EQUALITY OF CENTERED DECAGONAL NUMBER WITH SPECIAL M-GONAL NUMBERS

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Abstract

Explicit formulas for the ranks of Centered Decagonal numbers which are simultaneously equal to Triangular number,Square number,Pentagonal number,Hexagonal number,Octagonal number and Decagonal number in turn are presented.

Keywords

Centered Decagonal number, Triangular number,Square number,Pentagonal number,Hexagonal number,Octagonal number,Decagonal number

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Introduction

In [1],the equality of Triangular numbers which are simultaneously equal to Pentagonal numbers and Hexagonal numbers are illustrated through examples.

In [2], explicit formulas for the ranks of Triangular numbers which are simultaneously equal to Pentagonal,Octagonal,Decagonal and Dodecagonal numbers in turn are presented.

In [3],explicit formula for the ranks of centered Hexagonal numbers which are simultaneously equal to Triangular number,Pentagonal number,Hexagonal number,Heptagonal number,Decagonal number,Dodecagonal number in turn are presented.

In [4],explicit formula for finding the ranks n of Hex-numbers which are simultaneously equal to Centered m-gonal numbers such as Centered Triangular, Centered Square, Centered Pentagonal, Centered Heptagonal, Centered Octagonal, Centered Nonagonal, Centered Decagonal numbers of rank m are presented.

In [5],a few interesting relations among the Centered Hexagonal numbers are obtained.Also the ranks of Centered Hexagonal numbers which are simultaneously equal to Nonagonal numbers are presented.

Method of Analysis

Denoting the ranks of the Centered Decagonal number and Triangular number to be C and T respectively, the identity

Centered Decagonal number = Triangular number (1)

is written as

$$y^2 = 10x^2 - 1 \quad (2)$$

where $x = 2C + 1 \quad , \quad y = 2T + 1$ (3)

whose initial solution is $x_0 = 1, y_0 = 3$ (4)

Let $(\tilde{x}_n, \tilde{y}_n)$ be the general solution of the Pellian

$$y^2 = 10x^2 + 1$$

where $\tilde{x}_n = \frac{1}{2\sqrt{10}}\left\{\left(19+6\sqrt{10}\right)^{n+1} - \left(19-6\sqrt{10}\right)^{n+1}\right\}$

$$\tilde{y}_n = \frac{1}{2}\left\{\left(19+6\sqrt{10}\right)^{n+1} + \left(19-6\sqrt{10}\right)^{n+1}\right\} \quad ,n = 0,1,2, \dots$$

Applying Brahmagupta's Lemma between the solutions (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the sequences of values of x and y satisfying equation (2) is given by

$$x_{n+1} = \frac{1}{2\sqrt{10}}\left\{\left(19+6\sqrt{10}\right)^{n+1}\left(3+\sqrt{10}\right) - \left(19-6\sqrt{10}\right)^{n+1}\left(3-\sqrt{10}\right)\right\}$$

$$y_{n+1} = \frac{1}{2}\left\{\left(19+6\sqrt{10}\right)^{n+1}\left(3+\sqrt{10}\right) + \left(19-6\sqrt{10}\right)^{n+1}\left(3-\sqrt{10}\right)\right\}$$

In view of (3) the ranks of Centered Decagonal and Triangular numbers are respectively given by

$$C_{n+1} = \frac{1}{4\sqrt{10}}\left\{\left(19+6\sqrt{10}\right)^{n+1}\left(3+\sqrt{10}\right) - \left(19-6\sqrt{10}\right)^{n+1}\left(3-\sqrt{10}\right) - 2\sqrt{10}\right\}$$

$$T_{n+1} = \frac{1}{4}\left\{\left(19+6\sqrt{10}\right)^{n+1}\left(3+\sqrt{10}\right) + \left(19-6\sqrt{10}\right)^{n+1}\left(3-\sqrt{10}\right) - 2\right\}$$

and their corresponding recurrence relations are found to be

$$C_{n+3} = 38C_{n+2} - C_{n+1} + 18$$

$$T_{n+3} = 38T_{n+2} - T_{n+1} + 18$$

In a similar manner we present below the ranks of Centered Decagonal numbers which are simultaneously equal to Square number, Pentagonal number, Hexagonal number, Octagonal number, Decagonal number in tabular form:

S.No.	m-gonal number	General Forms of Ranks
1.	Centered Decagonal number (C) Square number(S)	$C_{n+1} = \frac{1}{4\sqrt{5}}\left\{\left(9+4\sqrt{5}\right)^{n+1}\left(2+\sqrt{5}\right) - \left(9-4\sqrt{5}\right)^{n+1}\left(2-\sqrt{5}\right) - 2\sqrt{5}\right\}$ $S_{n+1} = \frac{1}{4}\left\{\left(9+4\sqrt{5}\right)^{n+1}\left(2+\sqrt{5}\right) + \left(9-4\sqrt{5}\right)^{n+1}\left(2-\sqrt{5}\right)\right\}$ $n = 0,1,2, \dots$
2.	Centered Decagonal number (C) Pentagonal number (P)	$C_{n+1} = \frac{1}{4\sqrt{30}}\left\{\left(11+2\sqrt{30}\right)^{n+1}\left(5+\sqrt{30}\right) - \left(11-2\sqrt{30}\right)^{n+1}\left(5-\sqrt{30}\right) - 2\sqrt{30}\right\}$ $P_{n+1} = \frac{1}{12}\left\{\left(11+2\sqrt{30}\right)^{n+1}\left(5+\sqrt{30}\right) + \left(11-2\sqrt{30}\right)^{n+1}\left(5-\sqrt{30}\right) + 2\right\}$ $n = 0,1,2, \dots$
3.	Centered Decagonal number (C) Hexagonal number (H)	$C_{n+1} = \frac{1}{4\sqrt{10}}\left\{\left(19+6\sqrt{10}\right)^{n+1}\left(3+\sqrt{10}\right) - \left(19-6\sqrt{10}\right)^{n+1}\left(3-\sqrt{10}\right) - 2\sqrt{10}\right\}$ $H_{n+1} = \frac{1}{8}\left\{\left(19+6\sqrt{10}\right)^{n+1}\left(3+\sqrt{10}\right) + \left(19-6\sqrt{10}\right)^{n+1}\left(3-\sqrt{10}\right) + 2\right\}$ $n = 0,1,2, \dots$

4.	Centered Decagonal number (C) Octagonal number (M)	$C_n = \frac{1}{4\sqrt{15}}\left\{\left(4+\sqrt{15}\right)^{n+1} - \left(4-\sqrt{15}\right)^{n+1} - 2\sqrt{15}\right\}$ $M_n = \frac{1}{12}\left\{\left(4+\sqrt{15}\right)^{n+1} + \left(4-\sqrt{15}\right)^{n+1} + 4\right\}$ $n = 1,2,\ldots$
5.	Centered Decagonal number (C) Decagonal number (Q)	$C_{n+1} = \frac{1}{8\sqrt{5}}\left\{\left(9+4\sqrt{5}\right)^{n+1}\left(5+2\sqrt{5}\right) - \left(9-4\sqrt{5}\right)^{n+1}\left(5-2\sqrt{5}\right) - 4\sqrt{5}\right\}$ $Q_{n+1} = \frac{1}{16}\left\{\left(9+4\sqrt{5}\right)^{n+1}\left(5+2\sqrt{5}\right) + \left(9-4\sqrt{5}\right)^{n+1}\left(5-2\sqrt{5}\right) + 6\right\}$ $n = 0,1,2,\ldots$

The recurrence relations satisfied by the ranks of each of the m-gonal numbers presented in the table above are as follows:

S.No.	Recurrence Relations
1.	$C_{n+3} = 18C_{n+2} - C_{n+1} + 8\ , C_1 = 8\ , C_2 = 152$ $S_{n+3} = 18S_{n+2} - S_{n+1}\ , S_1 = 19\ , S_2 = 341$
2.	$C_{2n+4} = 482C_{2n+2} - C_{2n} + 240\ , C_2 = 230\ , C_4 = 111100$ $P_{2n+4} = 482P_{2n+2} - P_{2n} - 80\ , P_2 = 421\ , P_4 = 202841$
3.	$C_{2n+4} = 1442C_{2n+2} - C_{2n} + 720\ , C_2 = 702\ , C_4 = 1013004$ $H_{2n+4} = 1442H_{2n+2} - H_{2n} - 360\ , H_2 = 1111\ , H_4 = 1601701$
4.	$C_{2n+4} = 62C_{2n+2} - C_{2n} + 30\ , C_2 = 31\ , C_4 = 1952$ $M_{2n+4} = 62M_{2n+2} - M_{2n} - 20\ , M_2 = 41\ , M_4 = 2521$
5.	$C_{n+3} = 18C_{n+2} - C_{n+1} + 8\ , C_1 = 9\ , C_2 = 170$ $Q_{n+3} = 18Q_{n+2} - Q_{n+1} - 6\ , Q_1 = 11\ , Q_2 = 191$

Conclusion

To conclude, one may search for the other m-gonal numbers satisfying the relation under consideration.

References

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