INTRODUCING AN INTEGRATING FACTOR IN STUDYING THE WAGE EQUATION

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Abstract

In this paper a first order wage equation is solved by the method of integrating factor. The subsequent wage function is then analyzed and interpreted for stability. The function could initially stand off the equilibrium wage rate but in the long run, it asymptotically stabilizes in inter temporal sense. It is observed that use of an integrating factor in solving the wage equation is just as effective as Laplace transforms demonstrated in [6] but with an advantage of being simple with limited algebra.

Key words: wage equation, wage function, wage rate, stability, and equilibrium wage rate.

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1. Introduction

Wages mean the reward for labor services. In [7] wage is defined as a fixed regular payment earned for work or services, typically paid on a daily or weekly basis. In [10] wage is viewed as payment for labour services to a worker, especially remuneration on an hourly, daily, weekly or by piece. It also views wage as a portion of national product that represents aggregate paid for all contributing labour and services as distinguished from the portion retained by management or reinvested in capital goods. In [11] wage is defined according to wages act of 1986. It is the sum payable to an employee by an employer in connection with that employment. It includes fees, bonuses, commissions, holiday pay or other emolument relevant to the employment whether specified in the contract of employment or not.

In this paper, we consider solving a first order ordinary differential wage equation using an integrating factor. Similar method is used in [8] where a deterministic price adjustment model is considered. In the study, a fixed supply and demand functions at instantaneous price for security are is discussed. It is argued that at equilibrium asset price, the quantity demanded is equal to the quantity supplied. This is discussed using the assumptions of fixed demand and supply curves while the price is kept constant. It also asserts that away from the equilibrium, excess demand for security raises its price, and excess supply lowers its price. In this situation, it is argued that the sign for the rate of change of price with respect to time depends on the sign of excess demand. If the demand and supply functions are linearized about a constant equilibrium price, the deterministic model of price adjustment is realized with respective sensitivities. This model is also called the deterministic logistic first order differential equation in price. The equation is solved in this case by introducing an integrating factor. In the analysis of the solution, it was observed that in the long run the asset price settles at a constant steady state point at which no further change can occur.

In [4] a natural decay equation is developed. The equation describes a phenomenon where a quantity gradually decreases to zero. In the work, it is emphasized that convergence depends the sign of the change parameter. If the change parameter is negative then it turns into a growth equation and if it is positive it stabilizes in the long run. The study of slope fields for autonomous equations and qualitative properties of the decay equation are also demonstrated. It was found that the solution could be positive, negative or zero. In all the three cases, the solution approaches zero in limit as time approaches infinity. In the same work, models representing

Newton's law of cooling, depreciation, population dynamics of diseases and water drainage are also presented with similar property in the long run.

In [1], dynamics of market prices are studied. It was found out that if the initial price of the price function lies off the equilibrium point, then in the long run the price stability will be realized at equilibrium position. In [2], equilibrium solutions representing a special class of static solutions are discussed. The study found that if a system starts exactly at equilibrium condition, then it will remain there forever. The study further found that in real systems, small disturbances often a rise which moves a system away from the equilibrium state. Such disturbances, regardless of their origin give rise to initial conditions which do not coincide with the equilibrium condition. If the system is not at equilibrium point, then some of its derivatives will be non zero and the system therefore exhibits a dynamic behavior, which can be monitored by watching orbits involved in the phase space.

The resistance-inductance electric circuit for constant electromotive force is modeled into a differential equation in [3]. The equation is solved by introducing an integrating factor. The stability of the solution is studied in the long run and is found to be a constant, which is the ratio of the constant electromotive force to the resistance.

In [6], a first order differential wage equation

$$\frac{dW}{dt} + \xi(\lambda + \sigma)W = \xi(\eta + \theta) \tag{1.1}$$

is developed using linear demand function

$$N_d = \eta - \sigma W, \quad (\eta, \sigma > 0) \tag{1.2}$$

and linear supply function

$$N_s = -\theta + \lambda W, \quad (\theta, \lambda > 0) \tag{1.3}$$

where η is a parameter showing the number of laborers demanded that does not depend on the wage rate W and σ is a parameter that shows the proportion by which the number of laborers demanded N_d responds to variation in wage rate; θ is a parameter showing the number of laborers supplied that does not depend on the wage rate W and λ is a parameter that shows the proportion by which the number of laborers supplied N_s responds to variation in wage rate. At equilibrium, the number of laborers demanded must equal the number of laborers supplied resulting in an equilibrium wage rate

$$\hat{W} = \frac{\eta+\theta}{\lambda+\sigma}, \ \lambda \neq -\sigma \qquad (1.4)$$

Equation (1.1) was therefore solved by Laplace Transforms. The subsequent wage function was analyzed and interpreted for stability. The function was found to initially stand off the equilibrium wage rate but in the long run, it became asymptotically stable in inter temporal sense. It also found that free market forces cause uncertainties when volatility in wage rate was experienced and this was witnessed could affect both investments and employment if not controlled. The paper proposed creating a middle path in which wage rate is allowed to oscillate freely within a narrow band managed by employers in consultation with the workers under the watch of the government.

The current paper therefore proposes solving wage equation [1.1] by introducing an integrating factor; analyzing and interpreting the results. Numerous solutions of linear ordinary differential equations using an integrating factor are presented in [5; 9] and the current work has borrowed immensely from them.

2. Solution of first order wage equation

In this section, we consider solving the first order wage equation (1.1). We make use of the integrating factor to perform the operation. Suppose we let $a = \xi(\sigma+\lambda)$ and $b = \xi(\eta+\theta)$, then equation (1.1) can be written more simply as

$$\frac{dW}{dt} + aW = b \qquad (2.1)$$

Equation (2.1) can quickly be solved by the use of an integrating factor. Thus the integrating factor is

$$\exp\left(\int a dt\right) = \exp at. \qquad (2.2)$$

Equation (2.1) can therefore be written as

$$\frac{d}{dt}\left(W \exp at\right) = b \exp at. \qquad (2.3)$$

Integrating both sides of equation (2.3) with respect to t gives

$$W \exp at = \int b \exp at dt$$

$$= \frac{b}{a} \exp at + c$$

$$\therefore W(t) = \frac{b}{a} + c \exp(-at) \quad (2.4)$$

where c is a constant of integration. Considering that $a = \xi(\sigma+\lambda)$ *and* $b = \xi(\eta+\theta)$, solution (2.4) can be written as

$$W(t) = \left(\frac{\eta+\theta}{\sigma+\lambda}\right) + c \exp(-\xi(\sigma+\lambda)t) \quad (2.5)$$

Solution (2.5) is the time path function of the wage equation (1.1). Suppose we denote the initial wage rate as W_0; that is, $W(t)|_{t=0} = W_0$ then the constant of integration c in solution (2.5) is found as

$$\left(W_0 - \frac{\eta+\theta}{\sigma+\lambda}\right) \quad (2.6)$$

If we substitute the value of c expression (2.6) in solution (2.5), then the solution is written as

$$W(t) = \left(\frac{\eta+\theta}{\sigma+\lambda}\right) + \left(W_0 - \frac{\eta+\theta}{\sigma+\lambda}\right) \exp(-\xi(\sigma+\lambda)t) \quad (2.7)$$

If we consider the equilibrium wage rate $\hat{W} = \frac{\eta+\theta}{\sigma+\lambda}$, $\sigma \neq -\lambda$ in (1.4), then solution (2.7) can be written in terms of $\hat{W}$ as

$$W(t) = \hat{W} + (W_0 - \hat{W}) \exp(-\xi(\sigma+\lambda)t) \quad (2.8)$$

and this is the required particular solution of the wage equation (1.1). We now proceed to the next section to study the function (2.8) for stability.

3. Results, analysis and interpretation

In this section, the results of this study are presented, analyzed and interpreted.

In this paper, equation (1.1) has been solved by the use of an integrating factor. Fortunately, if we use the initial condition $W(t)|_{t=0} = W_0$ a particular solution

$$W(t) = \hat{W} + \left(W_0 - \hat{W}\right)\exp\left(-\xi(\sigma+\lambda)t\right), \quad where\ \hat{W} = \frac{\eta+\theta}{\sigma+\lambda}, \quad \sigma \neq -\lambda \qquad (3.2)$$

is obtained. We now investigate function (3.2) for stability by finding its path in the long run, i.e.

$$W(t) = \lim_{t\to\infty}\left[\hat{W} + \left(W_0 - \hat{W}\right)\exp\left(-\xi(\sigma+\lambda)t\right)\right] \qquad (3.3)$$

In this case, $\left(W_0 - \hat{W}\right)$ is a constant and the value of the limit function (3.3) depends on the key factor $\exp\left(-\xi(\sigma+\lambda)t\right)$. In view of the fact that $\xi(\sigma+\lambda) > 0$, $\left(W_0 - \hat{W}\right)\exp\left(-\xi(\sigma+\lambda)t\right) \to 0$ $as\ t \to \infty$. The limit function (3.3) therefore becomes

$$W(t) = \hat{W}, \quad where \quad \hat{W} = \frac{\eta+\theta}{\sigma+\lambda}, \quad \sigma \neq -\lambda \qquad (3.4)$$

This means that the time path of the wage function (3.3) moves towards the equilibrium position in the long run. This is interpreted in an inter-temporal sense rather than the market clearing sense. Further analysis of function (3.3) is possible by considering the relative positions of $W_0\ and\ \hat{W}$; that is, comparing the relative positions of the initial wage rate and the equilibrium wage rate. This may be discussed in three different cases.

CASE I: In this case, we let $W_0 = \hat{W}$. This means that the wage function (3.3) becomes $W(t) = \hat{W}$. The time path of the function is thus constant and parallel to the time axis. The wage function is therefore in a stable state.

CASE II: In this case, we let $W_0 > \hat{W}$. The second term on the right hand side of function (3.3) is positive but it decreases as $t \to \infty$ since it is lowered by the value of the exponential factor $\exp\left(-\xi(\sigma+\lambda)t\right)$. The time path thus asymptotically approaches the equilibrium value $\hat{W}$ from above and in the long run, the wage function becomes stable.

CASE III: In this case, we let $W_0 < \hat{W}$, i.e. the initial wage rate is taken to be less than the equilibrium wage rate. The second term of the right hand side of function (3.3) is negative and it infinitely makes W_0 to rise asymptotically towards the equilibrium wage $\hat{W}\ as\ t \to \infty$. These three cases are illustrated as shown in figure 3.1 below.

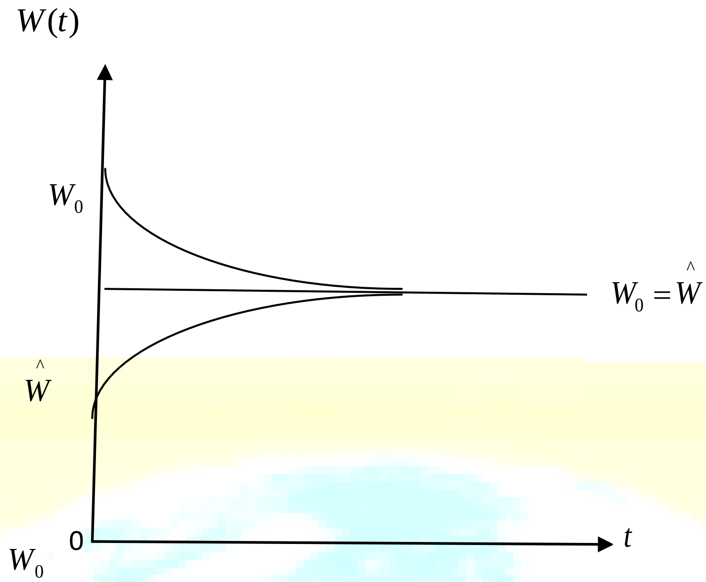

Figure 3.1: Stability Analysis of the Wage Function

Figure 3.1 shows that when $W_0 = \hat{W}$, *then* $W(t) = \hat{W}$, which is a constant function. If $W_0 > \hat{W}$ *then* $W(t)$ decreases asymptotically towards $\hat{W}$ and if $W_0 < \hat{W}$ then $W(t)$ increases asymptotically towards the equilibrium wage rate $\hat{W}$. The results therefore shows that as $t \to \infty$, the function (3.3) approaches the equilibrium wage rate asymptotically either from above or below and stabilizes at the equilibrium wage rate.

4. Conclusion

In this paper, a first order wage function has been solved using an integrating factor. The subsequent wage function is analyzed and interpreted for stability. It is found that the wage function could stand off the equilibrium wage rate initially and as time approaches infinity, it asymptotically stabilizes at the equilibrium wage rate in inter temporal sense. The study observed that use of an integrating factor in solving the wage equation is just as effective as Laplace transforms demonstrated in [6]. Integrating factor method is rather simple and fast with limited algebra than Laplace transforms.

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