

## OBSERVATIONS ON TRIACONTAGONAL NUMBER

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### Abstract

We obtain different relations among Triacontagonal number and other two, three and four dimensional figurate numbers.

### Introduction

The numbers that can be represented by a regular arrangement of points are called the polygonal numbers (also known as two dimensional figurate numbers). The polygonal number series can be summed to form solid three dimensional figurate numbers called Pyramidal numbers that be illustrated by pyramids[1].Numbers have varieties of patterns[2-16] and varieties of range and richness. In this communication we deal with triacontagonal numbers given by  $t_{30,n} = 14n^2 - 13n$  and various interesting relations among these numbers are exhibited by means of theorems involving the relations.

### Notation

$t_{m,n}$  = Polygonal number of rank n with sides m

$p_n^m$  = Pyramidal number of rank n with sides m

$so_n$  = Stella Octagonal number of rank n with sides m

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$F_{m,n,p}$  = m-dimensional figurate number of rank n where generated polygon is of p sides

$ja_n$  = Jacobsthal number

$ct_{m,n}$  = Centered Polygonal number of rank n with sides m

$g_n$  = Gnomonic number of rank n with sides m

$p_n$  = Pronic number

$carl_n$  = Carol number

$mer_n$  = Mersenne number, where n is prime

$cul_n$  = Cullen number

$Tha_n$  = Thabit ibn kurrah number

$PEN_n$  = Pentatope number

### Interesting Relations

$$1) \sum_{n=1}^N t_{30,n} = 14p_N^4 - 13t_{3,N}$$

**Proof:**

$$\begin{aligned} \sum_{n=1}^N t_{30,n} &= \sum_{n=1}^N (14n^2 - 13n) \\ &= 14 \sum_{n=1}^N n^2 - 13 \sum_{n=1}^N n \\ &= 14 \frac{N(N+1)(2N+1)}{2} - 13 \frac{N(N+1)}{2} \end{aligned}$$

$$\sum_{n=1}^N t_{30,n} = 14p_N^4 - 13t_{3,N}$$

$$2) t_{30,n} - t_{28,n} + 2n = 2t_{3,n}$$

**Proof:**

$$\begin{aligned} t_{30,n} - t_{28,n} + 2n &= 14n^2 - 13n - 13n^2 + 12n + 2n \\ &= n^2 - n + 2n \end{aligned}$$

$$t_{30,n} - t_{28,n} + 2n = 2t_{3,n}$$

3)  $t_{30,n+1} + t_{30,n-1} - n - 14 = t_{58,n}$

**Proof:**

$$\begin{aligned} t_{30,n+1} + t_{30,n-1} &= 14(n+1)^2 - 13(n+1) + 14(n-1)^2 - 13(n-1) \\ &= 14n^2 + 15n + 1 + 14 + 14n^2 - 41n + 13 \\ &= 28n^2 - 27n + n + 14 \end{aligned}$$

$$t_{30,n+1} + t_{30,n-1} = t_{58,n} + n + 14$$

4)  $t_{30,n} - t_{18,n} + 1 = s_n$

**Proof:**

$$\begin{aligned} t_{30,n} - t_{18,n} &= 14n^2 - 13n - 8n^2 + 7n \\ &= 6n^2 - 6n \\ t_{30,n} - t_{18,n} &= s_n - 1 \end{aligned}$$

5) Each of the following triplet are in arithmetic progression

a)  $(t_{29,n}, t_{30,n}, t_{31,n})$

**Proof:**

$$\begin{aligned} t_{29,n} + t_{31,n} &= \frac{1}{2}[27n^2 - 25n] + \frac{1}{2}[29n^2 - 27n] \\ &= 28n^2 - 26n \end{aligned}$$

$$t_{29,n} + t_{31,n} = 2t_{30,n}$$

b)  $(t_{20,n}, t_{30,n}, t_{40,n})$

**Proof:**

$$\begin{aligned} t_{20,n} + t_{40,n} &= 9n^2 - 8n + 19n^2 - 18n \\ &= 28n^2 - 26n \end{aligned}$$

$$t_{20,n} + t_{40,n} = 2t_{30,n}$$

c)  $(t_{10,n}, t_{30,n}, t_{50,n})$

**Proof:**

$$\begin{aligned} t_{10,n} + t_{50,n} &= 4n^2 - 3n + 24n^2 - 23n \\ &= 28n^2 - 26n \end{aligned}$$

$$t_{10,n} + t_{50,n} = 2t_{30,n}$$

d)  $(t_{30,n}, t_{32,n}, t_{34,n})$

**Proof:**

$$\begin{aligned} t_{30,n} + t_{34,n} &= 14n^2 - 13n + 16n^2 - 15n \\ &= 30n^2 - 28n \end{aligned}$$

$$t_{30,n} + t_{34,n} = 2t_{32,n}$$

6)  $t_{30,n} - t_{22,n} = t_{10,n} - n$

**Proof:**

$$\begin{aligned} t_{30,n} - t_{22,n} &= 14n^2 - 13n - 10n^2 + 9n \\ &= 4n^2 - 3n - n \end{aligned}$$

$$t_{30,n} - t_{22,n} = t_{10,n} - n$$

7) Each of the following equations represents a Nasty number

a)  $6\{t_{30,n} + 26t_{3,n} - 2n^2\}$

**Proof:**

$$\begin{aligned} 6\{t_{30,n} + 26t_{3,n} - 2n^2\} &= 6\{14n^2 - 13n + 13n^2 + 13n - 2n^2\} \\ &= 6\{5n\}^2 \end{aligned}$$

b)  $6\{2t_{30,n} - t_{58,n} + n^2\}$

**Proof:**

$$\begin{aligned} 6\{2t_{30,n} - t_{58,n} + n^2\} &= 6\{2(14n^2 - 13n) - 28n^4 + 26n^2\} \\ &= 6n^2 \end{aligned}$$

c)  $6\{t_{30,n} + 13p_n - 2n^2\}$

**Proof:**

$$6\{2t_{30,n^2} - t_{58,n^2} + n^2\} = 6\{2(14n^2 - 13n) - 28n^4 + 26n^2 + n^2\}$$

$$= 6n^2$$

8)  $n^2[t_{30,n} + 2] - nt_{10,n} = 2(t_{6,n} - t_{9,n})$

**Proof:**

$$2(t_{6,n} - t_{9,n}) = 2 \left\{ (2n^2 - n) \frac{(7n^2 - 5n)}{2} \right\}$$

$$= 14n^4 - 17n^3 + 5n^2$$

$$= n^2(14n^2 - 13n) - n(4n^2 - 3n) + 2n^2$$

$$2(t_{6,n} - t_{9,n}) = n^2(t_{30,n} + 2) - nt_{10,n}$$

9)  $2t_{30,n} = t_{54,n} + ng_n$

**Proof:**

$$2t_{30,n} = 28n^2 - 26n$$

$$= 26n^2 - 25n + 2n^2 - n$$

$$2t_{30,n} = t_{54,n} + ng_n$$

10)  $nt_{30,n} - t_{30,n+2} + n + t_{28,n} = 28p_n^5 + 16g_n + 14$

**Proof:**

$$nt_{30,n} - t_{30,n+2} = 14n^3 - 13n^2 + 14n^2 + 43n + 30$$

$$= 14(n^3 + n^2) - (13n^2 - 12n) + 16(2n - 1) - n + 14$$

$$nt_{30,n} - t_{30,n+2} = 28p_n^5 - t_{28,n} + 16g_n - n + 14$$

11)  $2(t_{30,n} + t_{32,n}) = t_{31,n}$

**Proof:**

$$t_{31,n} = \frac{1}{2} [29n^2 - 27n]$$

$$= \frac{1}{2} [14n^2 - 13n + 15n^2 - 14n]$$

$$t_{31,n} = 2[t_{30,n} + t_{32,n}]$$

$$12) t_{30,n^2+n} + 13n^2 + 1 = (ct_{4,n} * ct_{6,n}) + 24PEN_n + t_{16,n^2} + 24p_n^6 - 40t_{3,n}$$

**Proof:**

$$t_{30,n^2+n} + 1 = 14n^4 + 14n^2 + 28n^3 - 13n^2 - 13n + 1$$

$$= (6n^4 + 12n^3 + 11n^2 + 5n + 1) + (n^4 + 6n^3 + 11n^2 + 6n) + (7n^4 - 6n^3)$$

$$+ 4(4n^3 + 3n^2 - n) - 20(n^2 + n) - 13n^2$$

$$t_{30,n^2+n} + 13n^2 + 1 = (ct_{4,n} * ct_{6,n}) + 24PEN_n + t_{16,n^2} + 24p_n^6 - 40t_{3,n}$$

$$13) t_{30,2^n} - t_{26,2^n} = (carl)_n + (mer)_{2n} + 2$$

**Proof:**

$$t_{30,2^n} - t_{26,2^n} = 14(2^{2n}) - 13(2^n) - 12(2^{2n}) + 11(2^n)$$

$$= 2^{2n} - 2^{n+1} + 2^{2n}$$

$$t_{30,2^n} - t_{26,2^n} = (carl)_n + (mer)_{2n} + 2$$

$$14) t_{30,2^n} + 13p_{2^n} = 9((Tha)_{2^n} + 1)$$

**Proof:**

$$t_{30,2^n} + 13p_{2^n} - 9 = 14(2^{2n}) - 13(2^n) + 13(2^{2n}) + 13(2^n) - 9$$

$$= 9(3(2^{2n}) - 1)$$

$$t_{30,2^n} + 13p_{2^n} = 9((Tha)_{2^n} + 1)$$

$$15) n(t_{30,2^n} - 2t_{16,2^n}) + (cul)_n = 1$$

**Proof:**

$$n(t_{30,2^n} - 2t_{16,2^n}) - 1 = n(14(2^{2n}) - 13(2^n) - 14(2^{2n}) + 12(2^n)) - 1$$

$$= -(n2^n + 1)$$

$$n(t_{30,2^n} - 2t_{16,2^n}) - 1 = -(cul)_n$$

$$16) t_{30,2^n} + (Tha)_n + 3(mer)_n + 4 = 7t_{6,2^n}$$

**Proof:**

$$t_{30,2^n} - 7t_{6,2^n} + 4 = 14(2^{2n}) - 13(2^n) - 14(2^{2n}) + 7(2^n) + 4$$

$$= -(3(2^{2n}) - 1) - 3(2^n - 1)$$

$$t_{30,2^n} - 7t_{6,2^n} + 4 = -(Tha)_n - 3(mer)_n$$

$$17) t_{30,n} + 7 = ct_{10,n} + s_n + t_{8,n} - 5g_n$$

**Proof:**

$$t_{30,n} + 7 = 5n^2 + 9n^2 - 18n + 5n + 7$$

$$= (5n^2 + 5n + 1) + (6n^2 - 6n + 1) + (3n^2 - 2n) - 5(2n - 1)$$

$$t_{30,n} + 7 = ct_{10,n} + s_n + t_{8,n} - 5g_n$$

$$18) 2t_{30,n} = t_{58,n} + n$$

**Proof:**

$$2t_{30,n} = 28n^2 - 26n$$

$$= 28n^2 - 27n + n$$

$$2t_{30,n} = t_{58,n} + n$$

$$19) 4t_{30,n} = t_{10,n} + 2t_{26,n} + t_{58,n}$$

**Proof:**

$$2t_{30,n} = 2t_{26,n} + t_{10,n} - n \tag{1}$$

$$2t_{30,n} = t_{58,n} + n \tag{2}$$

Add (1) and (2) we get,

$$4t_{30,n} = t_{10,n} + 2t_{26,n} + t_{58,n}$$

$$20) t_{30,n} = t_{42,n} - t_{14,n} + n$$

**Proof:**

$$t_{30,n} = t_{18,n} + t_{14,n} - n \quad (1)$$

$$2t_{30,n} = t_{18,n} + t_{42,n} \quad (2)$$

Subtract (2) and (1) we get,

$$t_{30,n} = t_{42,n} - t_{14,n} + n$$

$$21) t_{30,n} * t_{6,n} = (t_{10,n} * t_{16,n}) + n(t_{16,n} - t_{6,n})$$

**Proof:**

$$\begin{aligned} t_{30,n} * t_{6,n} &= 28n^4 - 40n^3 + 13n^2 \\ &= 28n^4 - 45n^3 + 18n^2 + 5n^3 - 5n^2 \end{aligned}$$

$$t_{6,n}(t_{30,n} + n) = t_{16,n}(n + t_{10,n})$$

$$22) t_{38,2^n} - t_{30,2^n} + 5 = (ja)_{2n+2} - 4(mer)_n$$

**Proof:**

$$\begin{aligned} t_{38,2^n} - t_{30,2^n} + 5 &= 18(2^{2n}) - 17(2^n) - 14(2^{2n}) + 13(2^n) + 5 \\ &= 4(2^{2n}) + 1 - 4((2^n) - 1) \end{aligned}$$

$$t_{38,2^n} - t_{30,2^n} + 5 = (ja)_{2n+2} - 4(mer)_n$$

$$23) \frac{n(7t_{6,2^n} - t_{14,2^n})}{(wo)_n + 1} = 5$$

**Proof:**

$$7t_{6,2^n} - t_{14,2^n} = 5(2^n)$$

$$n(7t_{6,2^n} - t_{14,2^n}) = 5((n2^n - 1) + 1)$$



$$= 5((wo)_n + 1)$$

$$24) (n-1)t_{30,n} + n26p_5^n - 13p_n = 224F_{4,n,4}$$

**Proof:**

$$n(t_{30,n} + 26p_5^n) = 27n^4 \quad (1)$$

$$t_{30,n} + 13p_n = 27n^2 \quad (2)$$

Subtract (1) and (2), we get

$$\begin{aligned} n(t_{30,n} + 26p_5^n) - t_{30,n} - 13p_n &= 27(n^4 - n^2) \\ &= 27n^2(n^2 - 1) \end{aligned}$$

$$(n-1)t_{30,n} + n26p_5^n - 13p_n = 224F_{4,n,4}$$

$$25) n((so)_n - (2t_{16,n} - t_{30,n})) = 24(F_{4,n,4})$$

**Proof:**

$$\begin{aligned} (so)_n - (2t_{16,n} - t_{30,n}) &= 2n^3 - n - 14n^2 + 12n + 14n^2 - 13n \\ &= 2n(n^2 - 1) \end{aligned}$$

$$n((so)_n - (2t_{16,n} - t_{30,n})) = 24(F_{4,n,4})$$

$$26) t_{30,n} - t_{26,n} = ct_{4,n-1} - 1$$

**Proof:**

$$\begin{aligned} t_{30,n} - t_{26,n} &= 14n^2 - 13n - 12n^2 + 11n \\ &= 2n^2 - 2n \end{aligned}$$

$$t_{30,n} - t_{26,n} = ct_{4,n-1} - 1$$

$$27) 2(t_{26,n}) - t_{22,n} = t_{30,n}$$

**Proof:**

$$\begin{aligned} 2(t_{26,n}) - t_{22,n} &= 24n^2 - 22n - 10n^2 + 9n \\ &= 14n^2 - 13n \end{aligned}$$

$$2(t_{26,n}) - t_{22,n} = t_{30,n}$$

$$28) t_{30,n} - t_{14,n} - t_{18,n} + n = 0$$

**Proof:**

$$\begin{aligned} t_{30,n} - t_{14,n} &= 8n^2 - 8n \\ &= t_{18,n} - n \end{aligned} \tag{1}$$

$$\begin{aligned} t_{30,n} - t_{18,n} &= 6n^2 - 6n \\ &= t_{14,n} - n \end{aligned} \tag{2}$$

On adding (1) and (2) we get,

$$2t_{30,n} - 2t_{14,n} - 2t_{18,n} + 2n = 0$$

$$t_{30,n} - t_{14,n} - t_{18,n} + n = 0$$

$$29) t_{30,n} - t_{26,n} = 4t_{3,n-1}$$

**Proof:**

$$\begin{aligned} 2t_{30,n} - 2t_{26,n} &= 4n^2 - 4n + 1 - 1 \\ &= (2n-1)^2 - 1 \\ &= 8t_{3,n-1} \end{aligned}$$

$$t_{30,n} - t_{26,n} = 4t_{3,n-1}$$

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