FUZZY ASSIGNMENT PROBLEM BASED ON TRAPEZOIDAL APPROXIMATION

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Abstract

In the fuzzy optimization problem researchers used triangular, trapezoidal, LR fuzzy numbers etc. Most of the results depend upon the shape of the fuzzy numbers when operating with fuzzy numbers. Less regular membership functions leads to more complex calculations. Trapezoidal approximation of fuzzy number is adopted for any given fuzzy number. In this paper authors analysed which type of fuzzy number is best in zero's and one's method of assignment problem. Numerical examples show that the assignment problem handling LR fuzzy numbers with trapezoidal approximation is an effective one in zero's assignment.

Keywords: LR fuzzy number, Zero's and one's assignment, Trapezoidal approximation, Yager's ranking index.

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1. Introduction:

Our human system and our activities are not certain. They are not clear (vague) but they are certain up to a degree (tolerance level) and they may be studied and analyzed by representing in a system with varying degree. The first attempt was made by Prof .L .A. Zadeh. He formulated a system in which sets are represented by membership functions in the year 1965.Such systems are called Fuzzy Sets and systems .After the introduction of fuzzy sets, all the topics of Pure and Applied Mathematics are being reformulated by means of fuzzy sets. Assignment problem is one of the most-studied, well-known and important problems in mathematical programming. The assignment problem is a special type of Linear Programming problem in which our objective is to assign n number of jobs to n number of persons at a minimum cost (time).Two different types of assignment problems are in discussion; they are conventional and fuzzy assignment problems. In conventional assignment problem, cost is always certain.

Costs in many real life applications are not deterministic in nature. Linear programming with fuzzy cost coefficients is one of the most frequently applied fuzzy linear programming techniques. The aim of Fuzzy assignment problem is finding the least assignment fuzzy cost of workers with varying degrees of skill to jobs. The fuzzy assignment problem is more realistic than classical assignment problem because most real environments are uncertain. In recent years, many researchers have begun to investigate assignment problems under fuzzy environments. We try to propose a model to solve the fuzzy assignment problem, where cost is not deterministic number but imprecise one. The elements of the cost matrix of the assignment problem are any fuzzy number. In the classical assignment algorithm the assignment is based on creating zeros in the assignment matrix. Accordingly the optimal assignment is made. That is each row and each column has at least one zero. Instead of zeros assignment Hadi Basirzadeh [18] proposed ones assignment method for solving assignment problems. In this paper authors proposed a new method to solve fuzzy assignment problem based on ones (fuzzy) assignment and trapezoidal approximation of any fuzzy number to obtain an optimum solution and compare with the result of the conventional assignment problem.

Assigning membership functions corresponding to fuzzy numbers that represent linguistic values is an important point in fuzzy modeling. When operating with fuzzy numbers, the results strongly depend on the shape of the membership functions. Less regular membership functions lead to more complicated calculations. The fuzzy numbers with simpler shape of membership functions often have more natural interpretation. For these reasons we are in need of trapezoidal approximation of fuzzy numbers. Recently, there have been many research papers in fuzzy assignment problems. Votaw and Orden [12] introduced assignment problem. Kuhn [7] developed the Hungarian algorithm. Balinski and Gomory [4] introduced a labeling algorithm for solving assignment problems. Aggarwal et al.[2] developed an algorithm for bottleneck assignment problems. Chen [5] introduced a fuzzy assignment model that considers all individuals have same skills. Lin and Wen [8] investigated a fuzzy assignment problem in which the cost depends on the quality of the job. Liu and Gao [9] introduced fuzzy weighted equilibrium multi-job assignment problem and genetic algorithm. Yaakob and Watada [13] developed fuzzy approach for assignment problem. Yang and Liu [15] introduced a multi-objective fuzzy assignment problem. Mukherjee and Basu [10] proposed intuitionistic fuzzy assignment problem by solving similarity measures and score functions. Grzegorzewski and Mrowka [11] introduced trapezoidal approximation of fuzzy numbers. Adrian Ban [1] introducd trapezoidal approximation of fuzzy numbers preserving expected interval.

2. Preliminaries:

Definition 2.1: A fuzzy set $\tilde{A}$ defined on the universal set of real number $\mathbb{R}$, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0,1]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on (a, b) and strictly decreasing on (c, d)

(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$.

Definition 2.2: A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a}{b-a}\right), & \text{if } a < x < b \\ 1, & \text{if } b \leq x \leq c \\ \left(\frac{x-d}{c-d}\right), & \text{if } c < x < d \\ 0, & otherwise \end{cases}$$

Definition 2.3: The λ -cut of a fuzzy number $\tilde{A}$ is defined as

$$A_\lambda = \{x: \mu_{\tilde{A}}(x) \geq \lambda , \lambda \in [0,1]\}.$$

Definition 2.4 [1]: Let $a_1, a_2, a_3, a_4 \in \mathbb{R}$ such that $a_1 < a_2 \leq a_3 < a_4$. A fuzzy number $\tilde{A}$ defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)^r, & \text{if } a_1 \le x \le a_2 \\ 1, & \text{if } a_2 \le x \le a_3 \\ \left(\frac{a_4-x}{a_4-a_3}\right)^r, & \text{if } a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

where r > 0, is denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)_r$. If $\tilde{A} = (a_1, a_2, a_3, a_4)_r$ then

$A_\alpha = [A_L(\alpha), A_U(\alpha)] = [a_1 + \alpha^{1/r}(a_2 - a_1), a_4 - \alpha^{1/r}(a_4 - a_3)]$, $\alpha \in [0,1]$.

Definition 2.5 [3]: A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be LR fuzzy number if

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), for\ x \le m, \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), for\ x \ge n, \beta > 0 \\ 1, \quad otherwise \end{cases}$$

L and R are called reference functions which are continuous non-increasing functions that defines the left and right shapes of $\mu_{\tilde{A}}(x)$ respectively and L (0) = R (0) = 1.Two special cases are triangular and trapezoidal fuzzy number, for which L(x) = R(x) = maximum {0, 1–$|x|$}, are linear functions. Three commonly used nonlinear reference functions with parameters q, denoted as RF_q, are summarized as follows:

Power: $RF_q(x) = \text{maximum}(0, 1-|x|^q)$, $q \ge 0$

Exponential power : $RF_q(x) = e^{-|x|^q}$, $q \ge 0$

Rational : $RFq(x) = \frac{1}{(1-|x|^q)}$, $q \ge 0$

If $\tilde{A} = (m, n, \alpha, \beta)$ is a LR fuzzy number then

$A_\lambda = \{x \in X: \mu_{\tilde{A}}(x) \ge \lambda\} = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$.

2.1 Yager's ranking approach [3]:

A number of ranking approaches have been proposed for comparing fuzzy numbers. In this paper, Yager's ranking approach is used for ranking of fuzzy numbers. This approach involves a procedure for index $\Re(\tilde{A})$ is calculated for an LR fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ from its λ-cut $A_\lambda = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)]$ according to the following formula :

$$\Re(\tilde{A}) = \frac{1}{2}\left[\int_0^1 (m - \alpha L^{-1}(\lambda))\, d\lambda + \int_0^1 (n + \beta R^{-1}(\lambda))\, d\lambda\right]$$

Let $\tilde{A}$ and $\tilde{B}$ be two LR fuzzy numbers then

(i) $\tilde{A} \succcurlyeq_{\Re} \tilde{B}$ if $\Re(\tilde{A}) \geq \Re(\tilde{B})$

(ii) $\tilde{A} \succ_{\Re} \tilde{B}$ if $\Re(\tilde{A}) > \Re(\tilde{B})$

(iii) $\tilde{A} \approx_{\Re} \tilde{B}$ if $\Re(\tilde{A}) = \Re(\tilde{B})$

2.2 Linearity Property of Yager's ranking index [3]:

Let $\tilde{A} = (m_1, n_1, \alpha, \beta)_{LR}$ and $\tilde{B} = (m_2, n_2, \alpha, \beta)_{LR}$ be two fuzzy numbers and k_1, k_2 be two non-negative real numbers. The λ-cut A_λ and B_λ corresponding to $\tilde{A}$ and $\tilde{B}$ are :

$A_\lambda = [m_1 - \alpha_1 L^{-1}(\lambda), n_1 + \beta_1 R^{-1}(\lambda)]$ and $B_\lambda = [m_2 - \alpha_2 L^{-1}(\lambda), n_2 + \beta_2 R^{-1}(\lambda)]$.

Using the property, $(\delta_1 A_1 + \delta_2 A_2)_\lambda = \delta_1 (A_1)_\lambda + \delta_2 (A_2)_\lambda$ for all $\delta_1, \delta_2 \in \mathbb{R}$ ($\mathbb{R}$ is the set of all real numbers), the λ-cut $\left(k_1\tilde{A} + k_2\tilde{B}\right)_\lambda$ corresponding to $k_1 \tilde{A} \oplus k_2 \tilde{B}$ is :

$\left(k_1\tilde{A} + k_2\tilde{B}\right)_\lambda = [k_1 m_1 + k_2 m_2 - (k_1\alpha_1 + k_2\alpha_2)L^{-1}(\lambda)\,,\ k_1 n_1 + k_2 n_2 - (k_1\beta_1 + k_2\beta_2)\,R^{-1}(\lambda)]$

Using the Yager's ranking index $\Re(k_1 \tilde{A} \oplus k_2 \tilde{B})$ corresponding to fuzzy number $(k_1\tilde{A} \oplus k_2\tilde{B})$ is :

$$\Re(k_1\tilde{A} \oplus k_2\tilde{B}) = \frac{1}{2}k_1\left[\int_0^1 \left(m_1 - \alpha_1 L^{-1}(\lambda)\right) d\lambda + \int_0^1 \left(n_1 + \beta_1 R^{-1}(\lambda)\right) d\lambda\right] + \frac{1}{2}k_2\left[\int_0^1 \left(m_2 - \alpha_2 L^{-1}(\lambda)\right) d\lambda + \int_0^1 \left(n_2 + \beta_2 R^{-1}(\lambda)\right) d\lambda\right]$$

$$= k_1\Re(\tilde{A}) + k_2\,\Re(\tilde{B})$$

Similarly, it can be proved that $\Re(k_1\tilde{A} \oplus k_2\tilde{B}) = k_1\Re(\tilde{A}) + k_2\,\Re(\tilde{B})$, for all $k_1, k_2 \in \mathbb{R}$.

2.3 Arithmetic operations:

Let $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ be two-trapezoidal fuzzy numbers, then arithmetic operation on them are defined as follows:

Addition: $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

Subtraction: $\tilde{A} \ominus \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$

Scalar multiplication: $\lambda \tilde{A} = (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1)$; $\lambda \geq 0$

$\lambda \tilde{A} = (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1)$; $\lambda < 0$

Division : $\tilde{A} \oslash \tilde{B} = (a_1/a_2, b_1/b_2, c_1/c_2, d_1/d_2)$

Definition 2.6:

The expected interval EI (A) of a fuzzy number $\tilde{A}$ is given by [1]:

$$EI(A) = \left[\int_0^1 A_L(\alpha)\,d\alpha, \int_0^1 A_U(\alpha)\,d\alpha\right]$$

Definition 2.7:

A ranking function $\Re : F(\mathbb{R}) \rightarrow \mathbb{R}$, where $F(\mathbb{R})$ is a set of fuzzy numbers defined on set of real numbers , which maps each fuzzy number into the real line.

3. Linear Programming formulation of fuzzy assignment problem:

Suppose there are *n* jobs to be performed and *n* persons are available for doing these jobs. Assume that each person can do one job at a time and each job can be assigned to one person only. Let $\tilde{c}_{ij}$ be the fuzzy cost (payment) if j[th] job is assigned to i[th] person. The problem is to find an assignment x_{ij} so that the total cost for performing all the jobs is minimum. The assignment problem can be stated in the form of n × n cost matrix $[\tilde{c}_{ij}]$ of fuzzy numbers as follows:

Table1: Fuzzy assignment costs

	Job 1	Job 2	…	Job j	…	Job n
Worker 1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	…	$\tilde{c}_{1j}$	…	$\tilde{c}_{1n}$
Worker 2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	…	$\tilde{c}_{2j}$	…	$\tilde{c}_{2n}$
Worker 3	$\tilde{c}_{31}$	$\tilde{c}_{32}$	…	$\tilde{c}_{3j}$	…	$\tilde{c}_{3n}$
⋮	⋮		⋮	⋮	⋮	⋮
Worker n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	…	$\tilde{c}_{nj}$	…	$\tilde{c}_{nn}$

Table2.Crisp assignment costs

	Job 1	Job 2	…	Job j	…	Job n
Worker 1	$\Re(\tilde{c}_{11})$	$\Re(\tilde{c}_{12})$	…	$\Re(\tilde{c}_{1j})$	…	$\Re(\tilde{c}_{1n})$
Worker 2	$\Re(\tilde{c}_{21})$	$\Re(\tilde{c}_{22})$	…	$\Re(\tilde{c}_{2j})$	…	$\Re(\tilde{c}_{2n})$
Worker 3	$\Re(\tilde{c}_{31})$	$\Re(\tilde{c}_{32})$	…	$\Re(\tilde{c}_{3j})$	…	$\Re(\tilde{c}_{3n})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Worker n	$\Re(\tilde{c}_{n1})$	$\Re(\tilde{c}_{n2})$	…	$\Re(\tilde{c}_{nj})$	…	$\Re(\tilde{c}_{nn})$

Mathematically assignment problem can be stated as,

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$$

$$\text{subject to } \sum_{i=1}^{n} x_{ij} = 1, \; j = 1,2,3,\dots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \; i = 1, 2, 3,\dots,n , \; x_{ij} \in [0,1]$$

where, $x_{ij} = \begin{cases} 1, & \text{if the } i^{th} \text{ worker is assigned to the } j^{th} \text{ job} \\ 0, & \text{otherwise} \end{cases}$

is the decision variable denoting the assignment of the worker i to job j. $\tilde{c}_{ij}$ is the cost of assigning the j^{th} job to the i^{th} worker. The objective is to minimize the total cost of assigning all the jobs to the available persons (one job to one worker).When the costs $\tilde{C}_{ij}$ are any fuzzy number, then $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}\mathbf{x}_{ij}$ becomes a fuzzy number.

4. Trapezoidal approximation [1]:

Suppose we are looking for an approximation operator $T:F(\mathbb{R})\to F^T(\mathbb{R})$ which produces a trapezoidal fuzzy number that is the closest to given original fuzzy number among all trapezoidal fuzzy numbers having identical expected interval as the original one. Let $\tilde{A}$ denote a fuzzy number and $[A_L(\alpha), A_U(\alpha)]$ be its α–cut. Given $\tilde{A}$ we will find a trapezoidal fuzzy number $T(\tilde{A})$ which is the nearest to $\tilde{A}$ with respect to metric

$d(\tilde{A}, \tilde{B}) = \sqrt{\int_0^1 [A_L(\alpha) - B_L(\alpha)]^2 d\alpha + \int_0^1 [A_U(\alpha) - B_U(\alpha)]^2 d\alpha}$ for arbitrary fuzzy numbers $\tilde{A}$ & $\tilde{B}$ with α–cuts $[A_L(\alpha), A_U(\alpha)]$ and $[B_L(\alpha), B_U(\alpha)]$ respectively.

Let $[T_L(\alpha), T_U(\alpha)]$ denote the α-cut of $T(\tilde{A})$. Thus we have to minimize

$$d(\tilde{A}, T(\tilde{A})) = \sqrt{\int_0^1 [A_L(\alpha) - T_L(\alpha)]^2 d\alpha + \int_0^1 [A_U(\alpha) - T_U(\alpha)]^2 d\alpha}$$

with respect to $T_L(\alpha)$ and $T_U(\alpha)$. Moreover, since the desired operator should preserve the expected interval of a fuzzy number, the following condition should be fulfilled:

$$EI(T(\tilde{A})) = EI(\tilde{A}).$$

However, since a trapezoidal fuzzy number is completely described by four real numbers that are borders of its support and core , our goal reduces to finding such real numbers $t_1 \le t_2 \le t_3 \le t_4$ that characterize $T(\tilde{A}) = T(t_1, t_2, t_3, t_4)$.

According to [1] we have,

(a) if $-\int_0^1 A_L(\alpha)d\alpha + \int_0^1 A_U(\alpha)d\alpha + 3\int_0^1 \alpha A_L(\alpha)d\alpha - 3\int_0^1 \alpha A_U(\alpha)d\alpha \le 0,$

then the desired solution $T_1(\tilde{A}) = T(t_1, t_2, t_3, t_4)$ is given by

$t_1 = 4\int_0^1 A_L(\alpha)d\alpha - 6\int_0^1 \alpha A_L(\alpha)d\alpha,$

$t_2 = -2\int_0^1 A_L(\alpha)d\alpha + 6\int_0^1 \alpha A_L(\alpha)d\alpha,$

$t_3 = -2\int_0^1 A_U(\alpha)d\alpha + 6\int_0^1 \alpha A_U(\alpha)d\alpha,$

$t_4 = 4\int_0^1 A_U(\alpha)d\alpha - 6\int_0^1 \alpha A_U(\alpha)d\alpha,$

(b) if $-\int_0^1 A_L(\alpha)d\alpha + \int_0^1 A_U(\alpha)d\alpha + 3\int_0^1 \alpha A_L(\alpha)d\alpha - 3\int_0^1 \alpha A_U(\alpha)d\alpha > 0$

and $2\int_0^1 A_L(\alpha)d\alpha + \int_0^1 A_U(\alpha)d\alpha - 3\int_0^1 \alpha A_L(\alpha)d\alpha - 3\int_0^1 \alpha A_U(\alpha)d\alpha \le 0,$

$-\int_0^1 A_L(\alpha)d\alpha - 2\int_0^1 A_U(\alpha)d\alpha + 3\int_0^1 \alpha A_L(\alpha)d\alpha + 3\int_0^1 \alpha A_U(\alpha)d\alpha \le 0$

then we get $T_2(\tilde{A}) = T_2(t_1, t_2, t_3, t_4)$,where

$t_1 = 3\int_0^1 A_L(\alpha)d\alpha + \int_0^1 A_U(\alpha)d\alpha - 3\int_0^1 \alpha A_L(\alpha)d\alpha - 3\int_0^1 \alpha A_U(\alpha)d\alpha,$

$t_2 = -\int_0^1 A_L(\alpha)d\alpha - \int_0^1 A_U(\alpha)d\alpha + 3\int_0^1 \alpha A_L(\alpha)d\alpha + 3\int_0^1 \alpha A_U(\alpha)d\alpha,$

$t_3 = t_2,$

$t_4 = \int_0^1 A_L(\alpha)d\alpha + 3\int_0^1 A_U(\alpha)d\alpha - 3\int_0^1 \alpha A_L(\alpha)d\alpha - 3\int_0^1 \alpha A_U(\alpha)d\alpha,$

(c) if $2\int_0^1 A_L(\alpha)d\alpha + \int_0^1 A_U(\alpha)d\alpha - 3\int_0^1 \alpha A_L(\alpha)d\alpha - 3\int_0^1 \alpha A_U(\alpha)d\alpha > 0$

then we get $T_3(\tilde{A}) = T_3(t_1,t_2,t_3,t_4)$, is given by

$t_1 = t_2 = t_3 = \int_0^1 A_L(\alpha)d\alpha,$

$t_4 = 2\int_0^1 A_U(\alpha)d\alpha - \int_0^1 A_L(\alpha)d\alpha,$

(d) if $-\int_0^1 A_L(\alpha)d\alpha - 2\int_0^1 A_U(\alpha)d\alpha + 3\int_0^1 \alpha A_L(\alpha)d\alpha + 3\int_0^1 \alpha A_U(\alpha)d\alpha > 0$

then we obtain $T_4(\tilde{A}) = T_4(t_1,t_2,t_3,t_4)$ such that

$t_1 = 2\int_0^1 A_L(\alpha)d\alpha - \int_0^1 A_U(\alpha)d\alpha,$

$t_2 = t_3 = t_4 = \int_0^1 A_U(\alpha)d\alpha.$

5. A new approach for solving fuzzy assignment problem:

Consider a minimum (maximum) n × n assignment problem in which the costs are represented by fuzzy numbers.

Step 1: Using the trapezoidal approximation of fuzzy number [1] convert the given fuzzy number into trapezoidal fuzzy numbers.

Step 2: (Row reduction)

Choose the minimum (maximum) fuzzy number of each row in the assignment matrix, using ranking of fuzzy number and divide each fuzzy number of that row by the chosen fuzzy number.

Then we obtain at least one fuzzy ones in each row. In the reduced matrix, if each row and each column has at least one fuzzy ones then go to step 4.

Step 3: (Column reduction)

Choose the minimum (maximum) fuzzy number of each column in the assignment matrix, using ranking of fuzzy number and divide each fuzzy number of the column by the chosen fuzzy number. Then we obtain at least one fuzzy ones in each column. That is we have at least one fuzzy ones in each row and each column. Make assignment in terms of ones. If no feasible assignment can be achieved then go to step 4.

Step 4:

Draw the minimum number of lines to cover all the fuzzy ones of the matrix. If the number of drawn lines less than n, the order of the matrix, then the complete assignment is not possible , while if the number of lines is exactly equal to n , then the complete assignment is obtained .

Step 5:

If a complete assignment is not possible in step 4, then select the smallest (largest) fuzzy number out of those which do not lie on any of the lines in the above matrix. Then divide the entries of uncovered rows and columns by the chosen fuzzy number. This operation creates some new fuzzy ones. Repeat the steps 4 and 5 up to the optimal assignment.

6. Numerical Example:

Consider the following assignment problem. Assign four jobs to four workers so as to minimize the total cost.

Case (i): The costs are in trapezoidal form.

Consider the cost matrix,

$$\begin{pmatrix} (3,5,6,7) & (5,8,11,12) & (9,10,11,15) & (5,8,10,11) \\ (7,8,10,11) & (3,5,6,7) & (6,8,10,12) & (5,8,9,10) \\ (2,4,5,6) & (5,7,10,11) & (8,11,13,15) & (4,6,7,10) \\ (6,8,10,12) & (2,5,6,8) & (5.5,7,7,9) & (2,4,5,7) \end{pmatrix}$$

By Yager's ranking index the given cost matrix is reduced to

$$\begin{pmatrix} 5.25 & 9 & 11.25 & 8.5 \\ 9 & 5.25 & 9 & 8 \\ 4.25 & 8.25 & 11.75 & 6.75 \\ 9 & 5.25 & 7.1 & 4.5 \end{pmatrix}$$

By row reduction, column reduction and by steps 4 and 5 the optimal cost matrix is obtained as follows:

$$\begin{pmatrix} 1 & 1.36 & [1] & 1.19 \\ 1.71 & [1] & 1 & 1.40 \\ [1] & 1.54 & 1.29 & 1.6 \\ 1 & 1.16 & 1 & [1] \end{pmatrix}$$

i.e., the optimal assignment is A $\rightarrow$ 3, B $\rightarrow$ 2, C $\rightarrow$1, and D $\rightarrow$ 4.

Then the fuzzy optimal total cost is 25.25.

Case (ii): Trapezoidal approximation approach when the costs are in the form of (a_1, a_2, a_3, a_4) $_r$.

Consider the cost matrix,

$$\begin{pmatrix} (3,5,6,7)_2 & (5,8,11,12)_3 & (9,10,11,15)_2 & (5,8,10,11)_2 \\ (7,8,10,11)_4 & (3,5,6,7)_2 & (6,8,10,12)_1 & (5,8,9,10)_1 \\ (2,4,5,6)_3 & (5,7,10,11)_4 & (8,11,13,15)_3 & (4,6,7,10)_1 \\ (6,8,10,12)_1 & (2,5,6,8)_2 & (5.5,7,7,9)_4 & (2,4,5,7)_4 \end{pmatrix}$$

The fuzzy numbers in the cost matrix are converted into trapezoidal fuzzy numbers using the trapezoidal approximation of fuzzy number [1] as follows:

$$\begin{pmatrix} (2.43,5.56,5.56,7.1) & (6.29,8.21,10.93,11.57) & (9.27,10.06,10.73) & (4.93,9.06,9.06,11.6) \\ (7.53,8.06,9.93,10.47) & (2.43,5.56,5.56,7.1) & (6,8,10,12) & (5,8,9,10) \\ (2.86,4.14,4.93,5.57) & (6.06,7.13,9.93,10.46) & (9.28,11.21,12.85,14.14) & (4,6,7,10) \\ (6,8,10,12) & (2.8,5.2,5.86,7.46) & (6.7,6.7,6.7,8.1) & (3.06,4.13,4.86,5.93) \end{pmatrix}$$

By row reduction, column reduction and by steps 4 and 5 the optimal cost matrix is

$$\begin{pmatrix} (1,1,1,1) & (1.66,1.16,1.83,1.39) & [(1,1,1,1)] & (1.16,1.46,1.15,1.12) \\ (3.09,1.44,1.78,1.47) & [(1,1,1,1)] & (1,1,1,1) & (1.83,1.62,1.23,1.12) \\ [(1,1,1,1)] & (1.36,1.36,1.87,1.61) & (0.85,0.69,1.33,1.29) & (0.80,1.29,1.00,1.24) \\ (1.96,1.93,2.05,2.02) & (0.91,1.25,1.20,1.25) & (1,1,1,1) & [(1,1,1,1)] \end{pmatrix}$$

i.e., the optimal assignment is A $\rightarrow$ 3, B $\rightarrow$ 2, C $\rightarrow$ 1, and D $\rightarrow$ 4.

The fuzzy optimal total cost is = (17.62, 23.89, 26.08, 32.53).

Yager's ranking index corresponding to minimal total fuzzy cost is 25.03.

Case (iii): The costs are in LR fuzzy form:

Consider the cost matrix,

$$\begin{pmatrix} (3,5,6,7)_{LR} & (5,8,11,12)_{LR} & (9,10,11,15)_{LR} & (5,8,10,11)_{LR} \\ (7,8,10,11)_{LR} & (3,5,6,7)_{LR} & (6,8,10,12)_{LR} & (5,8,9,10)_{LR} \\ (2,4,5,6)_{LR} & (5,7,10,11)_{LR} & (8,11,13,15)_{LR} & (4,6,7,10)_{LR} \\ (6,8,10,12)_{LR} & (2,5,6,8)_{LR} & (5.5,7,7,9)_{LR} & (2,4,5,7)_{LR} \end{pmatrix}$$

where L(x) = maximum{0, $1–x^2$}, R(x) = maximum{0, 1–x}.

By row reduction, column reduction and by steps 4 and 5 the optimal cost matrix is

$$\begin{pmatrix} 1 & 1.3616 & [1] & 1.1933 \\ 1.7541 & [1] & 1 & 1.4173 \\ [1] & 1.5659 & 1.2821 & 1.1855 \\ 2.0384 & 1.1538 & 1 & [1] \end{pmatrix}$$

i.e., the optimal assignment is A $\to$ 3, B $\to$ 2, C $\to$1, and D $\to$ 4.

The fuzzy optimal total cost is = (23, 27, 7, 8).

Yager's ranking index corresponding to minimal total fuzzy cost is 24.66.

Case (iv): Trapezoidal approximation approach when the costs are in LR fuzzy form:

Consider the cost matrix,

$$\begin{pmatrix} (3,5,6,7)_{LR} & (5,8,11,12)_{LR} & (9,10,11,15)_{LR} & (5,8,10,11)_{LR} \\ (7,8,10,11)_{LR} & (3,5,6,7)_{LR} & (6,8,10,12)_{LR} & (5,8,9,10)_{LR} \\ (2,4,5,6)_{LR} & (5,7,10,11)_{LR} & (8,11,13,15)_{LR} & (4,6,7,10)_{LR} \\ (6,8,10,12)_{LR} & (2,5,6,8)_{LR} & (5.5,7,7,9)_{LR} & (2,4,5,7)_{LR} \end{pmatrix}$$

By trapezoidal approximation as in section 4, the given cost matrix is reduced to

$$\begin{pmatrix} (0.83,6.5,6.5,6.5) & (0.5,11.5,11.5,11.5) & (5.66,13,13,13) & (1.5,10.5,10.5,10.5) \\ (4.16,10.5,10.5,10.5) & (0.83,6.5,6.5,6.5) & (2.32,11,11,11) & (2.5,9.5,9.5,9.5) \\ (-0.18,5.5,5.5,5.5) & (0.82,10.5,10.5,10.5) & (4,14,14,14) & (0.82,8.5,8.5,8.5) \\ (2.32,11,11,11) & (-1,7,7,7) & (4,8,8,8) & (-0.68,6,6,6) \end{pmatrix}$$

By row reduction, column reduction and by steps 4 and 5 the optimum cost matrix is

$$\begin{pmatrix} 1 & 1.3616 & [1] & 1.1933 \\ 1.7541 & [1] & 1 & 1.4173 \\ [1] & 1.5659 & 1.2821 & 1.1855 \\ 2.0384 & 1.1538 & 1 & [1] \end{pmatrix}$$

i.e., the optimal assignment is A $\to$ 3, B $\to$ 2, C $\to$1, and D $\to$ 4.

Then the fuzzy optimal total cost is = (5.63, 31, 31, 31).

Yager's ranking index corresponding to minimal total fuzzy cost is 24.6575.

7. Results and Discussion:

Types of fuzzy number	Optimum cost	
	One's method	Zero's method

Normal trapezoidal	25.25	24.375
r fuzzy form	25.03	24.125
LR fuzzy	24.66	23.75
LR fuzzy to trapezoidal approximation	24.6575	23.745

We infer from the table that the trapezoidal approximation of LR fuzzy numbers has less assignment cost in both one's assignment and zero's assignment methods. While comparing one's assignment and zero's assignment in fuzzy environment zero's assignment has less assignment cost in all cases.

8. Conclusion:

Assigning different membership functions corresponding to considered fuzzy numbers results in different optimum costs. We noticed that simpler shape of fuzzy numbers have more natural interpretation. Numerical examples showed that converting LR fuzzy number into trapezoidal approximation preserving the expected interval leads to minimum assignment cost in zero's assignment method. The comparative result showed that in fuzzy environment zero's assignment is better than one's assignment.

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