TEMPERATURE PATTERNS OF NANOFLUIDS IN A SQUARE ENCLOSURE WITH NATURAL CONVECTION

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***Abstract*—**
This paper reports a numerical study on natural convection heat transfer and fluid flow in a square cavity filled with CuO–Water nanofluids. Both upper and lower surfaces are being insulated, whilst a uniform constant temperature field applied in horizontal walls. The governing equations of fluid flow are discretized using a finite volume method with a collocated grid arrangement. The numerical results are reported for the effect of Rayleigh number, solid volume fraction and both presence and absence of thermophoresis and Brownian motion effects.

Keywords*— *Natural convection; nanofluid; temperature patterns; cavity; numerical

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1. INTRODUCTION

Natural convection heat transfer in enclosures is an important phenomenon in engineering systems due to its wide applications in building heating, automotive technology, solar technology cooling of electronic equipment, etc [1–3]. Improvement in the heat transfer performance of these systems is an essential topic from an energy saving perspective. With this aim, an innovative technique to enhance heat transfer rate by using nanoscale particles (smaller than 100 nm) suspended in the base fluid, has emerged. Engineered suspensions of nanoparticles in liquids, known recently as nanofluid, have generated considerable interest for their potentials to enhance the heat transfer rate in engineering systems. Nanofluids are made from various materials, such as oxide ceramics (Al_2O_3, CuO), nitride ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu, Ag, Au), semiconductors, (TiO_2, SiC), carbon nanotubes, and composite materials such as alloyed nanoparticles or nanoparticle core–polymer shell composites [4]. The base media of nanofluids are usually water, oil, acetone, decene, ethyleneglycol, etc. Compared with conventional heat transfer such as oil, water and ethylene glycol mixture, nanofluids have significantly higher thermal conductivity that consequently enhances the heat transfer characteristics of these fluids.

Heat transfer from a localized heater inside a rectangular cavity is considered as a model of electronic devices and equipment cooling. An efficient cooling is essential for these electronic equipment and many researchers, have been investigated free convection in cavities with different shapes of heaters. Close et al. [5] conducted a numerical study to observe heat transfer from a vertically located heater in a cavity. They found that gas/vapor mixtures can provide an efficient cooling method for electronic devise problem. Sun and Emery [6] studied conjugate heat transfer inside a cavity with heat source and internal baffle, numerically and experimentally.

In the present study the effect of Cuo particles on temperature patterns of Cuo–Water nanofluid in square cavity has been investigated numerically.

2. PHYSICAL MODEL

The physical model under consideration is natural convection in a square enclosure of side length L schematically shown in Fig. 1. The left vertical wall is maintained at a constant temperature t_h higher than the constant temperature t_c of the of the right vertical wall. Other walls of the enclosures are all thermally insulated. The fluid in the enclosure is a water based nanofluid containing CuO nanoparticles. The nanofluid is assumed incompressible and Newtonian and only Brownian diffusion and thermophoresis are important slip mechanisms between the two media. Water and CuO nanoparticles are in thermal equilibrium and the flow is also conceived as laminar and two-dimensional.

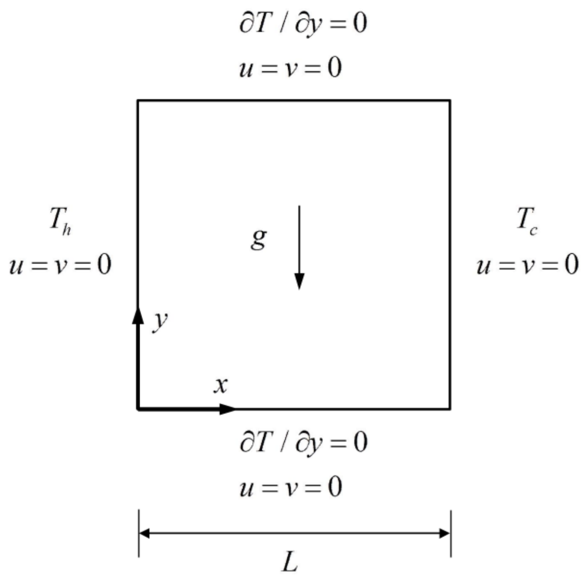

Fig. 1. A schematic diagram for the problem with boundary conditions.

3. Governing equations

Governing equations describing the conservation of total mass, momentum, thermal energy, and nanoparticles for the laminar, two-dimensional and steady state natural convection are written as:

$$\nabla . V^* = 0 \tag{1}$$

$$V^* . \nabla V^* = -\frac{1}{\rho_{nf}} \nabla p + \nabla \tau + g \tag{2}$$

$$V^* . \nabla T^* = -\frac{1}{c_{nf}\rho_{nf}} \nabla (k . \nabla T^*) + \frac{c_p \rho_p}{c_{nf}\rho_{nf}} \left(D_B^* \nabla \phi^* \nabla T^* + D_T^* \frac{\nabla T^* \nabla T^*}{T_C^*} \right) \tag{3}$$

$$V^* . \nabla \phi^* = \nabla . \left(D_B^* \nabla \phi^* + D_T^* \frac{\nabla T^*}{T_C^*} \right) \tag{4}$$

Where the stress tensor in Eq. 2 is given as [8]:

$$\tau = \mu_{nf} \left(\nabla V^* + \left(\nabla V^* \right)^t \right) \tag{5}$$

Here, ρ_{nf} is the effective density of the nanofluid, μ_{nf} is the effective dynamic viscosity of the nanofluid, ϕ^* is nanoparticle volume fraction and g is the acceleration due to gravity. The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T as reported are given by:

$$D_B^* = \frac{k_B T^*}{3\pi\mu_f d_p} \tag{6}$$

$$D_T^* = \left(\frac{\mu_f}{\rho_f}\right)\left(0.26\frac{k_f}{2k_f + k_p}\right)\phi^* \tag{7}$$

Where μ_f is the viscosity of the fluid, d_p is the nanoparticle diameter and k_f and k_p are the thermal conductivity of the fluid and particle materials, respectively. The effective density (ρ_{nf}) and heat capacitance $(\rho C_p)_{nf}$ of the nanofluid are defined as:

$$\rho_{nf} = (1-\phi^*)\rho_f + \phi^*\rho_p \tag{8}$$

$$(\rho c_p)_{nf} = (1-\phi^*)(\rho c_p)_f + \phi^*(\rho c_p)_p \tag{9}$$

Thermal diffusivity of the nanofluids is

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \tag{10}$$

The effective thermal conductivity of the nanofluid is calculated using an empirical model proposed by Chon et al. as follows:

$$\frac{k_{nf}}{k_f} = 1 + 64.7 \times \phi^{0.7460}\left(\frac{d_{bf}}{d_p}\right)^{0.3690}\left(\frac{k_p}{k_{bf}}\right)^{0.7476} \times \Pr^{0.9955} \times \mathrm{Re}^{1.2321} \tag{11}$$

Here Pr_T and Re are defined by:

$$\Pr = \frac{\mu_f}{\rho_f \alpha_f} \tag{12}$$

$$\mathrm{Re} = \frac{\rho_f k_B T}{3\pi\mu_f^2 l_f} \tag{13}$$

where $k_B = 1.3807 \times 10^{-23} J/K$ is the Boltzmann's constant, T is temperature (in °K) and l_f is mean–free path of base fluid molecules (in m) given as 0.17 nm for water .This model considers the Brownian random motion created due to the existence of nanoparticles and also the effect of nanoparticle size and temperature on nanofluids thermal conductivity. The correlation provided is valid for nanoparticle sizes ranging between 11 and 150 nm and for temperature, the associated validity range is 21–71°C. Minsta et al. experimentally showed that Chon et al. model predicts the thermal conductivity of nanofluid accurately up to a volume fraction of 9% for both CuO and Al_2O_3 nanoparticles.

In the current study, the correlation for dynamic viscosity of CuO–water nanofluid is derived using the available experimental data of Nguyen et al. as a function of temperature and volume fraction of nanoparticles as follows:

$$\mu_{cuo}(cp) = -0.6967 + \frac{15.937}{T} + 1.238\phi + \frac{1356.14}{T^2} - 0.259\phi^2 - 30.88\frac{\phi}{T} - \frac{19652.74}{T^3} + 0.01593\phi^3 + 4.38206\frac{\phi^2}{T} + 147.573\frac{\phi}{T^2} \quad (14)$$

The viscosity in Eq. (14) is expressed in centipoise and the temperature in °C. The R^2 of the regression is 99.8% and a maximum error is 5%.

It is worth noting that the following correlation [8] has been used to evaluating water dynamic viscosity:

$$\mu = A \times 10^{\frac{B}{T-C}}, C = 140,\ B = 247,\ A = 2.414e-5 \quad (15)$$

Where T is temperature in °K and μ is viscosity in centipoise. The above equations can be converted to non–dimensional form, using the following dimensionless parameters

$$X = \frac{x^*}{L}, \quad Y = \frac{y^*}{L}, \quad U,V = \frac{u,v\ L}{\alpha_{f_0}}, \quad \mu = \frac{\mu_{nf}}{\mu_{f_0}}$$
$$\theta = \frac{T^* - T_C}{T_H - T_C}, \quad F = \frac{\phi^*}{\phi_b}, \quad \alpha = \frac{\alpha_{nf}}{\alpha_{f_0}}, \quad P = \frac{pL^2}{\rho_{nf}\alpha_{f_0}^{\ 2}} \quad (16)$$
$$D_B = \frac{D_B^*}{D_{B_0}}, \quad D_T = \frac{D_T^*}{D_{T_0}}, \quad \kappa = \frac{k_{nf}}{k_{f_0}}$$

The governing equations can now be written in dimensionless form as follows

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (17)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\Pr}{1-\phi + \phi\frac{\rho_p}{\rho_f}}\left(\frac{\partial}{\partial X}\left(\mu\frac{\partial U}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\mu\frac{\partial U}{\partial Y}\right)\right) - \frac{\partial P}{\partial X} \quad (18)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=\frac{\Pr}{1-\phi+\phi\frac{\rho_p}{\rho_f}}\left(\frac{\partial}{\partial X}\left(\mu\frac{\partial V}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\mu\frac{\partial V}{\partial Y}\right)\right)$$
$$-\frac{\partial P}{\partial Y}+\Pr Ra\theta\left[\frac{1}{\frac{1-\phi}{\phi}\frac{\rho_f}{\rho_p}+1}\frac{\beta_p}{\beta_f}+\frac{1}{\frac{\phi}{1-\phi}\frac{\rho_f}{\rho_p}+1}\right] \qquad (19)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial}{\partial X}\left(\kappa\frac{\partial \theta}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\kappa\frac{\partial \theta}{\partial Y}\right)\right)$$
$$\times\frac{1}{(1-\phi)+\phi\left(\frac{(\rho c_p)_p}{(\rho c_p)_f}\right)}+\frac{\Pr Le D_T\xi}{Sc}\left(\left(\frac{\partial \theta}{\partial X}\right)^2+\left(\frac{\partial \theta}{\partial Y}\right)^2\right) \qquad (20)$$
$$\times\frac{\Delta T}{T_c}+\frac{\Pr D_B\phi_b\xi}{Sc}\left(\frac{\partial \theta}{\partial X}\frac{\partial F}{\partial X}+\frac{\partial F}{\partial Y}\frac{\partial \theta}{\partial Y}\right)$$

$$U\frac{\partial F}{\partial X}+V\frac{\partial F}{\partial Y}=\frac{\Pr D_B}{Sc}\left(\frac{\partial^2 F}{\partial X^2}+\frac{\partial^2 F}{\partial Y^2}\right)$$
$$+\frac{\Pr Le D_T}{Sc}\frac{\Delta T}{\phi_b T_c}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (21)$$

Where

$$Ra=\frac{\beta g(T_H-T_C)L^3}{\nu_{f_0}\alpha_{f_0}}\ ,\ \xi=\frac{\rho_p c_p}{\phi(\rho c_p)_p+(1-\phi)(\rho c_p)_f}$$
$$Le=\frac{D_{T_0}}{D_{B_0}}\ ,\ \Pr=\frac{\nu_{f_0}}{\alpha_{f_0}}\ ,\ Sc=\frac{\nu_{f_0}}{D_{B_0}} \qquad (22)$$

Where the subscript "o" stands for the reference temperature which is taken as 22°C in the present work, whereas the temperature difference between the hot and cold walls is kept constant at 1°C. The dimensionless boundary conditions are as follows

$$U=V=0 \quad at \quad X=0,1 \quad ,Y=0,1$$
$$\theta(0,Y)=1 \quad , \quad \theta(1,Y)=0$$
$$\frac{\partial\theta(X,0)}{\partial Y}=\frac{\partial\theta(X,1)}{\partial Y}=0 \qquad (23)$$
$$\frac{\partial F(X,0)}{\partial Y}=\frac{\partial F(X,1)}{\partial Y}=0$$

For relative nanoparticle volumetric fraction (F), the zero particles flux on the boundaries used as reported in [4] is given by

$$\frac{\partial^2\phi}{\partial y^2}=-\frac{1}{N_{BT}}\frac{D_T}{D_B}\left(\frac{\partial^2\theta}{\partial y^2}\right) \qquad (24)$$

Where

$$N_{BT}=\frac{T_C D_{B_0}\phi_b}{D_{T_0}\Delta T} \quad (25)$$

Is the ratio of Brownian diffusivity to thermophoretic diffusivity. This type of boundary conditions used for *F* is more realistic physically in comparison with the *F*–constant boundary conditions available in most of numerical simulations reported in literature [5, 7].

The heat transfer rate along the left wall is expressed in terms of the local and average Nusselt's numbers Nu and Nu_{avg}, respectively. The local Nusselt number can be expressed as:

$$\text{Nu}=\frac{hH}{k_f} \quad (26)$$

Where the heat transfer coefficient is

$$h=\frac{q}{T_h-T_c} \quad (27)$$

Where q is total heat flux per unit area. In addition to conduction heat flux there is another heat flux as nanoparticle diffusion heat flux, so the total nanofluid energy flux could be written as:

$$q=-k_{nf}\nabla T^{*}+h_p j_p \quad (28)$$

In equation (28), c_p and h_p are the specific heat and enthalpy of the nanoparticle material, respectively. Hence, in order to taking into account both Brownian and thermophoresis effects in heat transfer rates of nanofluids, the local Nusselt number can be defined as:

$$\text{Nu}=-k\frac{\partial\theta}{\partial y}-\frac{D_{T_0}\rho_p c_p}{k_f}\left(N_{BT}D_B\frac{\partial\phi}{\partial y}+D_T\frac{\partial\theta}{\partial y}\right) \quad (29)$$

Where

$$k=\frac{k_{nf}}{k_{f_0}} \quad (30)$$

The first term on the right–hand side of Eq. (26) is the heat flux due to conduction and the second term is the heat flux due to nanoparticle diffusion.

The average Nusselt number is obtained by integrating the above local Nusselt number over the hot vertical wall

$$\mathrm{Nu}_{avg} = \int_0^1 Nu(y)\,\mathrm{d}y \tag{31}$$

To evaluate Eq. (31), a 1/3 Simpson's rule of integration is implemented. It is appropriate to define a normalized average Nusselt number to evaluate the efficacy of using nanofluid rather than pure fluids for this case. This number is defined by dividing the Nusselt number at any volume fraction of nanoparticles to that of pure water:

$$\mathrm{Nu}^*_{avg}(\phi) = \frac{\mathrm{Nu}_{avg}(\phi)}{\mathrm{Nu}^*_{avg}(\phi = 0)} \tag{32}$$

If the normalized average Nusselt number is greater than one, then using nanofluid can enhance the heat transfer and values lesser than one shows the contrary.

4. Numerical procedure

The dimensionless governing equations were discretized by the dimensional finite volume method. The grid layout was arranged by utilizing collocated grid procedure, while the Hybrid and Quick scheme were adopted for the convection–diffusion terms for calculation in the fluid domain. The coupling between velocity and pressure is solved using the SIMPLEC algorithm and the Rhie and Chow interpolation is used to avoid the checker–board solutions for the pressure.
For these simulations, the convergence was considered to be reached when the relative error on the value of a property per unit mass denoted by Φ between two successive iterations, n and n+1 was smaller than a chosen tolerance:

$$\frac{\sum\left|\Phi^{n+1}_{i,j} - \Phi^{n}_{i,j}\right|}{\sum\left|\Phi^{n+1}_{i,j}\right|} \le 10^{-6} \tag{33}$$

To test and assess grid independence of the solution scheme, the average Nusselt number for seven different grid sizes is performed as shown in Table. 2 in case Ra= 10^5 and ϕ=5%. Based on the results illustrated in the table and considering the accuracy of the results required and computational time involved, an 51×51 uniform grid is used for all of the subsequent numerical calculations.

TABLE I. GRID INDEPENDENCE STUDY

Grid size	Nu_{avg}
20×20	3.28702
30×30	3.26587
40×40	3.25015
50×50	3.24556
60×60	3.24398

70×70	3.24283

5. Code validation

The present FORTRAN code is validated by comparing the present code results against the numerical simulation of Khanafer et al. and Oztab et al. for enclosures filled with a water–Cu nanofluid (Ra = 10^5, ϕ =5%, Pr = 6.2) a shown in Fig. 2. The agreement is found to be good and little differences could be duo to different models that used for nanofluids modeling.

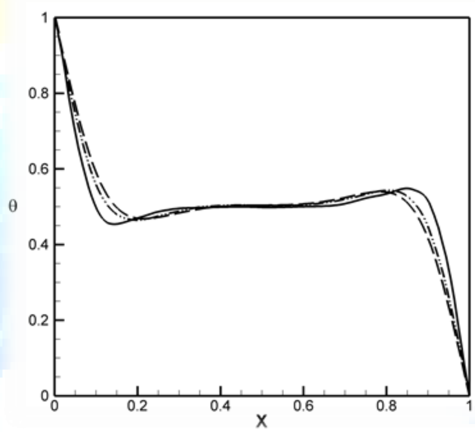

Fig. 2. Validation of the present code against other studies for a square enclosure filled with a Cu-water nanofluid (Ra = 10^5, ϕ =5%, Pr = 6.2)

6. Results

The overall objective of this current study is to investigate the nanoparticles distribution and heat transfer behavior of natural convection inside a cavity filled with CuO–water nanofluid in both presence and absence of Brownian and thermophoresis effects. The ranges of the magnitude of the Rayleigh numbers and volume fractions of nanoparticles used in this study are $Ra=10^4–10^5$ and $0 \leq \phi \leq 7\%$, respectively. It is worth mentioning that the difference between the hot and the cold walls is fixed to 1°C and it is assumed that the Prandtl number (Pr) equals 6.2.

A comparison of the isotherms contours between pure water and the CuO–water nanofluid is conducted for two different values of the Rayleigh number and in the range of $1\% \leq \phi \leq 7\%$ as shown in Fig. 3 (a)–(d). The field of isotherms indicates that these are mostly parallel to the heated and cooled boundaries and vertical to the isolated walls of enclosure. Furthermore, these figures clearly show temperature patterns are influenced by the presence of nanoparticles. That is to say, for low volume fractions (ϕ = 1%), the isotherms are almost identical for both nanofluids and water. Whereas, increasing the nanoparticle concentration for both Rayleigh number, increases deviations of nanofluid and pure water isotherms, especially at the central zone of the enclosure.

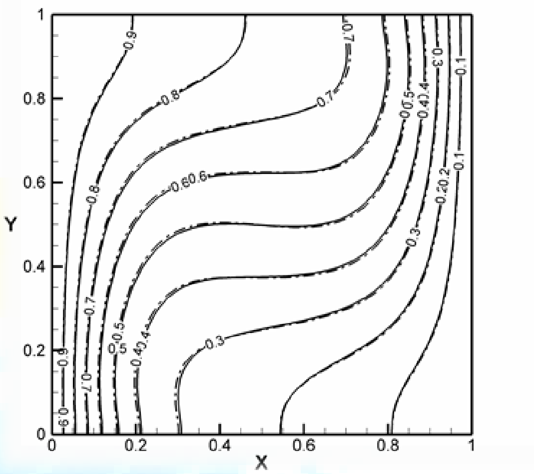

Fig. 3 a. Isotherms of CuO-nanofluid (--) with ϕ = 0.01 and pure water (—) at Ra = 10^4

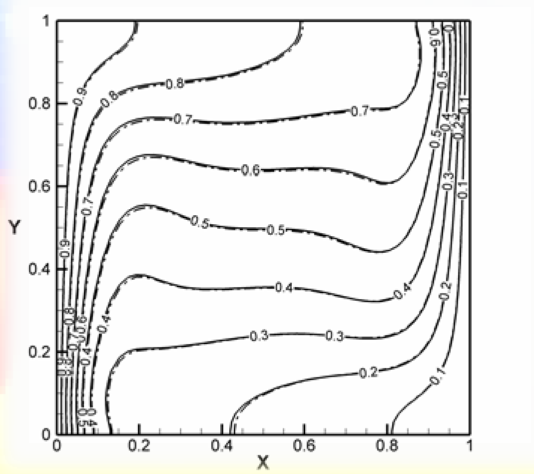

Fig. 3 b. Isotherms of CuO-nanofluid (--) with ϕ = 0.01 and pure water (—) at Ra = 10^5

Likewise, Fig. 3 illustrates that the thickness of thermal boundary layer next to the heated wall is influenced by the addition of nanoparticles. So that, addition of nanoparticles to the base fluid increases thermal conductivity of the nanofluid and higher values of thermal conductivity are accompanied by higher values of thermal diffusivity. The high value of thermal diffusivity increases the boundary thickness and this increase in thermal boundary layer thickness has opposite effects on the Nusselt number (according to Eq. (26)). For higher Rayleigh numbers, natural convection become more vigorous and the fluid moves faster and the isotherms become more packed beside adiabatic walls as shown in Fig. 3 (a)–(d).

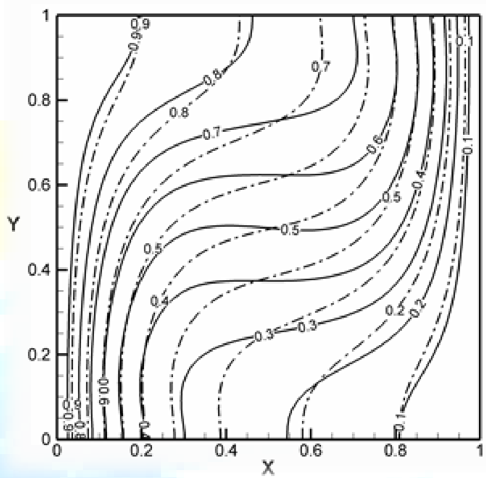

Fig. 3 c. Isotherms of CuO-nanofluid (--) with ϕ = 0.07 and pure water (—) at Ra = 10^4

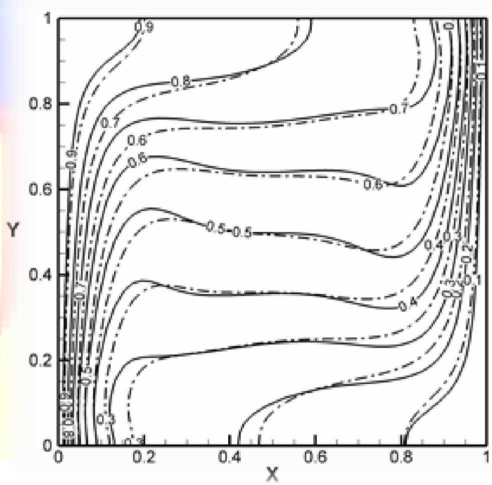

Fig. 3 d. Isotherms of CuO-nanofluid (--) with ϕ = 0.07 and pure water (—) at Ra = 10^5

7. Conclusion

Buoyancy induced flow and heat transfer in a square enclosure filled with a water–Cu nanofluid has been numerically investigated. Main efforts of this investigation were focused on influence of

Brownian and thermophoresis effects on the fluid flow and heat transfer characteristics. The following conclusions are drawn. For all values of the solid volume fraction, the heat transfer of water–CuO nanofluid in the presence of Brownian and thermophoresis effects is higher than the pure fluids and it is increased by increasing the Rayleigh.

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