills also improved an indication of which error analysis helped in developing student proving ability. Exposing students to making proofs in teaching mathematics lead them to acquire critical thinking skills and problem solving ability. The result supported the findings of [11] that engaging students in an inquiry based activities which includes error analysis in writing proof have improved their achievement level and enhanced proof technique.

Table 3. Means, standard deviation and descriptive level of students' conceptual understanding score

	Experimental group (n=48)		Control group (n=48)	
	Pretest	Posttest	Pretest	Posttest
Mean Score	0.27	31.21	0.18	12.34
SD	0.80	6.66	0.75	10.93
Descriptive level	Beginning	Moderately Strong	Beginning	Beginning

Table 4. One way ANCOVA summary for students' conceptual understanding score

Source	Df	Adjusted SS	Adjusted MS	F	P
Treatment within	1	8214.03	12621.9	84.77	0.0001*
Error	94	8984.14	140.9		
Total	95	17198.17			

*Significant at 0.05 level

Table 3 shows the experimental and control group means scores in the conceptual understanding pretest of 0.27 and 0.18 respectively which have a description of beginning level. Their pretest score is very low because the total number of score is 110. In the posttest, experimental group conceptual understanding level had improved to moderately strong with a mean of 31.21 but still low because it is only 28% of the total points while the control group remain at the beginning level with a corresponding mean score of 12.34 which only 11.22% of the total points.

Moreover, the table further shows the experimental and control group standard deviations in the pretest of 0.80 and 0.75 respectively. The very small standard deviations of both groups indicate that their scores are very close to each. After the instruction, their standard deviations have increase to 6.66 and 10.93 with a difference of 4.27. This means that the scores of both groups are now widely dispersed where some are high score while others are very low. To determine if there is significant difference of the students' mean scores ANCOVA was used.

Table 4 shows the analysis of covariance to the mean scores of the students' in terms of treatment they received. The table yielded a computed F ratio of 84.77 with a probability value of 0.0001 lesser than 0.05 level of significance. This led to the rejection of the null hypothesis which implies that there is a significant difference of the experimental group students mean scores 31.21 as influenced by the treatment they received which is error analysis technique.

Students' error analysis on geometric proofs and solutions made the mean scores of the experimental group significantly higher in conceptual understanding tests compare to the control group with mean score of 12.34. Engaging in the different activities by analyzing the errors made in the solutions enable students to have better assimilation of geometric concepts which resulted to a satisfactory level of conceptual understanding. The engaging power of writing geometric proofs and the effect of error analysis on students have helped the process of accommodating new information that produce better learning. These were observable when students' exchange and share ideas during the activities to come up with the proofs of the given problem. Thus error analysis has influenced students' performance level in the posttest. The result supported the findings of [12] that students develop better conceptual understanding and proving skills when they have the chance to analyze errors in writing geometric proof.

Students' optimism towards the subject were also evident in making their journals as they explicitly stated how their errors made positive impact on their learning progress. They become interested in Mathematics and enjoyed proving theorems even though it is difficult.

4. Conclusion

Based on the findings of the study, the researcher concludes that students' error analysis on geometric proofs and solutions in the class is effective in improving students' achievement and conceptual understanding. Hence, the researcher recommends the use of error analysis in classroom discussions. Similar study may be conducted to a different population in a wider scope to enhance the generalizability of using error analysis in the class.

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