A GENERALIZATION OF FUZZY SEMI-PRE OPEN SETS.

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Abstract

The authors introduced and studied fuzzy **C** -closed sets in fuzzy topological space where **C** : [0, 1]→ [0, 1] is a complement function. Let X be a non empty set. For any fuzzy subset λ of X, the complement **C** λ of λ is defined to be **C** λ (x)= **C** (λ (x)) for every x∈X. This **C** is called a complement function and **C** λ is called the complement of λ with respect to **C**. Using this, we introduced fuzzy **C** -regular closed sets, fuzzy **C** -α-closed sets, fuzzy **C** -semi closed sets and fuzzy **C** -pre closed sets and discussed their basic properties. The purpose of this paper is to introduce fuzzy **C** - semi pre open sets, fuzzy **C** -semi-pre closed sets and to discuss their properties.

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Key words: Fuzzy **C** -pre open, fuzzy **C** -pre closed, Fuzzy **C** -semi open, Fuzzy **C** -semi closed, fuzzy **C** -semi-pre open, fuzzy **C** - semi-pre closed, fuzzy **C** -semi-pre continuity and fuzzy topology.

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1. Introduction

S. S. Thakur, S.Singh [4] introduced the notion of fuzzy semi-pre open and fuzzy semi-pre closed. And their characterizations are studied by using the complement function $\lambda'(x) = 1 - \lambda(x)$, where the complement using is the standard complement function **C** $(x) = 1 - x$.

In this paper fuzzy **C** -semi-pre open and fuzzy **C** -semi-pre closed sets are introduced and characterized by using the arbitrary complement function.

In section 2, the basic concepts are discussed. In section 3, we introduce the notion of fuzzy **C** - semi- pre open and studied some of their properties using fuzzy **C** - closure, where **C**:$[0, 1]\to[0, 1]$ is a complement function.

In section 4, we give the notion of fuzzy **C** - semi- pre closed and studied some of their properties using fuzzy **C** - closure, where **C** :$[0, 1]\to[0, 1]$ is a complement function.

In section 5, we define the concept of fuzzy **C** –semi-pre interior and fuzzy **C** –semi-pre closure and investigate some of their basic properties.

In section 6, we give the concept of fuzzy **C** -semi -pre continuous functions.

Throughout this paper we assume that (X,τ) is a fuzzy topological space in the sense of Chang[7]. Let λ and μ denote the fuzzy subsets of X. For a fuzzy set λ in X, the operators $Int\,\lambda$ and $Cl_C\,\lambda$ denote the fuzzy interior and fuzzy **C** - closure of λ respectively.

2. Preliminaries

Definition 2.1 [Definition 2.2, [2]]

Let **C** : $[0, 1]\to[0, 1]$ be a complement function. If λ is a fuzzy subset of (X,τ) then the complement **C** λ of a fuzzy subset λ is defined by **C** $\lambda(x)$ = **C** $(\lambda(x))$ for all $x\in X$.

Lemma 2.2 [Lemma 2.9, [2]]

Let **C** : $[0, 1]\to[0, 1]$ be a complement function that satisfies the monotonic and involutive conditions. Then for any family $\{\lambda_\alpha: \alpha\in\Delta\}$ of fuzzy subsets of X, we have

(i) **C** $(\sup\{\lambda_\alpha(x): \alpha\in\Delta\}) = \inf\{$ **C** $(\lambda_\alpha(x)): \alpha\in\Delta\} = \inf\{($ **C** $\lambda_\alpha(x)): \alpha\in\Delta\}$ and

(ii) **C** $(\inf\{\lambda_\alpha(x): \alpha\in\Delta\}) = \sup\{$ **C** $(\lambda_\alpha(x)): \alpha\in\Delta\} = \sup\{($ **C** $\lambda_\alpha(x)): \alpha\in\Delta\}$ for $x\in X$.

Definition 2.3 [Lemma 2.9, [8]]

A complement function **C** is said to satisfy

(i) the boundary condition if **C** $(0) = 1$ and **C** $(1) = 0$,

(ii) monotonic condition if $x \le y \Rightarrow$ **C** $(x) \ge$ **C** (y), for all $x, y \in [0, 1]$,

(iii) involutive condition if **C** (**C** (x)) = x, for all x $\in$[0, 1].

The properties of fuzzy complement function **C** and **C** λ are given in Klir[8] and Bageerathi et al[2].

Definition 2.4 [Definition 3.1, [2]]

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then a fuzzy subset λ of X is fuzzy **C** -closed in (X,τ) if **C** λ is fuzzy open in (X,τ).

Definition 2.5 [Definition 4.1, [2]]

Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset λ of X, the fuzzy **C** -closure of λ is defined as the intersection of all fuzzy **C** -closed sets μ containing λ. The fuzzy **C** -closure of λ is denoted by $Cl_C\ \lambda$ that is equal to $\wedge\{\mu: \mu \geq \lambda$, **C** $\mu \in \tau\}$.

Lemma 2.6 [Lemma 4.2, [2]]

If the complement function **C** satisfies the monotonic and involutive conditions, then for any fuzzy subset λ of X, (i) **C** $(Int\ \lambda) = Cl_C$ (**C** λ) and (ii) **C** $(Cl_C \lambda) = Int$ (**C** λ).

Lemma 2.7 [Theorem 4.3, [2]]

Let **C** be a complement function that satisfies the monotonic and involutive conditions. Then for any two fuzzy subsets λ and μ of a fuzzy topological space, we have

(i) $\lambda \leq Cl_C\ \lambda$,

(ii) λ is fuzzy **C** -closed $\Leftrightarrow$ $Cl_C\ \lambda = \lambda$,

(iii) $Cl_C\ (Cl_C\ \lambda) = Cl_C\ \lambda$,

(iv) If $\lambda \leq \mu$ then $Cl_C\ \lambda \leq Cl_C\ \mu$,

(v) $Cl_C\ (\lambda \vee \mu) = Cl_C\ \lambda \vee Cl_C\ \mu$,

(vi) $Cl_C\ (\lambda \wedge \mu) \leq Cl_C\ \lambda \wedge Cl_C\ \mu$.

Lemma 2.8 [Theorem 4.4, [2]]

Let **C** be a complement function that satisfies the monotonic and involutive conditions. For any family $\{\lambda_\alpha\}$ of fuzzy sub sets of a fuzzy topological space we have

(i) $\vee Cl_C\ \lambda_\alpha \leq Cl_C\ (\vee\lambda_\alpha)$ and (ii) $Cl_C\ (\wedge \lambda_\alpha) \leq \wedge Cl_C\ \lambda_\alpha$.

Lemma 2.9 [Theorem 3.2, [2]]

Let (X,τ) be a fuzzy topological space. Let **C** be a complement function that satisfies the boundary, monotonic and involutive conditions. Then the following conditions hold.

(i) 0 and 1 are fuzzy **C** -closed sets,

(ii) arbitrary intersection of fuzzy **C** -closed sets is fuzzy **C** -closed and

(iii) finite union of fuzzy **C** -closed sets is fuzzy **C** -closed.

Lemma 2.10 [Lemma 2.10, [2]]

Let **C** : $[0, 1] \to [0, 1]$ be a complement function that satisfies involutive and monotonic conditions. Then for any family $\{\lambda_\alpha : \alpha \in \Delta\}$ of fuzzy subsets of X. we have (i) $\mathbf{C}(\vee\{\lambda_\alpha : \alpha\in\Delta\}) = \wedge\{\mathbf{C}\lambda_\alpha : \alpha\in\Delta\}$ and (ii) $\mathbf{C}(\wedge\{\lambda_\alpha : \alpha\in\Delta\}) = \vee\{\mathbf{C}\lambda_\alpha : \alpha\in\Delta\}$.

Definition 2.11 [Definition 2.15, [3]]

A fuzzy topological space (X, τ) is **C** -product related to another fuzzy topological space (Y, σ) if for any fuzzy subset ν of X and ζ of Y, whenever $\mathbf{C}\lambda \not\geq \nu$ and $\mathbf{C}\mu \not\geq \zeta$ imply $\mathbf{C}\lambda \times 1 \vee 1 \times \mathbf{C}\mu \geq \nu \times \zeta$, where $\lambda \in \tau$ and $\mu \in \sigma$, there exist $\lambda_1 \in \tau$ and $\mu_1 \in \sigma$ such that $\mathbf{C}\lambda_1 \geq \nu$ or $\mathbf{C}\mu_1 \geq \zeta$ and $\mathbf{C}\lambda_1 \times 1 \vee 1 \times \mathbf{C}\mu_1 = \mathbf{C}\lambda \times 1 \vee 1 \times \mathbf{C}\mu$.

Lemma 2.12 [Theorem 2.19, [3]]

Let (X, τ) and (Y, σ) be **C** -product related fuzzy topological spaces. Then for a fuzzy subset λ of X and a fuzzy subset μ of Y, $Cl_C(\lambda \times \mu) = Cl_C\lambda \times Cl_C\mu$.

Definition 2.13 [Definition 3.1, [5]]

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then λ is called fuzzy **C** -pre open if there exists a $\mu \in \tau$ such that $\mu \leq \lambda \leq Cl_C\,\mu$.

Lemma 2.14 [Proposition 6.2, [5]**]**

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involutive properties. Then for a fuzzy set λ of a fuzzy topological space (X,τ) is fuzzy **C** - preopen if and only if $\lambda \leq Int(Cl_C\lambda)$.

Definition 2.15 [Definition 6.1[4]]

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then a fuzzy subset λ of X is called a fuzzy **C** -pre closed set of X if $Cl_C(Int(\lambda)) \leq \lambda$.

Lemma 2.16 [Proposition 6.2, [4]]

Let λ be a fuzzy subset of a fuzzy topological space (X,τ) and **C** be a complement function that satisfies the monotonic and involutive conditions. Then λ is fuzzy **C** -pre closed if and only if $\mathbf{C}\lambda$ is fuzzy **C** -pre open.

Lemma 2.17 [Theorem 6.4, [5]]

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the monotonic and involutive properties. Then the arbitrary union of fuzzy **C** -preopen sets is fuzzy **C** -preopen.

Lemma 2.18 [Theorem 5.39, [5]]

Let (X, τ) and (Y, σ) be the fuzzy topological spaces such that X is **C** -product related to Y. Then the product $\lambda \times \mu$ of a fuzzy **C** -pre open set λ of X and a fuzzy **C** -pre open set μ of Y is fuzzy **C** -pre open of the fuzzy product space $X \times Y$.

Lemma 2.19 [Proposition 3.2, [5]]

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involutive properties. Then for a fuzzy set λ of a fuzzy topological space (X,τ) is fuzzy **C** - semiopen if and only if $\lambda \leq Cl_C(Int\, \lambda)$.

Lemma 2.20 [Proposition 5.4, [6]]

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the monotonic and involutive conditions. Then for a fuzzy sub set λ of a fuzzy topological space (X, τ) is fuzzy **C** -semi closed if and only if $Int\,(Cl_C\,(\lambda)) \leq \lambda$.

Lemma 2.21 [Lemma 5.1, [2]]

Suppose f is a function from X to Y. Then $f^{-1}(\mathbf{C}\,\mu) = \mathbf{C}\,(f^{-1}(\mu))$ for any fuzzy subset μ of Y.

Lemma 2.22 [Lemma 2.1, [1]]

Let $f : X \to Y$ be a function. If $\{\lambda_\alpha\}$ a family of fuzzy subsets of Y, then

(i) $f^{-1}(\vee \lambda_\alpha) = \vee f^{-1}(\lambda_\alpha)$ and

(ii) $f^{-1}(\wedge \lambda_\alpha) = \wedge f^{-1}(\lambda_\alpha)$.

3. Fuzzy C - semi pre-open sets

In this section, we introduce the concept of fuzzy **C** - semi pre open sets of a fuzzy topological space using fuzzy **C** -closure operator.

Definition 3.1

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then a fuzzy subset λ of X is called fuzzy **C** -semi-pre open if there exists a fuzzy **C** - pre open set μ such that $\mu \leq \lambda \leq Cl_C\ \mu$.

The class of all fuzzy semi-pre open sets coincides with the class of all fuzzy **C** -semi-pre open sets if the standard complement function coincides with the arbitrary complement function.

Proposition 3.2

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involutive conditions. Then a fuzzy subset λ of a fuzzy topological space (X,τ) is fuzzy **C** -semi-pre open if and only if $\lambda \leq Cl_C\ Int\ Cl_C\ \lambda$.

Proof.

Let λ be fuzzy **C** -semi-pre open. Then by using Definition 3.1, there exists a fuzzy **C** -pre open set μ such that $\mu \leq \lambda \leq Cl_C\ \mu$. Since **C** satisfies the monotonic and involutive condition, by applying Lemma 2.14, $\mu \leq Int\ (Cl_C\ \mu)$ that implies $\mu \leq \lambda \leq Cl_C\ \mu \leq Cl_C\ Int\ (Cl_C\ \mu)$. Thus we have $\lambda \leq Cl_C\ Int\ Cl_C\ \lambda$.

Conversely, we assume that $\lambda \leq Cl_C\ Int\ Cl_C\ \lambda$. Let $\mu = Int\ Cl_C\ \lambda$. Since μ is fuzzy **C** - pre open and **C** satisfies the monotonic and involutive condition by using Lemma 2.14, $\mu \leq Int\ (Cl_C\ \mu)$. From the above discussions, we have $\lambda \leq \mu \leq Cl_C\ (\lambda)$. By using Definition 3.1, λ is fuzzy **C** -semi-pre open.

Remark 3.3

It is clear that every fuzzy **C** - semi open set and every fuzzy **C** - pre open set is fuzzy **C** -semi-pre open. But the separate converses are not true as shown by the following example.

Example 3.4

Let X = {a, b} and τ = **{**0, $\{a_{.3}, b_{.8}\}$, $\{a_{.2}, b_{.5}\}$, $\{a_{.7}, b_{.05}\}$, $\{a_{.3}, b_{.5}\}$, $\{a_{.3}, b_{.05}\}$, $\{a_{.2}, b_{.05}\}$, $\{a_{.7}, b_{.8}\}$, $\{a_{.7}, b_{.5}\}$, 1**}**. Let $\mathbf{C}(x) = \frac{1-x}{1+2x}$, $0 \leq x \leq 1$, be the complement function. The family of all fuzzy **C** -closed sets **C** (τ) = **{**0, $\{a_{.4375}, b_{.077}\}$, $\{a_{.57}, b_{.25}\}$, $\{a_{.125}, b_{.86}\}$, $\{a_{.4375}, b_{.25}\}$, $\{a_{.4375}, b_{.86}\}$, $\{a_{.57}, b_{.86}\}$, $\{a_{.125}, b_{.077}\}$, $\{a_{.125}, b_{.25}\}$, 1**}**. Let $\lambda = \{a_{.3}, b_{.4}\}$.Then it can be computed that $Cl_C\ \lambda = \{a_{.4375}, b_{.86}\}$ and $Int\ Cl_C\ \lambda = \{a_{.3}, b_{.8}\}$ and $Cl_C\ Int\ Cl_C\ \lambda = \{a_{.4375}, b_{.86}\}$. Thus $\lambda \leq Cl_C\ Int\ Cl_C\ \lambda$.

By using Proposition 3.2, we see that λ is fuzzy **C** -semi-pre open.

Also $Int\,\lambda = \{a_{.3}, b_{.05}\}$ and $Cl_C\, Int\,\lambda = \{a_{.4375}, b_{.077}\}$ that implies $\lambda \not\leq Cl_C\, Int\,\lambda$.

That shows, by using Lemma 2.19, we see that λ is not fuzzy **C** -semi open.

Example 3.5

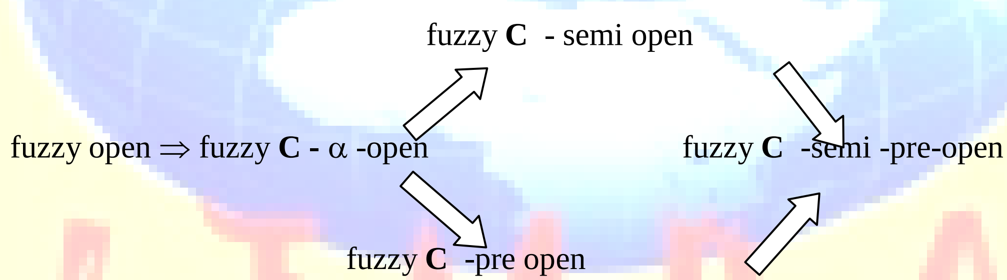
From Example 3.4, let X = {a, b} and $\tau = \{0, \{a_{.3}, b_{.8}\}, \{a_{.2}, b_{.5}\}, \{a_{.7}, b_{.05}\}, \{a_{.3}, b_{.5}\}, \{a_{.3}, b_{.05}\}, \{a_{.2}, b_{.05}\}, \{a_{.7}, b_{.8}\}, \{a_{.7}, b_{.5}\}, 1\}$. Let $\lambda = \{a_{.2}, b_{.85}\}$.Then it can be computed that $Cl_C\,\lambda = \{a_{.4375}, b_{.86}\}$ and $Int\, Cl_C\,\lambda = \{a_{.3}, b_{.8}\}$ and $Cl_C\, Int\, Cl_C\,\lambda = \{a_{.4375}, b_{.86}\}$. Thus $\lambda \leq Cl_C\, Int\, Cl_C\,\lambda$.

By using Proposition 3.2, we see that λ is fuzzy **C** -semi-pre open.

Also $Cl_C\,\lambda = \{a_{.4375}, b_{.86}\}$ and $IntCl_C\,\lambda = \{a_{.3}, b_{.8}\}$ that implies $\lambda \not\leq Int\, Cl_C\,\lambda$.

That shows, by Lemma 2.14, we see that λ is not fuzzy **C** -pre open.

It is clear from Remark 3.7 [5] and 6.4 [5] that the following diagram of implications is true.



S.S. Thakur and S.Singh**[9]** established that the intersection of fuzzy semi pre open sets is not a fuzzy semi pre open set. The next example shows that the intersection of any two fuzzy **C** -semi- pre open sets is not fuzzy **C** - semi-pre open.

Example 3.5

From Example 3.4, let X = {a, b} and $\tau = \{0, \{a_{.3}, b_{.8}\}, \{a_{.2}, b_{.5}\}, \{a_{.7}, b_{.05}\}, \{a_{.3}, b_{.5}\}, \{a_{.3}, b_{.05}\}, \{a_{.2}, b_{.05}\}, \{a_{.7}, b_{.8}\}, \{a_{.7}, b_{.5}\}, 1\}$. Let $\lambda = \{a_{.2}, b_{.6}\}$, it can be found that $Cl_C\,\lambda = \{a_{.4375}, b_{.86}\}$, $Int\, Cl_C\,\lambda = \{a_{.3}, b_{.8}\}$ and $Cl_C\, Int\, Cl_C\,\lambda = \{a_{.4375}, b_{.86}\}$. That is $\lambda \leq Cl_C\, Int\, Cl_C\,\lambda$.

And let $\mu = \{a_{.8}, b_{.25}\}$, it follows that $Cl_C\,\mu = \{1\}$, $Int\, Cl_C\,\mu = \{1\}$ and $Cl_C\, Int\, Cl_C\,\lambda = \{1\}$ that implies $\mu \leq Cl_C\, Int\, Cl_C\,\mu$. Now $\lambda\wedge\mu = \{a_{.2}, b_{.25}\}$, $Cl_C(\lambda\wedge\mu) = \{a_{.4375}, b_{.25}\}$, $Int\, Cl_C(\lambda\wedge\mu) = \{a_{.3}, b_{.05}\}$ and $Cl_C\, Int\, Cl_C(\lambda\wedge\mu) = \{a_{.4375}, b_{.077}\}$ that implies $\lambda\wedge\mu \not\leq Cl_C\, IntCl_C(\lambda\wedge\mu)$.

By using Proposition 3.2 shows that $\lambda \wedge \mu$ is not fuzzy **C** -semi -pre open, even though λ and μ are fuzzy **C** -semi-pre open.

S.S. Thakur and S.Singh**[9]** established that any union of fuzzy semi-pre open sets is a fuzzy semi-pre open set. The next example shows that the union of any two fuzzy **C** -semi-pre open sets is not fuzzy **C** -semi-pre open.

Example 3.6

Let X = {a, b, c} and τ = **{**0, {a.$_6$, b.$_3$}, {b.$_4$, c.$_6$}, {a.$_2$, c.$_5$}, {b.$_3$}, {a.$_6$, b.$_4$, c.$_6$}, {a.$_2$, b.$_4$, c.$_6$}, {c.$_5$}, {a.$_2$}, {a.$_6$, b.$_3$, c.$_5$}, {a.$_2$, b.$_3$}, {a.$_2$, b.$_3$, c.$_5$}, {b.$_3$, c.$_5$}, 1**}**. Then (X, τ) is a fuzzy topological space. Let **C** (x) = $\frac{2x}{1+x}$, $0 \le x \le 1$, be a complement function and **C** does not satisfy the monotonic and involutive conditions. The family of all fuzzy **C** -closed sets **C** (τ) = **{**0, {a.$_{75}$, b.$_{46}$}, {b.$_{571}$, c.$_{75}$}, {a.$_{33}$, c.$_{667}$}, {b.$_{462}$}, {a.$_{75}$, b.$_{571}$, c.$_{75}$}, {a.$_{33}$, b.$_{57}$, c.$_{75}$}, {c.$_{667}$}, {a.$_{33}$}, {a.$_{75}$, b.$_{462}$, c.$_{667}$}, {a.$_{33}$, b.$_{462}$}, {a.$_{33}$, b.$_{462}$, c.$_{667}$}, {b.$_{462}$, c.$_{667}$}, 1**}**. Let λ = {a.$_{75}$, b.$_{35}$}, it can be find that $Cl_C\,\lambda$ = {a.$_{75}$, b.$_{46}$}, $Int\ Cl_C\,\lambda$ = {a.$_6$, b.$_3$}. and $Cl_C\ Int\ Cl_C\,\lambda$= {a.$_{75}$, b.$_{46}$}. That is $\lambda \le Cl_C\ Int\ Cl_C\,\lambda$.

Let μ = {b.$_{45}$, c.$_{75}$}, it follows that $Cl_C\,\mu$ = {b.$_{57}$, c.$_{75}$}, $Int\ Cl_C\,\mu$ = {b.$_4$, c.$_6$}and $Cl_C\ Int\ Cl_C\,\mu$ = {b.$_{571}$, c.$_{75}$} that implies $\mu \le Cl_C\ Int\ Cl_C\,\mu$. Now $\lambda \vee \mu$ = {a.$_{75}$, b.$_{45}$, c.$_{75}$}, $Cl_C\,(\lambda \vee \mu)$ = {a.$_{75}$, b.$_{57}$, c.$_{75}$} and

$Int\ Cl_C\,(\lambda \vee \mu)$ = {a.$_6$, b.$_4$, c.$_6$} and $Cl_C\ Int\ Cl_C\,(\lambda \vee \mu)$ = {a.$_{75}$, b.$_{462}$, c.$_{667}$} that implies $\lambda \vee \mu \not\le Cl_C\ Int\ Cl_C\,(\lambda \vee \mu)$. By using Proposition 3.2, $\lambda \vee \mu$ is not fuzzy **C** - semi-pre open, even though λ and μ are fuzzy **C** - semi-pre open.

If the complement function **C** satisfies the monotonic and involutive conditions, then union of two fuzzy **C** - semi-pre open sets is again fuzzy **C** - semi-pre open as shown in the next proposition.

Theorem 3.7

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the monotonic and involutive conditions. Then the arbitrary union of fuzzy **C** -semi-pre open sets is fuzzy **C** -semi-pre open.

Proof

Let $\{\lambda_\alpha\}$ be a collection of fuzzy **C** - semi-pre open sets of a fuzzy space X. Then for each α, there exists a fuzzy **C** - pre open set μ_α such that $\mu_\alpha \leq \lambda_\alpha \leq Cl_C(\mu_\alpha)$. Thus $\vee\mu_\alpha \leq \vee\lambda_\alpha \leq \vee Cl_C(\mu_\alpha)$. Since **C** satisfies the monotonic and involutive properties, by using Lemma 2.8, we have $\vee Cl_C(\mu_\alpha) \leq Cl_C(\vee\mu_\alpha)$, that implies $\vee\mu_\alpha \leq \vee\lambda_\alpha \leq Cl_C(\vee\mu_\alpha)$. By using Lemma 2.17, we have arbitrary union of fuzzy **C** -pre pen sets is fuzzy **C** – semi open, that implies $\vee\mu_\alpha$is fuzzy **C** - pre open. By using Definition 3.1, we have $\{\vee\lambda_\alpha\}$ is a fuzzy **C** - semi- preopen set.

Proposition 3.8

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the monotonic and involutive conditions. If $\lambda \leq \mu \leq Cl_C\ \lambda$ and λ is fuzzy **C** -semi-pre open in (X, τ) then μ is also such that fuzzy **C** -semi-pre open in (X, τ)

Proof.

Let ν be fuzzy **C** -pre open such that $\lambda \leq \mu \leq Cl_C\ \lambda$. Clearly $\nu_1 \leq \mu$ and $\lambda \leq Cl_C\ \nu$ implies that $Cl_C\ \nu \leq Cl_C\ \lambda$. Consequently, $\nu \leq \mu\ Cl_C\ \nu$. Hence μ is fuzzy **C** -semi-pre open in (X, τ).

Theorem 3.9

Let (X, τ) and (Y, σ) be **C** -product related fuzzy topological spaces. Then the product $\lambda_1 \times \lambda_2$ of a fuzzy **C** -semi-pre open set λ_1 of X and a fuzzy **C** -semi-pre open set λ_2 of Y is a fuzzy **C** -semi-pre open set of the fuzzy product space X × Y.

Proof.

Let λ_1 be a fuzzy **C** -semi-pre open subset of X and λ_2 be a fuzzy **C** -semi-pre open subset of Y. Then by using Definition 3.1, there exists a **C** -pre open sets μ_1 in X and μ_2 in Y such that $\mu_1 \leq \lambda_1 \leq Cl_C\ \mu_1$ and $\mu_2 \leq \lambda_2 \leq Cl_C\ \mu_2$. That implies $\mu_1 \times \mu_2 \leq \lambda_1 \times \lambda_2 \leq Cl_C\ \mu_1 \times Cl_C\ \mu_2$. By applying Lemma 2.12, $\mu_1 \times \mu_2 \leq \lambda_1 \times \lambda_2 \leq Cl_C(\mu_1 \times \mu_2)$. Again by using Definition 3.1, $\lambda_1 \times \lambda_2$ is a fuzzy **C** -semi pre open set of the fuzzy product space X × Y.

4. Fuzzy C -semi-pre closed sets

This section is devoted to the concept of fuzzy **C** -semi-pre closed sets that are defined by using fuzzy **C** -closure operator.

Definition 4.1

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then a fuzzy subset λ of X is called fuzzy **C** -semi-pre closed in (X,τ) if there exists a fuzzy **C** -pre closed set μ such that $\mathbf{C}\,\mu \leq \mathbf{C}\,\lambda \leq Cl_C(\mathbf{C}\,\mu)$.

Remark 4.2

If **C** (x) = 1– x, then **C** μ ≤ **C** λ ≤ Cl_C (**C** μ) ⇒ 1 – μ ≤ 1 – λ ≤ **C** (*Int*μ)

⇒ 1 – μ ≤ 1 – λ ≤ 1 – *Int* μ

⇒ *Int* μ ≤ λ ≤ μ

So, the class of all fuzzy **C** -semi-pre closed sets coincides with the class of all fuzzy semi-pre closed sets if **C** (x) = 1–x.

The standard complement of fuzzy semi-pre open is fuzzy semi pre closed. The analogous result is not true for fuzzy **C** -semi-pre open. If the complement function **C** satisfies the involutive condition, then the arbitrary complement of fuzzy **C** -semi-pre open is fuzzy **C** -semi-pre closed.

Proposition 4.3

Let (X, τ) be a fuzzy topological space and **C** be a complement function. Then

(i) If λ is fuzzy **C** -semi-pre closed then **C** λ is fuzzy **C** -semi-pre open.

(ii) If λ is fuzzy **C** -semi-pre open then **C** λ is fuzzy **C** -semi-pre closed provided **C** satisfies the involutive condition.

Proof.

Let λ be fuzzy C -semi-pre closed. Then by using Definition 4.1, there exist a fuzzy C -pre closed set μ such that C μ ≤ C λ ≤ Cl_C (C μ). By replacing C μ = δ, δ ≤ C λ ≤ Cl_C (δ). By using Definition 3.1, C λ is fuzzy C -semi-pre open. This proves (i).

Let λ be fuzzy C -semi-pre open. Then by using Definition 3.1, there exists a fuzzy C -pre open η such that η ≤ λ ≤ Cl_C η. Let μ = C η. Since C satisfies the involutive condition, η = C (C η) = C μ. That is, C μ ≤ C (C λ) ≤ Cl_C (C μ). Thus, C λ is fuzzy C -semi-pre closed.

Proposition 4.4

Let (X, τ) be a fuzzy topological space and C be a complement function that satisfies the monotonic and involutive conditions. Then for a fuzzy sub set λ of a fuzzy topological space (X, τ) is fuzzy C -semi-pre closed if and only if *Int* Cl_C *Int*(λ) ≤ λ.

Proof.

Let λ be fuzzy C -semi-pre closed. Then by using Proposition 4.3, C λ is fuzzy C -semi-pre open that implies C λ ≤ Cl_C *Int*Cl_C **C** λ. Taking complement on both sides, we getC (C λ)≤ C

$(Cl_C\ Int\ Cl_C\ \mathbf{C}\ \lambda)$. Since **C** satisfies the monotonic and involutive conditions, by applying Lemma 2.6, $Int\ Cl_C\ Int\ \lambda \leq \lambda$.

S.S.Thakur and S.Singh **[9]** established that any union of fuzzy semi-pre closed sets is not fuzzy semi-pre closed set. However the following example shows that the union of any two fuzzy C -semi pre closed sets is not fuzzy C -semi -pre closed.

Example 4.5

Let X = {a, b, c} and $\tau = \{0, \{c_{.4}\}, \{a_{.7}\}, \{a_{.7}, c_{.4}\}, 1\}$. Let C (x) = $\frac{1-x}{1+2x}$, $0 \leq x \leq 1$, be a complement function. Then the family of all fuzzy C -closed sets is $C(\tau) = \{0, \{a_1, b_1, c_{.33}\}, \{a_{.125}, b_1, c_1\}, \{a_{.125}, b_1, c_{.33}\}, 1\}$. Let $\lambda = \{c_{.4}\}$, $\mu = \{a_{.7}\}$ and $\lambda \vee \mu = \{a_{.7}, c_{.4}\}$. Then $Int\ \lambda = \{c_{.4}\}$, $Cl_C\ Int\ \lambda = \{a_{.125}, b_1, c_1\}$ and $Int\ Cl_C\ Int\ \lambda = \{c_{.4}\} \leq \lambda$. Now $Int\ \mu = \{c_{.4}\}$, $Cl_C\ Int\ \mu = \{a_{.125}, b_1, c_1\}$ and $Int\ Cl_C\ Int\ \mu = \{c_{.4}\} \leq \mu$. By Proposition 4.4, shows that λ and μ are fuzzy C-semi- pre closed sets. Now, $Int\ (\lambda \vee \mu) = \{a_{.7}, c_{.4}\}$ $Cl_C\ Int\ (\lambda \vee \mu) = \{1\}$ and $Int\ Cl_C\ Int\ (\lambda \vee \mu) = 1 \nleq \lambda \vee \mu$. By using Proposition 4.4, $\lambda \vee \mu$ is not fuzzy C –semi-pre closed.

S.S.Thakur and S.Singh **[9]** established that the intersection of fuzzy semi-pre closed sets is fuzzy semi-pre closed. Moreover the following examples shows that the intersection of any two fuzzy C -semi-pre closed sets is not fuzzy C -semi-pre closed.

Example 4.6

Let X = {a, b, c} and $\tau = \{0, \{a_{.6}, b_{.3}\}, \{b_{.4}, c_{.6}\}, \{a_{.2}, c_{.6}\}, \{b_{.3}\}, \{a_{.6}, b_{.4}, c_{.6}\}, \{a_{.2}, b_{.4}, c_{.6}\}, \{c_{.5}\}, \{a_{.2}\}, \{a_{.6}, b_{.3}, c_{.6}\}, \{a_{.2}, b_{.3}\}, \{a_{.2}, b_{.3}, c_{.6}\}, \{b_{.3}, c_{.6}\}, 1\}$. Then (X,τ) is a fuzzy topological space. Let C (x) = $\frac{2x}{1+x}$, $0 \leq x \leq 1$, be a complement function. The family of all fuzzy C-closed sets $C(\tau) = \{0, \{b_{.57}, c_{.75}\}, \{a_{.75}, b_{.46}\}, \{a_{.33}, c_{.75}\}, \{b_{.46}\}, \{a_{.75}, b_{.57}, c_{.75}\}, \{a_{.33}, b_{.57}, c_{.75}\}, \{c_{.75}\}, \{a_{.33}\}, \{a_{.75}, b_{.46}, c_{.75}\}, \{a_{.33}, b_{.46}\}, \{a_{.33}, b_{.46}, c_{.75}\}, \{b_{.46}, c_{.75}\}, 1\}$. Let $\lambda = \{a_{.2}, b_{.3}, c_{.6}\}$ and $\mu = \{b_{.4}, c_{.6}\}$. Then it can be calculated that $Int\ \lambda = \{a_{.2}, b_{.3}, c_{.6}\}$, $Cl_C\ Int\ \lambda = \{a_{.33}, b_{.46}, c_{.75}\}$ and $Int\ Cl_C\ Int\ \lambda = \{a_{.2}, b_{.3}, c_{.6}\}$.

And $Int\ \mu = \{b_{.4}, c_{.6}\}$, $Cl_C\ Int\ \mu = \{b_{.46}, c_{.75}\}$ and $Int\ Cl_C\ Int\ \mu = \{b_{.4}, c_{.6}\}$. By Proposition 4.4, shows that λ and μ are fuzzy C-semi- pre closed sets. Now, $Int\ \lambda \wedge \mu = \{b_{.3}, c_{.6}\}$ and $Cl_C\ Int\ (\lambda \wedge \mu) \neq \{b_{.46}, c_{.75}\}$ and $Int\ Cl_C\ Int\ (\lambda \wedge \mu) = \{b_{.4}, c_{.6}\} \nleq (\lambda \wedge \mu)$. By using Proposition 4.4, $\lambda \wedge \mu$ is not fuzzy **C** - semi-pre closed.

Remark 4.7

Further, the Example 4.6 shows that the intersection of any two fuzzy C - semi-pre closed sets is not fuzzy C - semi-pre closed, even though the complement function satisfies the monotonic and involutive conditions.

If the complement function **C** satisfies the monotonic and involutive conditions. Then arbitrary intersection of fuzzy **C** -semi-pre closed sets is fuzzy **C** -semi-pre closed as shown in the following proposition.

Proposition 4.8

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the monotonic and involutive conditions. Then arbitrary intersection of fuzzy **C** -semi-pre closed sets is fuzzy **C** -semi-pre closed.

Proof.

Let $\{\lambda_\alpha\}$ be a collection of all fuzzy **C** -semi-pre closed sets of a fuzzy topological space X. Then for each α, there exists a fuzzy **C** -pre closed set μ_α such that **C**

$\mu_\alpha \leq$ **C** $\lambda_\alpha \leq Cl_C$ (**C** μ_α). Thus $\vee$ C $\mu_\alpha \leq \vee$ **C** $\lambda_\alpha \leq \vee Cl_C$ (**C** μ_α). Since **C** satisfies the monotonic and involutive conditions, by using Lemma 2.8, we have $\vee Cl_C$ (C μ_α) $\leq Cl_C$ ($\vee$C μ_α). This implies that $\vee$ C $\mu_\alpha \leq \vee$ C $\lambda_\alpha \leq Cl_C$ ($\vee$C μ_α). By using Lemma 2.2, **C** ($\wedge\mu_\alpha$) $\leq$ **C** ($\wedge\lambda_\alpha$) $\leq Cl_C$ **C** ($\wedge\mu_\alpha$). By using Lemma 2.17, arbitrary intersection of fuzzy **C** -pre closed sets is fuzzy **C** -pre closed. That implies $\wedge\mu_\alpha = \mu$ is fuzzy **C** -pre closed. Thus we see that **C** $\mu \leq$ **C** ($\wedge\lambda_\alpha$) $\leq Cl_C$ (**C** μ). By using Definition 4.1, $\wedge\lambda_\alpha$ is a fuzzy **C** -semi-pre closed.

It is clear that every fuzzy **C** -pre closed set is fuzzy **C** -semi-pre closed. But the converse is not true as shown by the following example.

Example 4.9

Let X = {a, b, c} and τ = **{**0, {a.$_2$, c.$_5$}, {b.$_3$}, {a.$_2$, b.$_3$, c.$_5$}, 1**}**. Let **C** (x) = $\frac{1-x}{1+3x}$, $0 \leq x \leq 1$, be a complement function. Then the family of all fuzzy **C** -closed sets **C** (τ) = **{**0, {a.$_5$, b$_1$, c.$_2$}, {a$_1$, b.$_{36}$, c$_1$}, {a.$_5$, b.$_{36}$, c.$_2$}, 1**}**. Let λ = {a.$_5$, b.$_5$, c$_1$}. Then *Int* λ = {a.$_2$, b.$_3$, c.$_5$}, Cl_C *Int* λ = {a$_1$, b.$_{36}$, c$_1$} and *Int* Cl_C *Int* λ= {a.$_2$, b.$_3$, c.$_5$}. This implies that *Int* Cl_C *Int* $\lambda \leq \lambda$.By using Proposition 4.4, λ is fuzzy **C** - semi-pre closed.

Also Cl_C *Int* λ= {a$_1$, b.$_{36}$, c$_1$} $\not\leq \lambda$, this shows that λ is not fuzzy **C** -pre closed.

It is clear that every fuzzy **C** -semi closed set is fuzzy **C** - semi-pre closed. But the converse is not true as shown in the following example.

Example 4.10

From Example 4.9, let X = {a, b, c} and τ = **{**0, {a.$_2$, c.$_5$}, {b.$_3$}, {a.$_2$, b.$_3$, c.$_5$}, 1**}**. Let μ = {a.$_2$, b.$_6$, c.$_8$}, it can be computed that *Int* λ = {a.$_2$, b.$_3$, c.$_5$}, Cl_C *Int* λ = {a.$_5$, b.$_{36}$, c$_1$} and *Int* Cl_C *Int* λ= {a.$_2$, b.$_3$, c.$_5$}. This implies that *Int* Cl_C *Int* $\lambda \leq \lambda$. By using Proposition 4.4, λ is fuzzy **C** - semi-pre closed. Also *Int* Cl_C λ= {1} $\not\leq \lambda$, this shows that λ is not fuzzy **C** - semi closed.

5. Fuzzy C -semi-pre interior and fuzzy C -semi-pre closure

In this section, we define the concept of fuzzy **C** -semi-pre interior and fuzzy **C** -semi-pre closure and investigate some of their basic properties.

Definition 5.1

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then for a fuzzy subset λ of X, the fuzzy **C** -semi-pre interior of λ (briefly spInt_C λ), is the union of all fuzzy **C** -semi-pre open sets of X contained in λ.

That is, spInt_C (λ) = $\vee$ {μ; $\mu \leq \lambda$, μ is fuzzy **C** -semi-pre open}.

Proposition 5.2

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involvtive conditions. Then for any fuzzy subsets λ and μ of a fuzzy topological space X, we have

(i) *Int* $\lambda \leq$ sp$Int_C\lambda$,

(ii) sp$Int_C\lambda$ is fuzzy **C** -semi-pre open,

(iii) λ is fuzzy **C** -semi-pre open $\Leftrightarrow$ spInt_C $\lambda = \lambda$,

(iv) spInt_C (spInt_C λ) = spInt_C λ,

(v) If $\lambda \leq \mu$ then spInt_C $\lambda \leq$ spInt_C μ.

Proof.

By using Remark, every fuzzy open set is fuzzy **C** -semi-pre open. So, we have *Int* $\lambda \leq$ spInt_C λ. This proves (i).

(ii) follows from Definition 5.1.

Let λ be fuzzy **C** -semi-pre open. Since $\lambda \leq \lambda$, by Definition 5.1, $\lambda \leq spInt_C\ \lambda$. By using (ii), we get $spInt_C\ \lambda = \lambda$. Conversely we assume that $spInt_C\ \lambda = \lambda$. By using Definition 5.1, λ is fuzzy **C** -semi -preopen. Thus (iii) is proved.

By using (iii), we get $spInt_C\ (sInt_C\ \lambda) = spInt_C\ \lambda$. This proves (iv).

Since $\lambda \leq \mu$, by using (i), $spInt_C\ \lambda \leq \lambda \leq \mu$. This implies that $spInt_C\ (spInt_C\ \lambda) \leq spInt_C\ \mu$. By using (iii), we get $spInt_C\ \lambda \leq spInt_C\ \mu$. This proves (v).

Proposition 5.3

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involutive conditions. Then for any two fuzzy subsets λ and μ of a fuzzy topological space, we have (i) $spInt_C\ (\lambda \vee \mu) \geq spInt_C\ \lambda \vee spInt_C\ \mu$ and (ii) $spInt_C\ (\lambda \wedge \mu) \leq spInt_C\ \lambda \wedge spInt_C\ \mu$.

Proof.

Since $\lambda \leq \lambda \vee \mu$ and $\mu \leq \lambda \vee \mu$. By using Proposition 5.2(iv), we get $spInt_C \lambda \leq spInt_C\ (\lambda \vee \mu)$ and $spInt_C\ \mu \leq spInt_C\ (\lambda \vee \mu)$. This implies that $spInt_C\ \lambda \vee spInt_C\ \mu \leq spInt_C\ (\lambda \vee \mu)$.

Since $\lambda \wedge \mu \leq \lambda$ and $\lambda \wedge \mu \leq \mu$. By using Proposition 5.2(v), we get $spInt_C\ (\lambda \wedge \mu) \leq spInt_C\ \lambda$ and $spInt_C\ (\lambda \wedge \mu) \leq spInt_C\ \mu$. This implies that $sInt_C\ (\lambda \wedge \mu) \leq spInt_C\ \lambda \wedge spInt_C\ \mu$.

Definition 5.4

Let (X,τ) be a fuzzy topological space. Then for a fuzzy subset λ of X, the fuzzy **C** - semi-pre closure of λ (briefly $spCl_C\ \lambda$), is the intersection of all fuzzy **C** - semi-pre closed sets containing λ.

That is $spCl_C\ \lambda = \wedge\{\mu : \mu \geq \lambda, \mu$ is fuzzy **C** - semi-pre closed$\}$.

The concepts of "fuzzy **C** - semi-pre closure" and "fuzzy semi-pre closure" are identical if **C** is the standard complement function.

Proposition 5.5

If the complement functions **C** satisfies the monotonic and involutive conditions. Then for any fuzzy subset λ of X, (i) **C** $(spInt_C\ \lambda) = spCl_C$ (**C** λ) and (ii) **C** $(spCl_C\ \lambda) = spInt_C$ (**C** λ), where $spInt_C\ \lambda$ is the union of all fuzzy **C** - semi-pre open sets contained in λ.

Proof.

By Definition 5.1, spInt_C λ = $\vee${μ: $\mu \leq \lambda$, μ is fuzzy **C** - semi-pre open}. Taking complement on both sides, we get **C** (spInt_C (λ)(x)) = **C** (sup{μ(x): $\mu(x) \leq \lambda(x)$, μ is fuzzy **C** - semi-pre open}). Since **C** satisfies the monotonic and involutive conditions, by using Lemma 2.2, **C** (spInt_C (λ)(x)) = inf { **C** (μ(x)); $\mu(x) \leq \lambda(x)$, μ is fuzzy **C** - semi-pre open}. By using Definition 2.1, **C** (spInt_C (λ)(x)) = inf { **C** μ(x)): **C** $\mu(x) \geq$ **C** λ(x), μ is fuzzy **C** - semi-pre open}. By using Proposition 4.3, **C** μ is fuzzy **C** - semi-pre closed, by replacing **C** μ by η, we see that **C** (spInt_C (λ)(x)) = inf{η(x): $\eta(x)\geq$ **C** λ(x), **C** η is fuzzy **C** - semi-pre open}. By using Definition 5.4, **C** (spInt_C (λ)(x)) = spCl_C (**C** λ) (x). This proves that **C** (spInt_C λ) = spCl_C (**C** λ).

By using Definition 5.4, spCl_C λ = $\wedge${μ: $\lambda\leq\mu$, μ is fuzzy **C** - semi-pre closed}. Taking complement on both sides, we get **C** (spCl_C λ (x)) = **C** (inf{μ(x); $\mu(x) \geq \lambda(x)$: μ is fuzzy **C** - semi-pre closed}). Since **C** satisfies the monotonic and involutive conditions, by using Lemma 2.2, **C** (spCl_C λ(x)) = sup { **C** (μ(x)): $\mu(x) \geq \lambda$ (x): μ is fuzzy **C** - semi-pre closed}. By Definition 2.1, **C** (spCl_C λ(x)) = sup { **C** μ(x): **C** μ(x)) $\leq$ **C** λ(x): μ is fuzzy **C** - semi-pre closed}. By using Proposition 4.3, **C** μ is fuzzy **C** - semi-pre open, by replacing **C** μ by η, we see that **C** (sCl_C (x)) = sup {η(x): $\eta(x) \leq$ **C** λ(x); η is fuzzy **C** - semi-pre open}. By using Definition 5.1, (spCl_C λ(x)) = spInt_C (**C** λ)(X). This proves **C** (spCl_C (λ)) = spInt_C (**C** λ).

Proposition 5.6

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involvtive conditions. Then for the fuzzy subsets λ and μ of a fuzzy topological space X, we have

(i) $\lambda \leq$ spCl_C λ,

(ii) λ is fuzzy **C** - semi-pre closed $\Leftrightarrow$ spCl_C $\lambda = \lambda$,

(iii) spCl_C (spCl_C λ) = spCl_C λ,

(iv) If $\lambda \leq \mu$ then spCl_C $\lambda \leq$ spCl_C μ.

Proof.

The proof for (i) follows from spCl_C λ= inf{μ: $\mu \geq \lambda$, μ is fuzzy **C** - semi-pre closed}.

Let λ be fuzzy **C** - semi-pre closed. Since **C** satisfies the monotonic and involvtive conditions. Then by using Proposition 4.3, **C** λ is fuzzy **C** - semi open. By using Proposition 5.2, sInt_C(**C** λ) = **C** λ. By using Proposition 5.5, we see that **C** (spCl_C λ) = **C** λ. Taking complement on both sides, we get **C** (**C** (s$Cl_C\lambda$)) = **C** (**C** λ). Since the complement function **C** satisfies the involutive condition, spCl_C $\lambda = \lambda$.

Conversely, we assume that spCl_C $\lambda = \lambda$. Taking complement on both sides, we get **C** (spCl_C λ)) = **C** λ. By using Proposition 5.5, spInt_C **C** λ = **C** λ. By using Proposition 5.2, **C** λ is fuzzy **C** -semi open. Again by using Proposition 4.3, λ is fuzzy **C** - semi-pre closed. Thus (ii) proved.

By using Proposition 5.5, **C** (spCl_C λ) = spInt_C (**C** λ). This implies that **C** (spCl_C λ) is fuzzy **C** - semi-pre open. By using Proposition 4.3, spCl_C (λ) is fuzzy **C** -semi closed. By applying (ii), we have spCl_C (spCl_C λ) = spCl_C λ. This proves (iii).

Suppose $\lambda \leq \mu$. Since **C** satisfies the monotonic condition **C** $\lambda \geq$ **C** μ. This implies that spInt_C **C** $\lambda \geq$ spInt_C **C** μ.Taking complement on both sides, we get C(spInt_C Cλ)$\leq$ **C** (spInt_C **C** μ). Then by using Proposition 5.5, spCl_C $\lambda \leq$ spCl_C μ. This proves (iv).

Proposition 5.7

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involutive conditions. Then for any two fuzzy subsets λ and μ of a fuzzy topological space, we have (i) spCl_C ($\lambda \vee \mu$) = spCl_C $\lambda \vee$ spCl_C μ and (ii) spCl_C ($\lambda \wedge \mu$) $\leq$ spCl_C λ $\wedge$ spCl_C μ.

Proof.

Since **C** satisfies the involutive condition, spCl_C ($\lambda \vee \mu$) = spCl_C (**C** (**C** ($\lambda \vee \mu$))). Since **C** satisfies the monotonic and involutive conditions, by using Proposition 5.5, spCl_C ($\lambda \vee \mu$) = **C** (spInt_C (**C** ($\lambda \vee \mu$))). By using Lemma 2.2, we have spCl_C ($\lambda \vee \mu$) = **C** (spInt_C (**C** $\lambda \wedge$ **C** μ)). Again by using Lemma 2.2, Cl_C ($\lambda \vee \mu$) $\leq$ **C** ((spInt_C **C** λ) $\wedge$(spInt_C **C** μ)) = **C** (spInt_C **C** λ) $\vee$ **C** (spInt_C **C** μ). By using Proposition 5.5, spCl_C ($\lambda \vee \mu$) $\leq$ spCl_C $\lambda \vee$spCl_C μ. Also spCl_C (λ) $\leq$ spCl_C ($\lambda \vee \mu$) and spCl_C (μ) $\leq$ spCl_C ($\lambda \vee \mu$) that implies spCl_C (λ)$\vee$spCl_C (μ) $\leq$ spCl_C ($\lambda \vee \mu$). Then it follows that spCl_C ($\lambda \vee \mu$) =

spCl_C $\lambda \vee$ spCl_C μ. Since spCl_C $(\lambda \wedge \mu) \leq$ spCl_C λ and spCl_C $(\lambda \wedge \mu) \leq$ sCl_C μ, it follows that spCl_C $(\lambda \wedge \mu) \leq$ spCl_C $\lambda \wedge$ spCl_C μ.

Proposition 5.8

Let **C** be a complement function that satisfies the monotonic and involutive conditions. Then for any family $\{\lambda_\alpha\}$ of fuzzy subsets of a fuzzy topological space, we have (i) $\vee$(spCl_C λ_α) $\leq$ spCl_C $(\vee\lambda_\alpha)$ and (ii) spCl_C $(\wedge\lambda_\alpha) \leq \wedge$(sp$Cl_C$ λ_α)

Proof.

For every β, $\lambda_\beta \leq \vee\lambda_\alpha \leq$ spCl_C $(\vee\lambda_\alpha)$. By using Proposition 5.6, spCl_C $\lambda_\beta \leq$ spCl_C $(\vee\lambda_\alpha)$ for every β. This implies that $\vee$spCl_C $\lambda_\beta \leq$ spCl_C $(\vee\lambda_\alpha)$. This proves (i). Now $\wedge\lambda_\alpha \leq \lambda_\beta$ for every β. Again using Proposition 5.6, we get spCl_C $(\wedge\lambda_\alpha) \leq$ spCl_C λ_β. This implies that spCl_C $(\wedge\lambda_\alpha) \leq \wedge$sp$Cl_C$ λ_α. This proves (ii).

6. Fuzzy C- semi-pre continuous functions

This section is devoted to the concept of fuzzy **C** -semi -pre continuous functions.

Definition 6.1

f: $(X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy **C** - semi-pre continuous function if $f^{-1}(\mu)$ is a fuzzy **C** – semi-pre open set in X for each fuzzy open subset μ in Y.

Proposition 6.2

Let X_1, X_2, Y_1 and Y_2 be fuzzy topological spaces such that X_1 is **C** -product related to X_2 and $f_1 : X_1 \rightarrow Y_1$ and $f_2 : X_2 \rightarrow Y_2$ be functions. If f_1 and f_2 are fuzzy **C** - semi-pre continuous then $f_1 \times f_2$ is also a fuzzy **C** -semi-pre continuous function.

Proof.

Let λ_α and μ_β are fuzzy open subsets in Y_1 and Y_2 respectively and let $\lambda = \vee(\lambda_\alpha \times \mu_\beta)$ be a fuzzy open set in $Y_1 \times Y_2$. Then by using Lemma 2.21, we have $(f_1 \times f_2)^{-1}(\lambda) = \vee(f_1 \times f_2)^{-1}(\lambda_\alpha \times \mu_\beta) = \vee[f_1^{-1}(\lambda_\alpha) \times f_2^{-1}(\mu_\beta)]$. Since f_1 and f_2 are fuzzy **C** -semi-pre continuous, by using Definition 6.1, $f_1^{-1}(\lambda_\alpha)$ and $f_2^{-1}(\mu_\beta)$ are fuzzy **C** -sem-pre open sets. Also by using Theorem 3.9, $f_1^{-1}(\lambda_\alpha) \times f_2^{-1}(\mu_\beta)$ is a fuzzy **C** -semi-pre open set and since arbitrary union of fuzzy **C** -semi-pre open sets is fuzzy **C** -semi-pre open, $\vee(f_1^{-1}(\lambda_\alpha) \times f_2^{-1}(\mu_\beta))$ is fuzzy **C** -semi-pre open. This shows that $(f_1 \times f_2)^{-1}$

(λ) is fuzzy **C** -semi -pre open. By using Definition 6.1, ($f_1 \times f_2$) is fuzzy **C** -semi-pre continuous function.

Proposition 6.3

Let f: (X, τ) $\rightarrow$ (Y, σ) be a function and g : X $\rightarrow$ X $\times$ Y be the graph of f. If g is fuzzy **C** -semi -pre continuous then f is also fuzzy **C** -semi -pre continuous.

Proof.

Let μ be a fuzzy open set in Y. Since $f^{-1}(\mu) = 1 \wedge f^{-1}(\mu) = g^{-1}(1\times\mu)$, g is a fuzzy **C** -semi-pre continuous and $1 \times \mu$ is a fuzzy open set in X×Y, $f^{-1}(\mu)$ is a fuzzy **C** -semi -pre open set of X. By using Definition 6.1, f is a fuzzy **C** -semi -pre continuous function.

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