SELECTION OF MIXED SAMPLING PLAN WITH QSS-1(N,MN;C$_0$) PLAN AS ATTRIBUTE PLAN INDEXED THROUGH MAPD AND AQL

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Abstract

This paper presents the procedure for the construction and selection of the mixed sampling plan using MAPD as a quality standard with the QSS-1 (n,mn;c_0) plan as attribute plan. The plans indexed through MAPD and AQL are constructed and compared for their efficiency. Tables are constructed for easy selection of the plan.

Key words and Phrases: *acceptable quality level, maximum allowable percent defective, operating characteristic, tangent intercept.*

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1. Introduction

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, then the plan is said to be independent otherwise dependent. The principal advantage of a mixed sampling plan over pure attribute sampling plan is a reduction in sample size for a similar amount of protection.

The second stage attribute inspection becomes more important to discriminate the lot if the first stage variable inspection fails to accept the lot. If rejection occurs during the normal inspection then tightened inspection is recommended in the mixed system and vice versa in the second stage. Hence Quick Switching System is imposed in the second stage to sharpen the sampling situation and to insist the producer to manufacture goods within the Acceptable Quality Level. Dodge (1967) proposed a sampling system called a 'Quick Switching System' (QSS) consisting of pairs of normal and tightened plans.

Romboski(1969) introduced a system designated as QSS-1(n,mn;c_0) refers to Quick Switching System, where in the normal and tightened plan have the same acceptance number but on tightened inspection the sample size is a multiple of m, m>1 of the sample size on normal inspection. If m = 1, then the system reduces to single sampling plan.

Schilling (1967) proposed a method for determining the operating characteristics of mixed variables – attributes sampling plans, single sided specification and standard deviation known using the normal approximation. Devaarul (2003), Sampath Kumar (2007), Radhakrishnan and Sampath Kumar (2006, 2007a, 2007b, 2009) have made contributions to mixed sampling plans for independent case. Radhakrishnan, et.al (2009) studied mixed sampling plan for dependent case. QSSs were originally proposed by Dodge (1967) and investigated by Romboski (1969) and Govindaraju (1991). Romboski (1969) has developed QSS by attributes is a reduction in the sample size required to achieve approximately the same operating characteristic curve.

In this paper, using the operating procedure of mixed sampling plan with QSS-1 ($n;mn,c_0$) plan as attribute plan, tables are constructed for the mixed sampling plan indexed through (i) MAPD (ii) AQL (acceptable quality level). This plan indexed through MAPD is compared with the plan indexed through AQL. Suitable suggestions are also provided for future research.

2. Conditions of Applications of QSS-1 Mixed Sampling Plan

- Production process should be steady and continuous.
- Lots are submitted substantially in the order of their production.
- Inspection is by variable criteria in the first stage and by attribute criteria in the second stage with quality defined as fraction defective.

3. Glossary of Symbols

The symbols used in this paper are as follows:

p : submitted quality of lot or process

$P_a(p)$: probability of acceptance for given quality 'p'

p_1 : the submitted quality such that $P_a(p_1) = 0.95$ (also called AQL)

p_* : maximum allowable percent defective (MAPD)

h_* : relative slope at 'p_*'

n_1 : sample size of variable sampling plan

n_2 : sample size of attribute sampling plan

c_0 : Attribute acceptance number

m : multiplicity factor for sample size

β_j : probability of acceptance for lot quality 'p_j'

β_j' : probability of acceptance assigned to first stage for percent defective 'p_j'

β_j'' : probability of acceptance assigned to second stage for percent defective 'p_j'

d : observed number of nonconforming units in a sample of n units

z(j) : 'z' value for the j^{th} ordered observation

k : variable factor such that a lot is accepted if $\overline{X} \leq A = U - k\sigma$

4. Operating procedure of Mixed Sampling Plan with QSS- 1(n,mn;c_0) as Attribute plan

Schilling (1967) has given the following procedure for the independent mixed sampling plan with Upper specification limit (U) and known standard deviation (σ).

- Determine the parameters of the mixed sampling plan n_1, n_2, k and c_0.
- Take a random sample of size n_1 from the lot.
- If a sample average $\overline{X} \leq A = U - k\sigma$, accept the lot.
- If a sample average $\overline{X} > A = U - k\sigma$, go to step 1.

Step 1: From a lot, take a random sample of size 'n_2'at the normal level. Count the number of defectives 'd'

- If $d \leq c_0$, accept the lot and repeat step 1.
- If $d > c_0$, reject the lot and go to step 2.

Step 2: From the next lot, take a random sample of size 'mn_2' at the tightened level. Count the number of defectives 'd'

- If $d \leq c_0$, accept the lot and use step 1 for the next lot.
- If $d > c_0$, reject the lot and repeat step 2 for the next lot.

5. Construction of Mixed Sampling Plan having QSS-1(n,mn;c_0) as Attribute Plan

The operation of mixed sampling plans can be properly assessed by the OC curve for given values of the fraction defective. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit 'U'. A parallel discussion can be made for lower specification limits.

The procedure for the construction of mixed sampling plans is provided by Schilling (1967) for a given 'n_1' and a point 'p_j' on the OC curve is given below.

- Assume that the mixed sampling plan is independent
- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage. Let it be β_j'.
- Decide the sample size n_1 (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as A = U - kσ = U – [z (p_j) + {z (β_j')/$\sqrt{n_1}$ }]σ , where U is the upper specification limit and z (t) is the standard normal variate corresponding to t' such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$
- Determine the sample average $\bar{X}$. If a sample average $\bar{X}$ > A = U - kσ , take a second stage sample of size 'n_2' using attribute sampling plan.
- Split the probability of acceptance β_j as β_j' and β_j'', such that $\beta_j = \beta_j' + (1- \beta_j')\beta_j''$. Fix the value of β_j'.
- Now determine β_j'', the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_j'' = (\beta_j - \beta_j') / (1-\beta_j')$
- Determine the appropriate second stage sample of size 'n_2' from

$$Pa(p) = \beta_j'' \text{ for } p = p_j.$$

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with QSS-1(n, mn;c_0) plan as attribute plan indexed through MAPD and AQL.

According to Soundararajan and Arumainayagam (1988), the operating characteristic function of QSS-1 is given below.

$$P_a(p) = \frac{b}{1-a+b} \qquad (1)$$

Where $$a = \sum_{i=0}^{c_0} \frac{e^{-n_2 p}(n_2 p)^i}{i!} \qquad (2)$$

$$b = \sum_{j=0}^{c_0} \frac{e^{-mn_2 p}(mn_2 p)^j}{j!} \qquad (3)$$

(for sample size tightening)

6. Construction of the plans indexed through MAPD

MAPD (p_*), introduced by Mayer (1967) and further studied by Soundararajan (1975) is the quality level corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level 'p_*' is measured by 'p_t', the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis. For designing, Soundararajan (1975) proposed a selection procedure for mixed sampling plan indexed with MAPD and $R = p_t / p_*$.

Using the probability mass function of QSS-1, given in expression (1), the inflection point (p_*) is obtained by using $\frac{d^2 p_a(p)}{dp^2} = 0$ and $\frac{d^3 p_a(p)}{dp^3} \neq 0$. The relative slope of the OC curve $h_* = \left[\frac{-p}{p_a(p)}\right]\frac{dp_a(p)}{dp}$ at $p = p_*$. The inflection tangent of the OC curve cuts the 'p' axis at $p_t = p_* + (p_*/h_*)$. The values of n_2p_*, h_*, n_2p_t and $R = p_t/p_*$ are calculated for different values of 'm' and 'c_0' for $\beta_*' = 0.40$ using c++ program and presented in Table 1.

6.1 Selection of the plan

For the given values of p_* and p_t, the ratio $R = \frac{p_t}{p_*}$ is found and the nearest value of R is located in Table 1.The corresponding value of c_0, m and np_* values are noted and the value of n_2 is obtained using $n_2 = \frac{n_2 p_*}{p_*}$.

6.2 Example: Given $p_* = 0.080$, $p_t = 0.095$ and $\beta_*' = 0.40$, the ratio $R = \frac{p_t}{p_*} = 1.1875$. In Table 1, the nearest R value is 1.1887 which is corresponding to $c_0 = 3$ and $m = 2.75$. The value of $n_2p_* = 2.6076$ is found and hence the value of n_2 is determined as $n_2 = \frac{n_2 p_*}{p_*} = \frac{2.6076}{0.080} = 33$. Thus $n_2 = 33$, $c_0 = 3$ and $m = 2.75$ are the parameters selected for the mixed sampling plan having QSS-1(n,mn;c_0) as attribute plan using Poisson distribution as a baseline distribution, for the given values of $p_* = 0.080$ and $p_t = 0.095$.

7. Construction of Mixed Sampling Plan Indexed Through AQL

The procedure given in Section 5 is used for constructing the mixed sampling plan indexed through AQL (p_1). By assuming the probability of acceptance of the lot be $\beta_1 = 0.95$ and $\beta_1' = 0.40$, the n_2p_1 values are calculated for different values of 'c_0' and 'm' using c++ program and is presented in Table 1.

7.1 Selection of the plan

Table 1 is used to construct the plans when AQL (p_1), 'c_0' and 'm' are given. For any given values of p_1, 'c_0' and 'm' one can determine n_2 value using $n_2 = \frac{n_2 p_1}{p_1}$.

7.2 Example: Given $p_1 = 0.07$, $c_0 = 7$, $m = 2.75$ and $\beta_1' = 0.40$. Using Table 1, find $n_2 = \frac{n_2 p_1}{p_1} = \frac{3.4447}{0.07} = 49$. Thus $n_2 = 49$, $c_0 = 7$ and $m = 2.75$ are the parameters selected for the mixed sampling plan having QSS-1(n,mn;c_0) as attribute plan for a specified $p_1 = 0.07$, $c_0 = 7$ and $m = 2.75$.

8. Comparison of Mixed Sampling Plan Indexed Through MAPD and AQL

In this section mixed sampling plan indexed through MAPD is compared with mixed sampling plan indexed through AQL by fixing the parameters c_0, m and β_j'.

For the specified values of p_* and p_t with the assumption $\beta_*' = 0.40$, one can find the values of c_0, m and n_2 indexed through MAPD. By fixing the values of c_0 and m, find the value of p_1 by equating $Pa(p) = \beta_1 = 0.95$. Using $\beta_1' = 0.40$, c_0 and m one can find the value of n_2 using $n_2 =$

$\frac{n_2 p_1}{p_1}$ from Table 1. For different combinations of p_*, p_t, c_0 and m the values of n_2 (indexed through MAPD) and n_2 (indexed through AQL) are calculated and presented in Table 2.

Table 2: Comparison of plans indexed through MAPD and AQL

p_*	p_t	c_0	m	INDEXED THROUGH MAPD	INDEXED THROGH AQL
				n_2	n_2
0.033*	0.037	6	2.75	135	145
0.035	0.040	4	3.00	88	97
0.048	0.066	2	1.50	56	68
0.054	0.081	1	1.75	28	37

***OC curve is drawn**

8.1 Construction of OC curve

The OC curves for the plans $n_2 = 135$, $c_0 = 6$, $m = 2.75$ (indexed through MAPD) and $n_2 = 145$, $c_0 = 6$, $m = 2.75$ (indexed through AQL) based on the different values of n_2p_1 and $p_a(p)$ are presented in Figure 1.

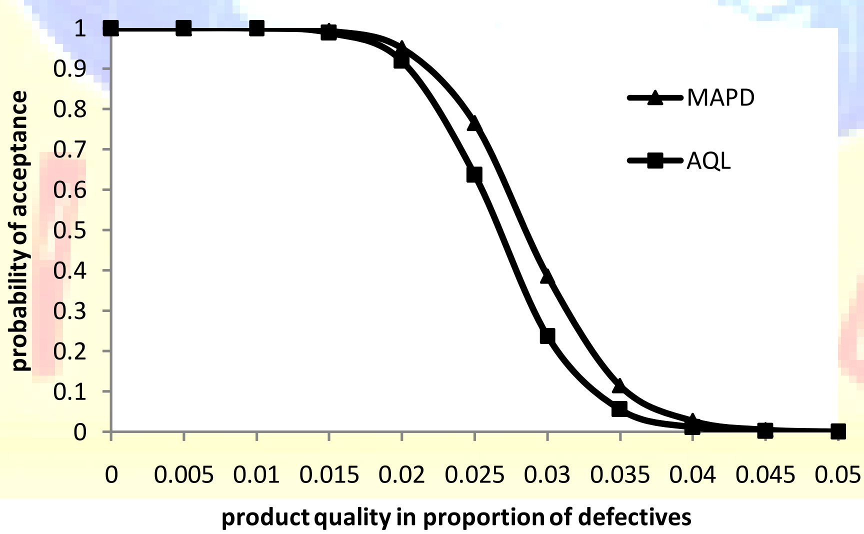

Figure 1: OC Curves for QSS-2(135,371.25;6) and (145,398.75;6)

9. Conclusion

In this paper using the operating procedure of mixed sampling plan with QSS-1(n, mn;c_0) as attribute plan, tables are constructed for the mixed sampling plan indexed through the parameters MAPD and AQL by taking Poisson distribution as a baseline distribution. It is concluded from the study that the second stage sample size required for QSS-1(n,mn;c_0) plan indexed through MAPD is less than that of the second stage sample size of the QSS-1(n,mn;c_0) plan indexed through AQL. Examples are provided for a specified value of $\beta_j' = 0.40$. If the floor engineers know the levels of MAPD or AQL, they can have their sampling plans on the floor itself by referring to the tables. This provides the flexibility to the floor engineers in deciding their sampling plans. Various plans can also be constructed to make the system user friendly by changing the first stage probabilities (β_*', β_1') and can also be compared for their efficiency.

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Table 1: Various characteristics of the mixed sampling plan when $\beta_*' = \beta_1' = 0.40$ and β_1 = 0.95

c_0	m	n_2p_1	β_*''	n_2p_*	h_*	n_2p_t	$R = p_t/p_*$
0	1.75	0.0820	0.7895	0.2058	0.2654	0.9812	4.7677
0	2.00	0.0805	0.6757	0.3033	0.4743	0.9428	3.1085
0	2.25	0.0791	0.6035	0.3526	0.6452	0.8991	2.5499
0	2.50	0.0778	0.5528	0.3778	0.7905	0.8557	2.2650
0	2.75	0.0763	0.5155	0.3891	0.9147	0.8145	2.0933
0	3.00	0.0753	0.4862	0.3930	1.0251	0.7764	1.9756
1	1.25	0.4561	0.4632	1.5936	1.2975	2.8218	1.7707
1	1.50	0.4455	0.4005	1.5836	1.6590	2.5382	1.6028
1	1.75	0.4455	0.3600	1.5269	1.9628	2.3048	1.5095
1	2.00	0.4350	0.3323	1.4560	2.2196	2.1120	1.4505
1	2.25	0.4248	0.3122	1.3840	2.4411	1.9510	1.4097
1	2.50	0.4149	0.2970	1.4121	2.9724	1.8872	1.3364
1	2.75	0.4054	0.2852	1.2519	2.8067	1.6979	1.3563
1	3.00	0.3963	0.2755	1.1940	2.9624	1.5971	1.3376
2	1.50	0.9683	0.3198	2.6782	2.5805	3.7161	1.3875
2	1.75	0.9424	0.2898	2.5063	2.9929	3.3437	1.3341
2	2.00	0.9164	0.2702	2.3428	3.3381	3.0446	1.2996
2	2.25	0.8909	0.2562	2.1964	3.6370	2.8003	1.2749
2	2.50	0.8663	0.2457	2.0670	3.9007	2.5969	1.2564
2	3.00	0.8200	0.2312	1.8511	4.3488	2.2768	1.2300
3	1.25	1.5941	0.3250	3.9714	2.7068	5.4386	1.3694
3	1.50	1.5506	0.2822	3.6888	3.3393	4.7935	1.2995
3	1.75	1.5046	0.2575	3.4109	3.8536	4.2960	1.2595
3	2.00	1.4582	0.2417	3.1645	4.2872	3.9026	1.2332
3	2.25	1.4128	0.2305	2.9520	4.6657	3.5847	1.2143
3	2.50	1.3690	0.2223	2.9748	5.7677	3.4906	1.1734
3	2.75	1.3272	0.2162	2.6076	5.2989	3.0997	1.1887

3	3.00	1.2876	0.2113	2.4673	5.5714	2.9102	1.1795
4	1.50	2.1757	0.2598	4.6557	4.0116	5.8163	1.2493
4	2.25	1.9663	0.2158	3.6780	5.6054	4.3342	1.1784
4	2.50	1.9002	0.2092	3.4426	6.0102	4.0154	1.1664
4	2.75	1.8375	0.2042	3.2396	6.3752	3.7478	1.1569
4	3.00	1.7784	0.2000	3.0634	6.7114	3.5198	1.1490
5	2.75	2.3637	0.1962	3.8592	7.4050	4.3804	1.1351
5	3.00	2.2835	0.1930	3.6472	7.7988	4.1149	1.1282
6	2.75	2.9005	0.2008	4.4422	8.2534	4.9804	1.1212
7	2.75	3.4447	0.5662	4.3211	4.4095	5.3011	1.2268
8	2.75	3.9945	0.1842	5.6727	10.3117	6.2228	1.0970
9	2.25	4.9420	0.1877	7.1318	9.7576	7.8627	1.1025
9	2.50	4.7377	0.1842	6.6664	10.5397	7.2989	1.0949
9	3.00	4.3738	0.1797	5.9271	11.9053	6.4250	1.0840