MATHEMATICAL MODELLING OF POWER OUTPUT IN A WIND ENERGY CONVERSION SYSTEM

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Abstract

The developments of wind energy systems have enabled an efficient production and use of wind energy. Three widely used control schemes for wind energy systems are Pitch control, Rotor resistance control and Vector control. A traditional wind energy system consists of a stall regulated or pitch control turbine connected to a synchronous generator through gearbox. The synchronous generator operates at fixed speed and one of the earliest rotor control schemes was the rotor resistance control. The speed of an induction machine is controlled by the external resistance in the rotor circuit. The drawback of the above methods is the inability of wind turbine to capture at low wind speeds. This paper develops a model which maximizes wind energy output. This model asses the effects of friction coefficient and the height of wind above ground(given by the height of turbine from the ground i. e the hub height) on power output. The study considers an already existing model, that is the turbine model in which the study incorporates height and friction coefficient. The three methods used in wind energy systems and the output variations for the different control techniques for a change in the input wind velocity and a constant desired power output reference are compared and the methods evaluated based on the response time and the magnitude of change in the power output compared to the desired power output and also compared by simulation. The results of this study may be useful in aiding in the efficient production of electricity in a wind energy conversion system.

Mathematics Subject Classification: Primary: 76N76; Secondary: 76M77.

Keywords: Moddelling, Wind energy, Output.

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1 Introduction

Mankind has used the wind as a source of energy for several thousands of years. It was one of the most utilized sources of energy together with hydro power during the seventeenth and eighteenth centuries[18]. By the end of the 19th century the first experiments were carried out on the use of windmills for generating electricity. The international oil crisis of 1972 initiated the utilization of renewable resources on a large scale, wind power among others. At this moment in time, the world is going the way of green energy (renewable energies) in its energy consumption. Wind energy or wind power describe the process by which wind is used to generate mechanical or electric power. Use of wind energy for electricity generation purposes is becoming an increasingly attractive energy source partly due to the increase in energy demand worldwide and environmental concerns[18]. Burning of fossil fuels emit gases such as carbon dioxide into the atmosphere that lead to global warming. Wind energy does not rely on fossil fuels for energy generation. Today, wind power is a fully established branch on the electricity market and it is treated accordingly. Electricity is traded like any other commodity on the market and there are, therefore, standards which describe its quality. In the case of electric power they are commonly known as the Power Quality Standards. Any device connected to the electric grid must fulfill these standards and this is a particularly interesting and important issue to be considered in the case of wind power installations, since the stochastic nature of wind and standardized parameters of electricity are joined together there. A mathematical model of a wind power output capable of predicting its interactions with the grid is, thus, an important topic. Wind is a fluid in motion and thus is a complex phenomenon, as it possesses potential energy, kinetic energy, gravitational energy besides causing frictional viscous forces. This paper is limited to the effects of wind height from the ground (represented by the hub height of the wind turbine) and frictional viscous forces on power output.

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The study evaluates how this parameters affects the final power output P_o in a turbine model as given by [18] is;

$P_o = \frac{1}{2}(\rho.A.V^3)$ 1

Where ρ is the density of the moving air (wind), A is the area swept by the rotor blade and V is the velocity of the wind. When the height and friction coefficient are introduced, the power model P_i becomes;

$$P_i = \frac{1}{2}\rho.A\left[V\left(\frac{h2}{h1}\right)^{1/\alpha}\right]^3 \qquad 2$$

Where P_i is the new power output with respect to friction coefficient and height adjustment, α is the ground surface friction coefficient and h_1 and h_2 are heights at lower levels and higher levels respectively. This is the model of the study in this paper.

1.1 Friction effect of the earth's surface

The velocity profile near the surface is determined mostly by turbulence, which is very effective in transporting momentum. When a wind is blowing, a laminar boundary layer is formed near the surface which is dominated by the molecular viscosity of the air. There is a relatively large velocity gradient in this layer, but it is not very thick. If irregularities on the surface are completely immersed in this laminar boundary layer, their size does not matter, and the surface is aerodynamically smooth. Larger bodies may project through the layer, and retard the wind by forming drag like any large body in an airstream. If these bodies dominate, then a turbulent boundary layer is formed, and the surface is classed as aerodynamically rough. Meteorologically, the earth's surface is generally rough, except that a calm sea surface may be smooth.

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1.2 Wind Speed Distribution

The wind speed is the most critical data needed to appraise the power potential of a given site. The wind is never steady at any site. It is influenced by the weather system, the local land terrain, and the height above the ground surface. The wind speed varies by the minute, hour, day, season, and year. When modeling these are some of facts to be considered. The annual mean speed is usually averaged over 10 or more years. Such a long term average raises the confidence in assessing the energy-capture potential of a site. However, long-term measurements are expensive, and most projects cannot wait that long . Thus in most cases, the short term, say one year, data is usually compared with a nearby site having a long term data to predict the long term annual wind speed at the site under consideration. This is known as the measure, correlate and predict (mcp) technique. The wind-speed variations over a given period can be described by a probability distribution function.

1.3 Weibull Probability Distribution

The variation in wind speed are best described by the Weibull probability distribution function h_r with two parameters, the shape parameter k, and the scale parameter c. The probability of wind speed being v during any time interval is given by the following:

$$h_r(v)=(\tfrac{k}{c})(\tfrac{v}{c})(k-1)e^{-}(\tfrac{v}{c})^{k} \text{ for } 0 < v < \infty \qquad 3$$

In the probability distribution chart, h_r is plotted against v over a chosen time period,

Where;

$$h_r=[1/\Delta v]x[\text{fraction of time wind speed is between v and}(v+\Delta v)] \qquad 4$$

By definition of the probability function, probability that the wind speed will be between zero and infinity during that period is unity, i.e.:

$$\int_0^\infty h_r . dv = 1 \qquad 5$$

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If we choose the time period of one year, then express the probability function in terms of the number of hours in the year, such that:

h_r=[1/Δv]*x* [number of hours the wind is between v and(v+Δv)]

The unit of h_r is hours per year per meter/second, and the integral 5 becomes 8,760 (the total number of hours in the year) instead of unity. The curve on the left with k = 1 (see diagram 1) has a heavy bias to the left, where most days are windless (v=0). The curve on the right with k = 3 looks more like a normal bell shape distribution, where some days have high wind and equal number of days have low wind. The curve in the middle with k = 2 is a typical wind distribution found at most sites. In this distribution, more days have lower than the mean speed, while few days have high wind. The value of k determines the shape of the curve, hence is called the shape parameter. The Weibull distribution with k = 1 is called the exponential distribution which is generally used in the reliability studies. For k > 3, it approaches the normal distribution, often called the Gaussian or the bell-shape distribution. The weibull probability function and the wind speed distribution in general helps us understand the fluctuation of wind speed irrespective of the height of turbine tower which is our main focus. This will definitely make the final power output to fluctuate depending on the time of the day or the weather system. Apart from the weibull probability function we can simply predict the speed of wind.

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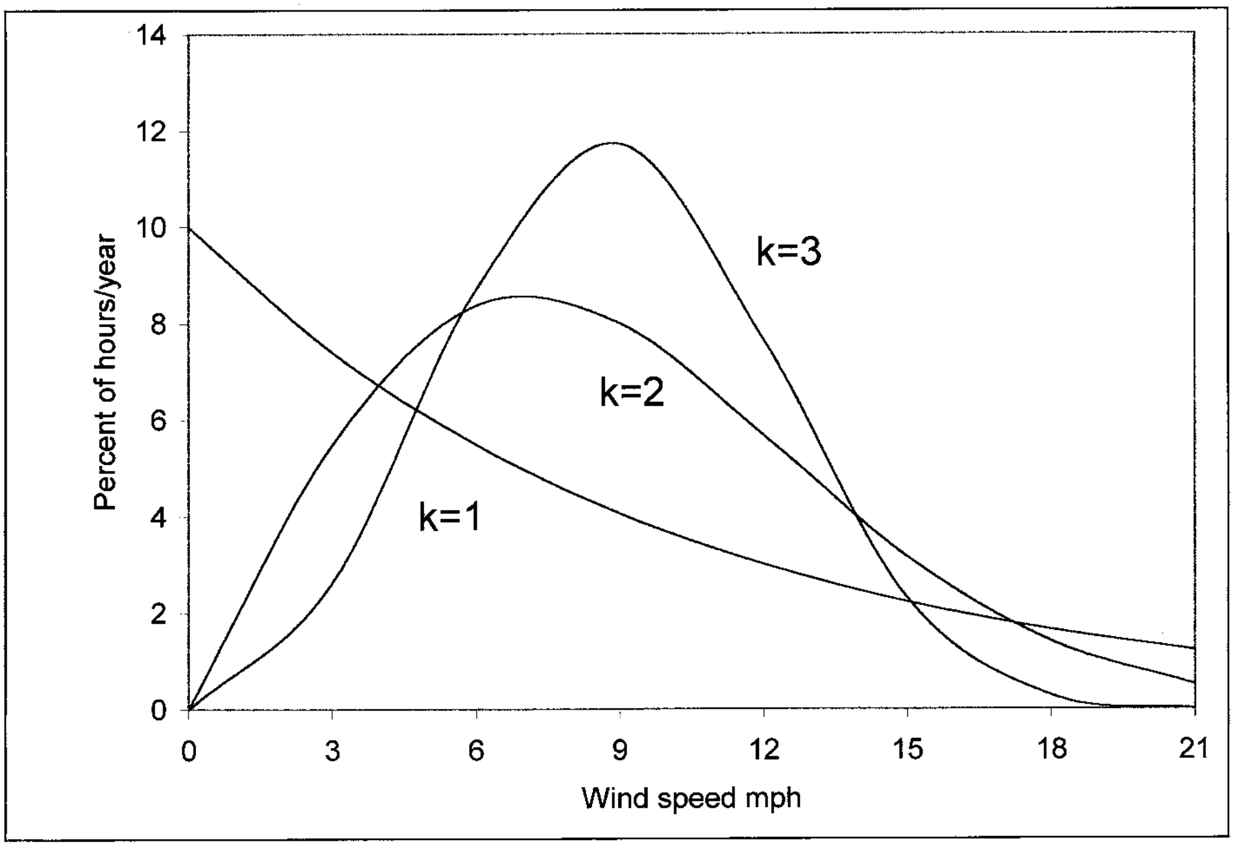

Figure 1: Weibull probability distribution function with scale parameter c=10 and shape parameters k=1,2 and 3.

1.4 Wind Speed Prediction

The available wind energy depends on the wind speed, which is a random variable as explained above. For the wind-farm operator, this poses difficulty in the system scheduling and energy dispatching, as the schedule of the wind-power availability is not known in advance. However, if the wind speed can be reliably forecasted up to several hours in advance, the generating schedule can efficiently accommodate the wind generation. Alexiadis et al2 have proposed a new technique for forecasting wind speed and power output up to several hours in advance. The technique is based on cross-correlation at neighboring sites and artificial neutral networks. The proposed technique can significantly improve forecasting accuracy compared to the persistence forecasting model. The new proposed method is calibrated at different sites over one year period.

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1.5 Effect of height on wind speed

The same small turbine can increase its power output by 30 percent or more if the height of its tower is 30m/100 feet instead of 18m/60 feet. The turbine towers are an important element of any wind system (except in the experimental urban micro-wind systems). Wind speed increases sharply with the tower height, causing a major increase in the electricity Output of the system .The wind speed does not increase with height indefinitely. The data collected at Merida airport in Mexico show that typically the wind speed increases with height up to about 450 meter of height, and then decreases. The wind speed at 450 meters high can be four to five times greater than that near the ground surface.

Table 1: Friction Coefficient of Various Terrain

Terrain type	Friction coefficient (α)
Lake, ocean and smooth hard ground	0.1
High grass on level ground	0.15
Tall crops, hedges and shrubs	0.20
Wooded country with many stones	0.25
Small town with some trees and shrubs	0.3
City area with tall buildings	0.4

1.6 Effect of friction coefficient on wind speed

The wind shear at ground surface causes the wind speed increases with height in accordance with the expression.

$$V_2 = V_1 \left(\frac{h2}{h1}\right)^{\alpha} \qquad 7$$

Where V_1 is the wind speed measured at the reference height h_1, V_2 is the wind speed estimated at height h_2, and α is the ground surface friction coefficient.The friction coefficient is low for smooth terrain and high for rough ones. The friction coefficient α is a function of the topography at a specific site and frequently assumed as a value of 1/7 for open land [6,20]. However, this parameter can vary for one place with 1/7 value during the day up to 1/2 during the night time [8]. Equation 7 is also known as the power law when

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the value of α is equal to 1/7 and is commonly referred to as the one-seventh power law. Provided there are no significant ground level obstacles, the friction coefficient α (equation 7) is set empirically and the equation

can be used to adjust the data reasonably well in the range of 10 up to 150 metres. The coefficient varies with the height, hour of the day, time of the year, land features, wind speeds and temperature. All such findings have emerged from the analysis undertaken at several locations worldwide [13,16,7,22]. Table 1 shows the friction coefficients of various land spots that, in each case, are given in function of the land roughness [14] and[20].

1.7 Modeling and Control of the Power Output; Height and friction coefficient

The hub height of a wind turbine is the distance from the turbine platform to the rotor of the an installed wind turbine. It indicates how high your turbine stands above the ground not including the length the turbine blades. The study has adopted this hub height to represent height in the study. Commercial turbines(greater than 1mw) are typically installed at 80m (262 ft) or higher while smaller scale wind turbines(appx 10 kw) are installed on shorter towers. For the evaluation of the model, the study takes into account the following; The average wind speed is usually 10m/s, The average diameter of the rotor of a commercial wind turbine is 42m (NB: The area swept by the rotor blades defines the area(A) in equation 1.2 which is the study model.), The study also assumes a value for air density of 1.225kg/m^3. From the equation 2, we can rewrite it as;

$$2 \log (P) = \log \rho + \log A + 3\log V + 3(1/\alpha)\log(x); \; x = h_2/h_1$$

$y = (\log \rho)/2 + (\log A)/2 + (3\log V)/2 + [3(1/\alpha) \log(x)]/2.$

where; $y = \log(P)$

Thus $P = \exp(y)$

When we use MATLAB to simulate and perform an analysis of the variation of P which represents power output against x which represent hub height we obtain parabolic curve showing that power output increases with increase in the hub height. Changing the value of α i.e α=0.1, 0.15, 0.2, and 0.25, the power output decreases as the friction coefficient increases. This shows that more power is captured in areas that have smooth terrains.

1.8 Conclusion

In conclusion, this paper has modeled power output in a wind turbine with consideration of some design parameters. The study is able to show that power output is dependent on the hub height of the turbine and friction plays a major role in power production in the turbine. The turbine operation exhibits reasonable range of tip speed ratio with β = 11 giving a good result according to the study. Turbine blade undergoes some forces as it goes round, one of which is centripetal force. Further research needs to be done to find out how centripetal force affects the power output in the turbine.

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