# ON PAIRS OF M- GONAL NUMBERS WITH UNIT DIFFERENCE

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## Abstract

We obtain the ranks of m-gonal numbers such that the difference between any two m-gonal numbers is unity. The recurrence relations satisfied by the ranks of each m-gonal numbers are also presented.

## Introduction

Number is the essence of mathematical calculation. Variety of numbers has variety of range and richness. Many numbers exhibit fascinating properties, they form sequences, they form patterns and so on [1,2,3]. In [4] explicit formulas for the ranks of Triangular numbers which simultaneously equal to Pentagonal, Octagonal, Decagonal and Dodecagonal numbers in turn are presented. Denoting the ranks of Triangular, Pentagonal, Hexagonal, Octagonal, Heptagonal, Decagonal and Dodecagonal number by the symbols N, P, Q, M, H, D and T respectively. In [5], the following relations are studied:

1. $N - P = 1$ 2. $N - M = 1$ 3. $N - H = 1$ 4. $N - D = 1$
5. $N - T = 1$ 6. $P - M = 1$ 7. $P - Q = 1$ 8. $P - H = 1$

In [6], the ranks of m-gonal numbers such that the difference between any two m-gonal numbers is unity. The recurrence relations satisfied by the ranks of each m-gonal number in turn are

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presented. In this communication, we make an attempt to obtain the ranks of other pairs of m-gonal numbers such that the difference between any two m-gonal numbers is unity. The recurrence relations satisfied by the ranks of m-gonal number are also presented.

## Method of Analysis

1. Tridecagonal number – Triangular number = 1

Denoting the ranks of the Tridecagonal number and Triangular number to be A and N respectively, the identify

Tridecagonal number – Triangular number = 1 (1)

is written as

$$y^2 = 11x^2 + 158 \tag{2}$$

where $x = 2N+1,\ y = 22A-9$ (3)

whose intial solutions is $x_0 = 1,\ y_0 = 13$

Let $(x_n, y_n)$ be the general solution of the Pellian $y^2 = 11x^2 + 1$

$$\tilde{x}_s = \frac{1}{2\sqrt{11}}\left[\left(10+3\sqrt{11}\right)^{s+1} - \left(10-3\sqrt{11}\right)^{s+1}\right]$$

$$\tilde{y}_s = \frac{1}{2}\left[\left(10+3\sqrt{11}\right)^{s+1} + \left(10-3\sqrt{11}\right)^{s+1}\right],\ s = 0,1,2,\ldots$$

Applying Brahmagupta's lemma between the solutions $(x_0, y_0)$ and $(x_n, y_n)$ the sequence of values of x and y satisfying equation (2) is given by

$$x_s = \frac{1}{2\sqrt{11}}\left[\left(10+3\sqrt{11}\right)^{s+1}\left(13+\sqrt{11}\right) - \left(10-3\sqrt{11}\right)^{s+1}\left(13-\sqrt{11}\right)\right]$$

$$y_s = \frac{1}{2}\left[\left(10+3\sqrt{11}\right)^{s+1}\left(13+\sqrt{11}\right) + \left(10-3\sqrt{11}\right)^{s+1}\left(13-\sqrt{11}\right)\right],\ s = 0,1,2,\ldots$$

In view of (3), the ranks of Tridecagonal and Triangular numbers are respectively given by

$$A_{2s-1} = \frac{1}{44}\left[\left(10+3\sqrt{11}\right)^{2s}\left(13+\sqrt{11}\right) + \left(10-3\sqrt{11}\right)^{2s}\left(13-\sqrt{11}\right) + 18\right]$$

$$N_{2s-1}=\frac{1}{4\sqrt{11}}\left[\left(10+3\sqrt{11}\right)^{2s}\left(13+\sqrt{11}\right)-\left(10-3\sqrt{11}\right)^{2s}\left(13-\sqrt{11}\right)-2\sqrt{11}\right],$$

$$s=1,2,\ldots$$

and their corresponding recurrence relations are found to be

$$A_{2s+3}-398A_{2s+1}+A_{2s-1}=-162,\ A_1=148, A_3=58741$$

$$N_{2s+3}-398N_{2s+1}+N_{2s-1}=-162,\ N_1=489, N_3=194820$$

In similar manner, we present below the ranks of other pairs of m-gonal numbers in a tabular form

| S.NO | M-Gonal number | General form of Ranks |
|---|---|---|
| 1 | Heptadecagonal number(E)<br>Triangular number(N) | $E_{2s-1}=\frac{1}{60}\left[\left(4+\sqrt{15}\right)^{2s}\left(17+\sqrt{15}\right)+\left(4-\sqrt{15}\right)^{2s}\left(17-\sqrt{15}\right)+26\right]$<br>$N_{2s-1}=\frac{1}{4\sqrt{15}}\left[\left(4+\sqrt{15}\right)^{2s}\left(17+\sqrt{15}\right)-\left(4-\sqrt{15}\right)^{2s}\left(17-\sqrt{15}\right)-2\sqrt{15}\right]$<br>$s=1,2,\ldots$ |
| 2 | Nanodecagonal number(F)<br>Triangular number(N) | $F_{2s-1}=\frac{1}{68}\left[\left(33+8\sqrt{17}\right)^{2s}\left(19+\sqrt{17}\right)+\left(33-8\sqrt{17}\right)^{2s}\left(19-\sqrt{17}\right)+30\right]$<br>$N_{2s-1}=\frac{1}{4\sqrt{17}}\left[\left(33+8\sqrt{17}\right)^{2s}\left(19+\sqrt{17}\right)-\left(33-8\sqrt{17}\right)^{2s}\left(19-\sqrt{17}\right)-2\sqrt{17}\right]$<br>$s=1,2,\ldots$ |
| 3 | Nanogonal number(M)<br>Triangular number(N) | $M_s=\frac{1}{28}\left[\left(8+3\sqrt{7}\right)^{s+1}\left(9+\sqrt{7}\right)+\left(8-3\sqrt{7}\right)^{s+1}\left(9-\sqrt{7}\right)+10\right]$<br>$N_s=\frac{1}{4\sqrt{7}}\left[\left(8+3\sqrt{7}\right)^{s+1}\left(9+\sqrt{7}\right)-\left(8-3\sqrt{7}\right)^{s+1}\left(9-\sqrt{7}\right)-2\sqrt{7}\right]$<br>$s=0,1,2,\ldots$ |
| 4 | Nanogonal number(M)<br>Hexagonal number(H) | $M_s=\frac{1}{28}\left[\left(8+3\sqrt{7}\right)^{s+1}\left(9+\sqrt{7}\right)+\left(8-3\sqrt{7}\right)^{s+1}\left(9-\sqrt{7}\right)+10\right]$<br>$H_s=\frac{1}{8\sqrt{7}}\left[\left(8+3\sqrt{7}\right)^{s+1}\left(9+\sqrt{7}\right)-\left(8-3\sqrt{7}\right)^{s+1}\left(9-\sqrt{7}\right)+2\sqrt{7}\right]$<br>$s=0,1,2,\ldots$ |

The recurrence relations satisfied by the ranks of each of the m-gonal numbers presented in the table above are as follows

| S.No | Recurrence Relations |
|---|---|
| 1 | $E_{2s+3}-62E_{2s+1}+E_{2s-1}=-26$, $E_1=22, E_3=1337$<br>$N_{2s+3}-62N_{2s+1}+N_{2s-1}=-26$, $N_1=83, N_3=5176$ |
| 2 | $F_{2s+3}-4354F_{2s+1}+F_{2s-1}=-1920$, $F_1=1481, F_3=6446353$<br>$N_{2s+3}-4354N_{2s+1}+N_{2s-1}=-1920$, $N_1=6104, N_3=26578992$ |
| 3 | $M_{s+2}-16M_{s+1}+M_s=-140$, $M_0=7, M_1=106$<br>$N_{s+2}-16N_{s+1}+N_s=-140$, $N_0=17, N_1=279$ |
| 4 | $M_{s+2}-16M_{s+1}+M_s=-140$, $M_0=7, M_1=106$<br>$H_{s+2}-16H_{s+1}+H_s=-140$, $H_0=9, H_1=140$ |

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