EFFECT OF SUCTION / INJECTION ON AN OSCILLATORY MHD FLOW IN A ROTATING HORIZONTAL POROUS CHANNEL

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ABSTRACT:

An analysis is made to study the effects of injection / suction on an oscillatory flow of an incompressible electrically conducting viscous fluid in a porous channel filled with a porous material. The porous channel with constant injection / suction rotates about an axis perpendicular to the plates. A uniform magnetic field is applied normally to the plates. The upper plate is allowed to oscillate in its own plane whereas the lower plate is kept at rest. Effects of typical results are illustrated to reveal the tendency of the solutions. Representative results are presented for resultant velocities, phase angles and amplitude and phase difference of shear stress.

KEY WORDS: MHD Flow, rotating channel, porous medium, suction / injection.

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1. INTRODUCTION:

Unsteady flow problems have wide applications in various technological fields like aeronautics, astrophysics, space science and chemical engineering. Unsteady flows through porous media have been extensively studied because of their various applications in science and engineering. An extensive overview on this topic has been documented in the books of Nield and Bejan [1] and Pop and Ingham [2]. Some remarkable investigations on flows of viscous fluid through porous media are reported by Postelnicu et al. [3] and Mahapatra et al. [4]. Hydromagnetic flows through porous media have gained considerable importance because of wide ranging applications. Various aspects of this type of problem under different physical situations have been reported by Dash [5], Israel-Cookey [6], Dash et al. [7], Hassanien and Obied Allah [8] and Panda et al. [9]. Moreover, several authors [10-15] studied the fluid flows in rotating systems.

The objective of the present work is to study the hydromagnetic oscillatory flow in a horizontal porous channel filled with a porous material in a rotating system.

2. MATHEMATICAL ANALYSIS:

An oscillatory flow of a viscous incompressible and electrically conducting fluid between two insulating infinite parallel porous plates at a distance d apart, is considered. A constant injection velocity, w_0, is applied at the lower stationary plate and the same constant suction velocity, w_0, is applied at the upper plate which is oscillating in its own plane with a velocity $U^*(t^*)$ about a non zero constant mean velocity U_0. Choose the origin on the lower plate lying in x^*-y^* plane and x^*- axis parallel to the direction of motion of the upper plate. The z^*-axis taken perpendicular to the planes of the plates, is the axis of rotation about which the entire system is rotating with a constant angular velocity Ω^*. A transverse uniform magnetic field of strength B_0 is applied along the axis of rotation. The magnetic Reynolds number of the flow is taken to be small enough, so that the induced distortion of the applied magnetic field can be neglected. It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. Since the plates are infinite in extent, all the physical quantities except the pressure, depend only on z^* and t^*. Assuming the velocity components u^*, v^* and w^* in the x^*, y^* and z^* directions respectively, the equations governing the rotating system are

$$\frac{\partial w^*}{\partial z^*} = 0, \qquad (1)$$

$$\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} = -\frac{P_x^*}{\rho} + \nu \frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* - \frac{\sigma B_0^2}{\rho} u^* - \frac{\nu}{k^*} u^*, \qquad (2)$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{P_y^*}{\rho} + \nu \frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* - \frac{\sigma B_0^2}{\rho} v^* - \frac{\nu}{k^*} v^* \qquad (3)$$

where ν is the kinematic viscosity, t is the time, ρ is the density and P^* is the modified pressure, σ is the electrical conductivity of the fluid. The boundary conditions of the problem are

$$\left.\begin{array}{l} u^* = v^* = 0 \qquad w^* = w_0 \qquad at\ z^* = 0, \\ u^* = U^*(t^*) = U_0(1 + \in \cos \omega^* t^*) \end{array}\right\} \qquad (4)$$

$$v^* = 0, \quad w^* = w_0, \quad \text{at} \quad z^* = d,$$

where ω^* is the frequency of oscillations and $\in$ is a very small positive constant.

From equation (1), it is clear that w^* is a constant, so, we assume that $w^* = w_0$. Substituting $w^* = w_0$ and eliminating the modified pressure gradient, under the usual boundary layer approximations i.e. from equation (2), we get

$$\frac{\partial U^*}{\partial t^*} + \frac{\sigma B_0^2}{\rho} U^* + \frac{\nu}{k^*} U^* = -\frac{P_x^*}{\rho}$$

Also from equation (3), we get

$$2\,\Omega^* U^* = -\frac{P_y^*}{\rho}$$

Substituting the above pressure gradients in the equations (2) and (3), we get

$$\frac{\partial u^*}{\partial t^*} + w_0 \frac{\partial u^*}{\partial z^*} = \frac{\partial U^*}{\partial t^*} + \nu \frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* - \frac{\sigma B_0^2}{\rho}(u^* - U^*) - \frac{\nu}{k^*}(u^* - U^*) \quad (5)$$

$$\frac{\partial v^*}{\partial t^*} + w_0 \frac{\partial v^*}{\partial z^*} = \nu \frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^*(u^* - U^*) - \frac{\sigma B_0^2}{\rho} v^* - \frac{\nu}{k^*} v^* \quad (6)$$

In order to transform the equations (5) and (6) into the non-dimensional form, the following non-dimensional parameters are introduced.

$$\eta = \frac{z^*}{d},\ t = \omega^* t^*,\ u = \frac{u^*}{U_0},\ v = \frac{v^*}{U_0},\ \Omega = \Omega^* \frac{d^2}{\nu} \text{ (the rotation parameter)}$$

$\omega = \frac{\omega^* d^2}{\nu}$ (the frequency parameter), $S = \frac{w_0 d}{\nu}$ (the injection / suction parameter), $M = B_0\, d \sqrt{\frac{\sigma}{\mu}}$ (the Hartmann number) and $K = \frac{k^*}{d^2}$ (the permeability parameter). Consequently the equations (5) & (6) are

$$\omega \frac{\partial u}{\partial t} + S \frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} + \omega \frac{\partial U}{\partial t} + 2\Omega v - \left(M^2 + \frac{1}{K}\right)(u - U) \quad (7)$$

$$\omega \frac{\partial v}{\partial t} + S \frac{\partial v}{\partial \eta} = \frac{\partial^2 v}{\partial \eta^2} - 2\Omega\,(u - U) - \left(M^2 + \frac{1}{K}\right) v \quad (8)$$

The corresponding transformed boundary conditions are

$$\left.\begin{aligned} u = v = 0 \quad &\text{at} \quad \eta = 0 \\ u = U(t) = 1 + \in \cos t,\ v = 0 \quad &\text{at} \quad \eta = 1 \end{aligned}\right\} \quad (9)$$

Equations (7) and (8) can now be combined into a single equation, by introducing a complex function $q = u + iv$, as

$$\omega\frac{\partial q}{\partial t} + S\frac{\partial q}{\partial \eta} = \frac{\partial^2 q}{\partial \eta^2} + \omega\frac{\partial U}{\partial t} - 2i\Omega(q-U) - (M^2 + \frac{1}{K})(q-U) \quad (10)$$

and the boundary conditions (9) can also be written in complex notations as $q = 0$ at $\eta = 0$

$$q = U(t) = 1 + \frac{\in}{2}\left(e^{it} + e^{-it}\right) \text{at} \quad \eta = 1 \quad (11)$$

In order to solve equation (10) subject to the boundary conditions (11), we look for a solution of the form

$$q(\eta, t) = q_0(\eta) + \frac{\in}{2}\left\{q_1(\eta)e^{it} + q_2(\eta)e^{-it}\right\}. \quad (12)$$

Substituting equation (12) into equations (10) and (11), and comparing the harmonic and non-harmonic terms, we get

$$q_0'' - Sq_0' - \left(l^2 + M^2 + \frac{1}{K}\right)q_0 = -\left(l^2 + M^2 + \frac{1}{K}\right), \quad (13)$$

$$q_1'' - Sq_1' - \left(m^2 + M^2 + \frac{1}{K}\right)q_1 = -\left(m^2 + M^2 + \frac{1}{K}\right), \quad (14)$$

$$q_2'' - Sq_2' - \left(n^2 + M^2 + \frac{1}{K}\right)q_2 = -\left(n^2 + M^2 + \frac{1}{K}\right). \quad (15)$$

where $l^2 = 2i\Omega$, $m^2 = i(2\Omega + \omega)$ and $n^2 = i(2\Omega - \omega)$.

The corresponding transformed boundary conditions are

$$\left.\begin{aligned} q_0 = q_1 = q_2 = 0 \quad &\text{at} \quad \eta = 0, \\ q_0 = q_1 = q_2 = 1 \quad &\text{at} \quad \eta = 1 \end{aligned}\right\} \quad (16)$$

The solutions of equations (13) to (15) under the boundary conditions (16) are obtained as

$$q_0(\eta) = 1 - \left[\frac{e^{(r_1 + r_2\eta)} - e^{(r_2 + r_1\eta)}}{e^{r_1} - e^{r_2}}\right], \quad (17)$$

$$q_1(\eta) = 1 - \left[\frac{e^{(r_3 + r_4\eta)} - e^{(r_4 + r_3\eta)}}{e^{r_3} - e^{r_4}}\right], \quad (18)$$

$$q_2(\eta) = 1 - \left[\frac{e^{(r_5 + r_6 \eta)} - e^{(r_6 + r_5 \eta)}}{e^{r_5} - e^{r_6}} \right]. \qquad (19)$$

where

$$r_1 = \frac{\left(S + \sqrt{S^2 + 4(l^2 + M^2 + \frac{1}{k})} \right)}{2}, \quad r_2 = \frac{\left(S + \sqrt{S^2 + 4(l^2 + M^2 + \frac{1}{k}} \right)}{2},$$

$$r_3 = \frac{\left(S + \sqrt{S^2 + 4(m^2 + M^2 + \frac{1}{k}} \right)}{2}, \quad r_4 = \frac{\left(S - \sqrt{S^2 + 4(m^2 + M^2 + \frac{1}{k}} \right)}{2},$$

$$r_5 = \frac{\left(S + \sqrt{S^2 + 4(n^2 + M^2 + \frac{1}{k}} \right)}{2}, \quad r_6 = \frac{\left(S - \sqrt{S^2 + 4(n^2 + M^2 + \frac{1}{k}} \right)}{2}$$

3. RESULTS AND DISCUSSION:

Now for the resultant velocities and the shear stresses of the steady and unsteady flow, we write

$$u_0(\eta) + i v_0(\eta) = q_0(\eta) \qquad (20)$$

and $$u_1(\eta) + i v_1(\eta) = q_1(\eta)\, e^{it} + q_2(\eta)\, e^{-it} \qquad (21)$$

The solution (17) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity. For steady flow

$$R_0 = \sqrt{u_0^2 + v_0^2} \quad , \quad \theta_0 = \tan^{-1}(v_0 / u_0). \qquad (22)$$

The resultant velocity or amplitude and the phase difference of the unsteady flow are given by

$$R_1 = \sqrt{u_1^2 + v_1^2}\,, \quad \theta_1 = \tan^{-1}(v_1 / u_1) \qquad (23)$$

In the present study, the effects of suction and injection on an oscillatory flow of a viscous electrically conducting fluid in a rotating horizontal porous channel filled with a porous material has been highlighted.

The most interesting aspects are due to the existence of homogeneous porous matrix coupled with the interaction of transverse magnetic field as well as coriolis force contributed by the rotating horizontal porous channel.

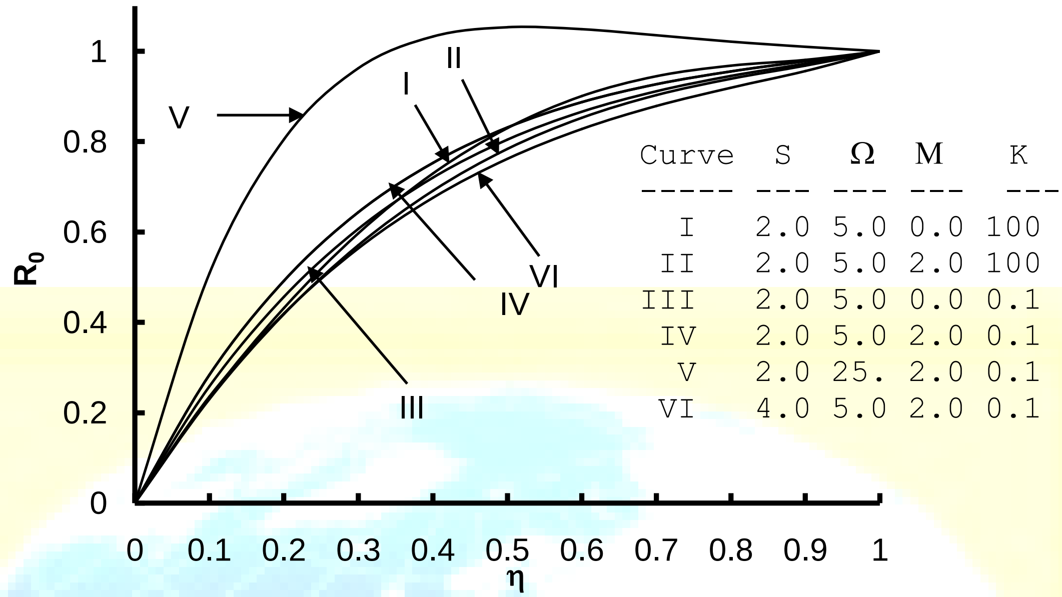

Fig. 1 Resultant velocity R_0 due to u_0 and v_0.

Fig. 1 exhibits the resultant velocity R_0 bringing out the effects of the permeability parameter K, the suction / injection parameter S, the rotation parameter Ω and the Hartmann number M. It is observed that for high value of Ω ($\Omega = 25.0$), the velocity increases significantly at all points of the flow field (curve V). In all other cases, the thining of boundary layer thickness is marked. Either in the absence or presence of porous media, an increase in Hartmann number decreases the resultant velocity R_0. Thus, it may be concluded that magnetic force has no discriminatory role on flow through porous media. Now, in the presence of porous media, an increase in rotation parameter increases R_0 but the reverse effect is observed in case of suction / injection. Comparing the cases of with and without magnetic field (curves III and IV), we conclude that the magnetic force enhances the velocity at all points of the flow field. Moreover, one interesting point is due to the presence or absence of porous media affecting the flow field at $\eta = 0.4$ (approximately) where both the curves intersect with each other (curve I and III). The flow profile due to the presence of porous media assumes higher value for $0 < \eta < 0.4$.

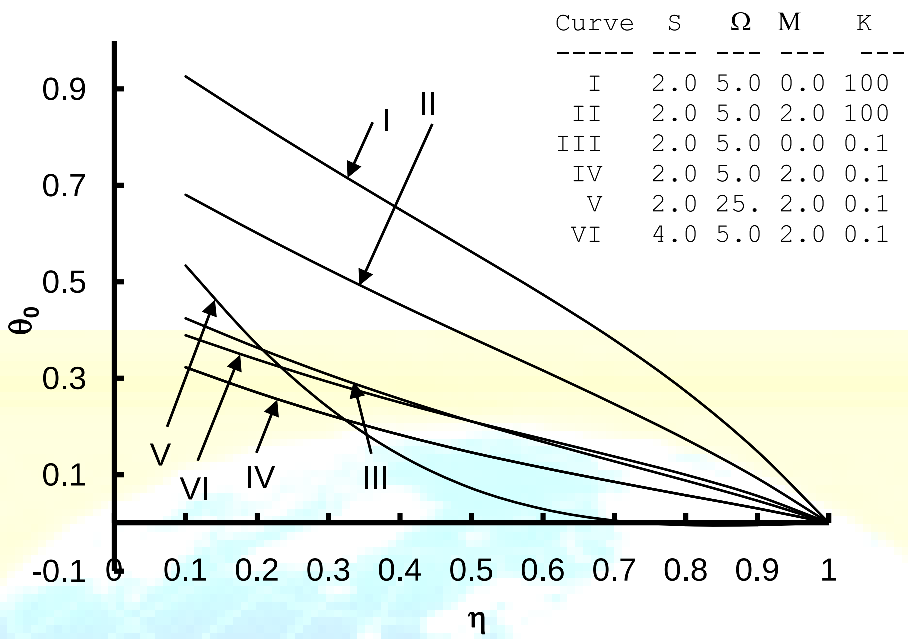

Fig. 2 Phase angle θ_0 due to u_0 and v_0.

Fig. 2 shows the effects of pertinent parameters on the phase angle θ_0. It is seen that the phase angle decreases as the Hartmann number increases i.e. both in presence or absence of porous media. This corroborates the effect on the resultant velocity. It is to note that the phase angle exhibited by the curve I assumes higher value at all points of the channel. The effect due to the absence of porous media, phase angle θ_0 decreases gradually from the stationary plate to the oscillating plate in all the cases. A point of intersection occurs at η=0.5 between the curves III and VI exhibiting the dominating effects of suction / injection parameter and Hartmann number with their higher values in curve VI than the lower values exhibited through the curve III. Another point of intersection occurs nearly at η=0.3, where the curves IV and V intersect. The sudden increase in the phase angle is marked near the stationary plate due to higher value of rotation parameter, then decreases when η>0.3. Further, it is noted that there is always phase lead at all points of the channel.

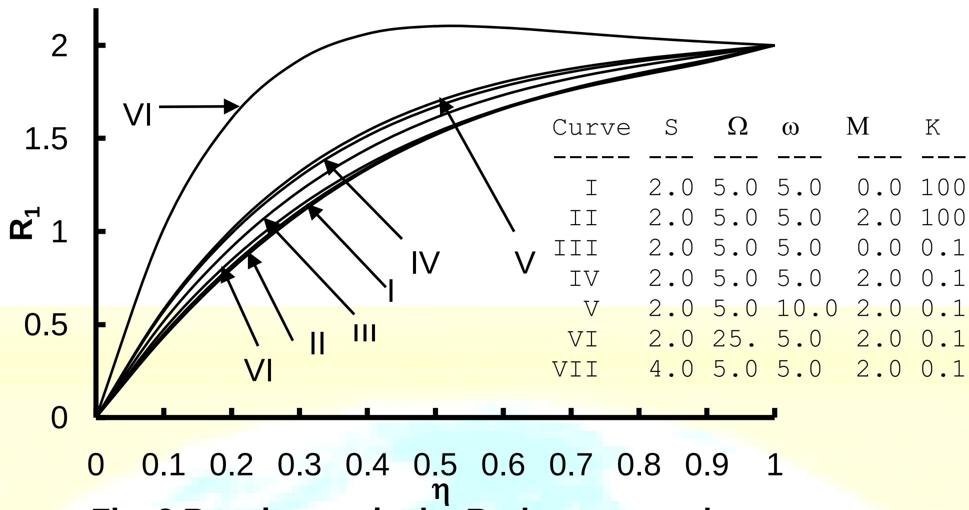

Fig. 3 Resultant velocity R_1 due to u_1 and v_1.

Fig. 3 depicts the resultant velocity R_1 of the unsteady primary component u_1 and secondary component v_1 for the fluctuating flow. It is observed that in the absence of porous media, the Hartmann number M has no significant effect on the resultant velocity but in the presence of the porous media, an increase in M leads to an increase in the resultant velocity R_1. Further, it is interesting to note that an increase in rotation parameter and frequency of oscillation, increases R_1 but suction velocity affects adversely. One striking feature is that for high rotation (Ω = 25), the unsteady resultant velocity increases significantly.

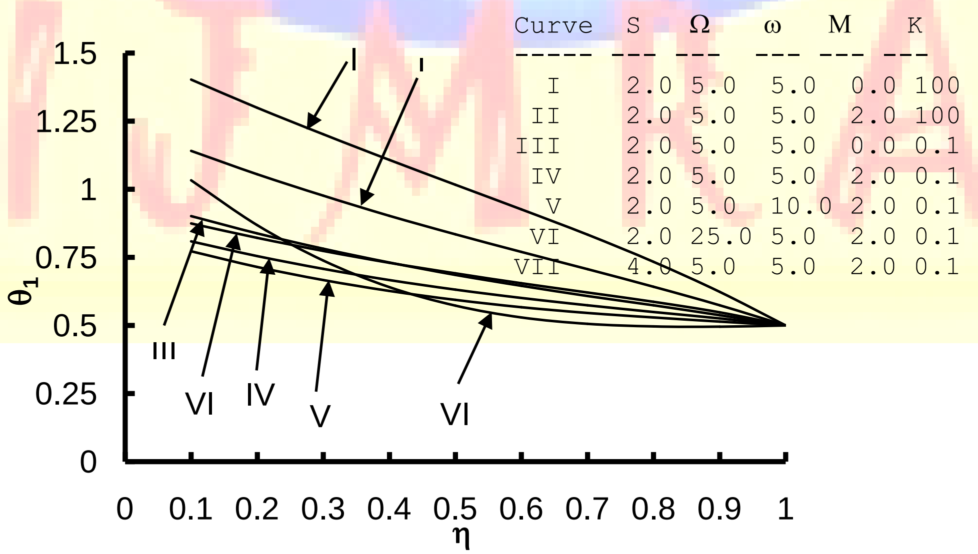

Fig. 4 Phase angle θ_1 due to u_1 and v_1.

Fig. 4 exhibits the effects on phase angle due to time varying fluctuation velocity. An increase in Hartmann number decreases the phase angle irrespective of the presence or absence of porous media. Thus it is concluded that Lorentz force is responsible to decrease the phase angle with or without porous matrix. Again the curves IV and VI also intersect for η>0.3 nearly due to high value of Ω = 25. For larger value of rotation parameter the phase angle initially assumes smaller value than that of lower rotation. The curve I shows that the phase angle becomes maximum in the absence of both porous media and magnetic field but in the presence of magnetic field, the phase angle decreases at any point of the flow field.

Table 1 : Values of τ_{0r} and θ_{0r} for different values of S, Ω, ω, M and K.

S	Ω	ω	M	K	τ_{0r}	θ_{0r}
2	5	5	0	100	2.508748	1.026874
2	5	5	2	100	2.529451	0.765710
2	5	5	0	0.1	2.939459	0.491191
2	5	5	2	0.1	3.299481	0.381462
2	5	10	2	0.1	3.299481	0.381462
2	5	5	2	0.1	3.299481	0.381462
4	5	5	2	0.1	2.648547	0.443713
4	10	5	2	0.1	3.457132	0.656790

Table 1 presents the numerical values of amplitude and the phase difference of shear stress at the stationary plate (η=0) for the steady part, which can be obtained as

$$\tau_{0r} = \sqrt{\tau_{0x}^2 + \tau_{0y}^2} \quad , \quad \theta_{0r} = \tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right) \tag{24}$$

where

$$\tau_{0x} + i\,\tau_{0y} = \left(\frac{\partial q}{\partial \eta}\right)_{\eta=0} = \left(\frac{r_1 e^{r_2} - r_2 e^{r_1}}{e^{r_1} - e^{r_2}}\right) \tag{25}$$

Here τ_{0x} and τ_{0y} are the shear stresses at the stationary plate due to the primary and secondary velocity component respectively. Presence of Lorentz force (M=2.0) leads to increase the amplitude τ_{0r} and decrease the phase angle θ_{0r}, but the presence of porous matrix affects adversely. No significant change in both τ_{0r} and θ_{0r} is observed due to an increase in frequency parameter (ω), but for high value of rotation both of them increase. Rotation causes higher shearing stress and larger phase angle.

Table 2 : Values of τ_{1r} and θ_{1r} for different values of S, Ω, ω, M and K.

S	Ω	ω	M	K	τ_{1r}	θ_{1r}
2	5	5	0	100	0.000000	1.207359
2	5	5	2	100	4.344791	0.904230
2	5	5	0	0.1	4.163930	0.568264
2	5	5	2	0.1	4.764009	0.433541
2	5	10	2	0.1	5.379962	0.463543
2	5	5	2	0.1	5.028723	0.433541
4	5	5	2	0.1	5.379962	0.509548
4	10	5	2	0.1	4.274333	0.719925
4	5	20	2	100	4.1700	-1.16751

Table 2 presents the amplitude and the phase difference in case of unsteady flow.

For the unsteady part of the flow, the amplitude and the phase difference of shear stresses at the stationary plate (η=0) can be obtained as

$$\tau_{1x} + i\tau_{1y} = \left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} + i\left(\frac{\partial v_1}{\partial \eta}\right)_{\eta=0} \qquad (26)$$

which gives

$$\tau_{1r} = \sqrt{\tau_{1x}^2 + \tau_{1y}^2} \quad , \quad \theta_{1r} = \left(\tau_{1y} / \tau_{1x}\right) \qquad (27)$$

In the presence or absence of porous media, Hartmann number increases τ_{1r} but decreases θ_{1r} significantly. Hence, Lorentz force is responsible for higher amplitude and lower phase angle in the presence or absence of porous media but the reverse effect is observed by increasing the rotation parameter (*Ω*).

In case of higher frequency of oscillation, both τ_{1r} and θ_{1r} assume higher values. One striking feature of all the entries in case of both steady and unsteady case is that no phase lag is observed but the absence of porous matrix with a higher frequency a phase lag is marked with a negative entry.

4. CONCLUSION:

The above results lead to the following conclusions of physical interest.

(i) In the lower part of the channel, i.e. near the stationary plate, porosity of the medium enhances the resultant velocity R_0 of the flow field but the reverse effect is observed in the upper half i.e. (near the oscillating plate).

(ii) The suction / injection parameter and Hartmann number enhance the phase angle in the layer near to the stationary plate but the reverse effect is observed near the oscillating plate.

(iii) Increasing values of rotation parameter and frequency of oscillation enhance the resultant velocity R_1 where as the suction / injection parameter is to decrease it.

(iv) The Lorentz force is responsible to decrease the phase angle with or without the porous matrix. This force also increases the amplitude τ_{0r} and decreases the phase angle θ_{0r}, but the presence of porous media affects adversely.

(v) Rotation parameter possesses higher shearing stress and larger phase angle.

5. REFERENCES:

1. Nield, D.A. and Bejan, A., Convection in Porous Media, 2nd ed. Springer, NY, (1999).
2. Pop, I. and Ingham, D.B., Convective Heat Transfer : Mathematical and Computational Modelling of viscous Fluids and Porous Media, Pergamon, Oxford, (2001).
3. Postelnicu, A., Grosan, T. and Pop, I., *Int. Comm. Heat Mass Transfer*, 27, 729 (2000).
4. Mahapatra, N., Dash, G.C., Panda, S. and Acharya, M., *J. Engg. Phys. And Therm. Phys*., 83(1), 130 (2010).
5. Dash, S., *Mod. Meas. Cont.* (B), 76, 83 (2007).
6. Israel-Cookey, C., Ogulu, A. and Omubo-Pepple, V.B., *Int. J. Heat Mass Transfer*, **46**, 2305 (2003).
7. Dash, S., Dash, G.C. and Mishra, D.P., *Proc. Nat. Acad. Sci*., *India*(A), **78**(I), 49 (2008).
8. Hassanien, I.A. and Obied Allah, M.H., *Int. Comm. Heat Mass Transfer*, **29**, 567 (2002).
9. Panda, J.P., Pattanaik, A.B. and Acharya, A., *Mod. Meas. Cont.* (B), **75**, 31 (2006).
10. Ganapathy, R., *J. Appl. Mech*., **61**, 1104 (1994).
11. Kulshrestha, S.P. and Singh, N.P., *Acta Ciencia Indica*, **19**, 143 (1993).
12. Singh, B.B., *Ind. J. Theo. Phys*., **43**, 193 (1995).
13. Singh, N.P., Gupta, S.K. and Singh, A.K., *Proc. Nat. Acad. Sci., India* (A), **71** (II), 149 (2001).
14. Singh, N.P., Singh. A.K., Yadav, M.K. and Singh, A.K., *Ind. J. Theo. Phys*., **50**, 37 (2002).
15. Singh, K.D., Gorla, M.G. and Hans Raj, *Ind. J. Pure Appl. Math*., **36**, 151 (2005).