

A LABEL FIELD FUSION USING STOCHASTIC ICM OPTIMIZATION TECHNIQUE IN IMAGE SEGMENTATION

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Abstract-

This paper presents a new, simple and efficient segmentation approach based on an ICM Optimization Technique to obtain an accurate segmentation result. ICM means Iterative Conditional Modes Optimization procedure. ICM is one of the Greedy algorithms that performed at the full resolution to get the final result. This method is accurate and also it should be computationally efficient. In existing methods, so many clustering techniques are available. These techniques are also very expensive and also not accurate. In the proposed method, PRand is calculated for the segmentation result. Among that result, the best one is taken as the starting point of the ICM optimization procedure. It improves the clarity of the image. This technique is successfully applied in Berkeley image database, and which is very simple to implement. The experiments reported in this paper illustrate the potential approach compared to the state-of-the-art segmentation methods recently proposed in the literature.

Index Terms:- Iterated Conditional Modes (ICM) , Greedy Algorithm, Berkeley Image Database, Probabilistic Rand Index (PRand)

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I. INTRODUCTION

Image segmentation is an important primitive step in many computer vision related tasks. Segmentation is dividing an image into coherent regions [1]. And also image segmentation means partitioning the input image into non overlapping regions such that each region is homogeneous and the union of any two adjacent regions is heterogeneous [2]. The main aim of image segmentation is divided into number of pixels in different image regions i.e., regions corresponding to individual surfaces, objects, or natural parts of objects. The result of image segmentation is a set of segments that collectively cover the entire image or a set of contours extracted from the image. Each of the pixels in a region is similar with respect to some characteristic or computed property, such as color, intensity, or texture. Adjacent regions are significantly different with respect to the same characteristics.

In image analysis and computer vision, image segmentation and object extraction plays an important role. In the last decades, so many methods have been proposed to solve the difficult problem. Among them, we can cite clustering algorithms [3], spatial-based segmentation methods which exploit the connectivity information between neighboring pixels and have led to Markov Random Field (MRF)-based statistical models [4], mean-shift-based techniques [5], [6], graph-based [7], [8], variational methods [9], [10], or by region-based split and merge procedures, sometimes directly expressed by a global energy function to be optimized [11].

Years of research in segmentation, so many methods have been demonstrated such as texture features [12][13], labels, region process or no. of classes [11][14][15]. These methods are computationally very expensive and some of the energy based models are costly optimization technique.

Basically, Image segmentation is divided into three categories (i) Manual segmentation (ii) Automatic (iii) Semi Automatic [2]. The segmentation approach, proposed in this paper, is conceptually different and explores another strategy initially introduced in [16]. Our proposed technique explores an Stochastic ICM optimization method that gives an accurate segmentation result to get an clear image. The optimization task is performed by a full resolution strategy. This new strategy is applied in the Berkeley image database.

This paper describe that the proposed method, is simple and often better (in terms of visual evaluations and quantitative performance measures) than the best existing state-of-the-art recent

segmentation methods on the Berkeley natural image database(containing also, for quantitative evaluations, ground truth segmentations obtained from human subjects) [17].

II. RELATED WORK

In this section, so many related measures have been proposed in this literature. This broadly categorizes and they are all used in the existing work. These categorize are previously measured and they are as follows

1. Region Differencing: Several steps operate by calculating the degree of representation of common coincidence of the cluster associated with each pixel in one segmentation and find the “closest” approximation in the other segmentation and select the closest one. Some of them are measurably intolerant of label improvement or elaboration [17].

2. Boundary matching: Several measures work by matching boundaries between the segmentations, and computing some summary statistic of match quality [18], [19]. Work in [17] proposed solving an approximation to a bipartite graph matching problem for matching segmentation boundaries, computing the percentage of matched edge elements, and using the harmonic mean of precision and recall, termed the F-measure as the statistic. However, since these measures are not tolerant of refinement, it is possible for two segmentations that are perfect mutual refinements of each other to have very low precision and recall scores. Furthermore, for a given matching of edge elements between two images, it is possible to change the locations of the unmatched edges almost arbitrarily and retain the same precision and recall score.

3. Information-based: Work in [17], [20] proposes to formulate the problem as that of evaluating an affinity function that gives the probability of two pixels belonging to the same segment. They compute the mutual information score between the classifier output on a test image and the ground-truth data, and use the score as the measure of segmentation quality. Its application in [19], [20] is however restricted to considering pixel pairs only if they are in complete agreement in all the training images. Work in [21] computes a measure of information content in each of the segmentations and how much information one segmentation gives about

the other. The proposed measure, termed the variation of information (VI), is a metric and is related to the conditional entropies between the class label distributions of the segmentations. The measure has several promising properties [22] but its potential for evaluating results on natural images where there is more than one ground-truth clustering is unclear. Several measures work by recasting the problem as the evaluation of a binary classifier [17], [23] through false-positive and false-negative rates or precision and recall, similarly assuming the existence of only one ground-truth segmentation. Due to the loss of spatial knowledge when computing such aggregates, the label assignments to pixels may be permuted in a combinatorial number of ways to maintain the same proportion of labels and keep the score unchanged

III. SEGMENTATION ALGORITHM

There is several segmentation algorithms are used in image segmentation. Among that several segmentation algorithm some of them are used in the existing i) mean shift based segmentation algorithm ii) Efficient graph based segmentation algorithm iii) Hybrid segmentation algorithm iv) Expectation Maximization algorithm v) Clustering algorithm

i) Mean Shift Based Segmentation: The mean shift-based segmentation technique was introduced in [5] and is one of many techniques under the heading of “feature space analysis.” The technique is comprised of two basic steps: a mean shift filtering of the original image data (in feature space), and a subsequent clustering of the filtered data points.

ii) Efficient Graph Based Segmentation: This segmentation, introduced in [8], is another method of performing clustering in feature space. This method works directly on the data points in feature space, without first performing a filtering step, and uses a variation on single linkage clustering. The key to the success of this method is adaptive thresholding. To perform traditional single linkage clustering, a minimum spanning tree of the data points is first generated (using Kruskal’s algorithm), from which any edges with length greater than a given hard threshold are removed. The connected components become the clusters in the segmentation. The method in [8] eliminates the need for a hard threshold, instead replacing it with a data dependent term.

The merging criterion allows Efficient graph-based clustering to be sensitive to edges in areas of low variability, and less sensitive to them in areas of high variability. However, the results it gives do not have the same degree of correctness with respect to the ground truth as mean shift based segmentation, as demonstrated in Fig. 1. This algorithm also suffers somewhat from sensitivity to its parameter, k .



Fig. 1. Example of changing scores for different parameters using efficient graph-based segmentation: (a) Original image, (b), (c), and (d) efficient graph-based segmentations using spatial normalizing factor $h_s = 7$, color normalizing factor $h_r = 7$, and k values 5, 25, and 125, respectively.

iii) Hybrid Segmentation Algorithm: An obvious question emerges when describing the mean shift-based segmentation method [5] and the efficient graph-based clustering method [8]: Can we combine the two methods to give better results than either method alone? More specifically, can we combine the two methods to create more stable segmentations that are less sensitive to parameter changes and for which the same parameters give reasonable segmentations across multiple images? In an attempt to answer these questions, the third algorithm we consider is a combination of the previous two algorithms:

First, we apply mean shift filtering and then we use efficient Graph-based clustering to give the final segmentation. The result of applying this algorithm with different parameters can be seen in Fig. 2. Notice that for $h_r=15$, the quality of the segmentation is high. Also, notice that the rate of granularity change is slower than either of the previous two algorithms, even though the parameters cover a wide range.

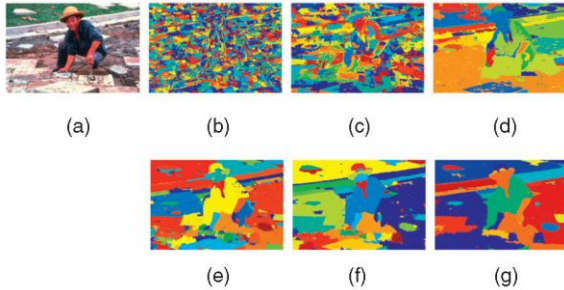


Fig. 2. Example of changing scores for different parameters using a hybrid segmentation algorithm which first performs mean shift filtering and then efficient graph-based segmentation: (a) Original image, (b), (c), (d), (e), (f), and (g) segmentations using spatial bandwidth $h_s = 7$, and color bandwidth δ_{hr} and k value combinations (3, 5), (3, 25), (3, 125), (15, 5), (15, 25), and (15, 125), respectively.

iv) EM Segmentation Algorithm: Our final algorithm is the classic Expectation Maximization (EM) algorithm [28], with the Bayesian Information Criterion (BIC) used to select the number of Gaussians in the model. By minimizing the BIC, we attempt to minimize model complexity while maintaining low error. The BIC is formulated as follows:

$$\text{BIC} = n \ln \left(\frac{\text{RSS}}{n} \right) + g \ln(n), \quad (1)$$

Where n is the sample size, g is the number of parameters, and RSS is the residual sum of squares. We present graphical results for the EM algorithm as a baseline for each relevant experiment; however, we omit it in the detailed performance discussion.

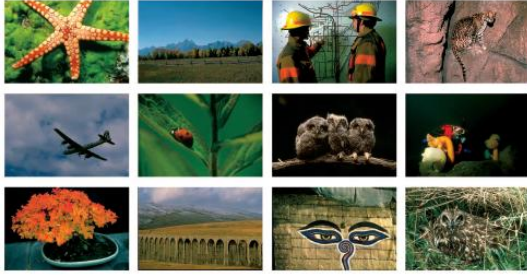


Fig. 3 Examples of images from the Berkeley image segmentation database [1].

V. PROPOSED FUSION MODEL

i) Clustering algorithm: In proposed system, also the clustering algorithm is used. In existing system the clustering algorithm was only suggest. Clustering is one of the basic techniques for the entire mentioned algorithm. In order to be more accurate while being computationally efficient, the proposed method fuses the results of k-means clustering applied with various values of its parameters to get an accurate segmentation

The K-means clustering algorithm is suggest by varying the parameter by different values that will give several segmentation result. K-means clustering algorithm applied in an input image that is possibly expressed in different color values or by other means[1].

The final result is obtained by maximizing its similarity to the k-means results which are taken to be ground truth segmentations. That segmentation result will act as the ground truth images in proposed system. What is mean by ground truth images? Ground truth images are also called as hand segment images or human segmented images.

ii) Rand Index: The proposed fusion model also suggests the Rand Index. The Rand index [25] is a clustering quality metric that measures the agreement of the clustering result with a given ground truth. This non-parametric statistical measure was recently used in image segmentation [28] as a quantitative and perceptually interesting measure to compare automatic segmentation of an image to a ground truth segmentation (e.g., a manually hand-segmented image given by an expert) and/or to objectively evaluate the efficiency of several unsupervised segmentation methods.

Let n_s be the number of pixels assigned to the same region (i.e., matched pairs) in both the segmentation to be evaluated (S^{test}) and the ground truth segmentation, S^{gt} and n_d be the number of pairs of pixels assigned

to different regions (i.e., mismatched pairs) in S^{test}

and S^{gt} . The Rand index is defined as the ratio of $(n_s + n_d)$ to the total number of pixel pairs, i.e.

$$\frac{N(N-1)/2}$$

for an image of size N pixels. More formally [29], if $\{l_i^{S^{\text{test}}}\}$ and $\{l_i^{S^{\text{gt}}}\}$ designate the set of region labels respectively associated to the segmentation map S^{test} and S^{gt} at pixel location \mathbf{x}^i and where \mathcal{I} is an indicator function, the Rand index is given by the following relation:

$$\begin{aligned} \text{Rand}(S^{\text{test}}, S^{\text{gt}}) &= \frac{1}{\frac{N(N-1)}{2}} \\ &\times \sum_{i,j \ i < j} \left[\overbrace{\mathcal{I}(l_i^{S^{\text{test}}} = l_j^{S^{\text{test}}} \text{ and } l_i^{\text{gt}} = l_j^{\text{gt}})}^{n_s} \right. \\ &\quad \left. + \overbrace{\mathcal{I}(l_i^{S^{\text{test}}} \neq l_j^{S^{\text{test}}} \text{ and } l_i^{S^{\text{gt}}} \neq l_j^{S^{\text{gt}}})}^{n_d} \right] \quad (2) \end{aligned}$$

which simply computes the proportion (value ranging from 0 to 1) of pairs of pixels with compatible region label relationships between the two segmentations to be compared. A value of 1 indicates that the two segmentations are identical and a value of 0 indicates that the two segmentations do not agree on any pair of points (e.g., when all the pixels are gathered in a single region in one segmentation whereas the other segmentation assigns each pixel to an individual region). When the number of labels (S^{test}) and S^{gt} in and are much smaller than the number of data points N , a computationally inexpensive estimator of the Rand index can be found in [29].

iii) Probabilistic Rand Index: The PRI was recently introduced by Unnikrishnan [28] to take into account the inherent variability of possible interpretations between human observers of an image, i.e., the multiple acceptable ground truth segmentations associated with each natural image. This variability between observers, recently highlighted by the Berkeley segmentation dataset [29] is due to the fact that each human chooses to segment an image at different levels of detail. This variability is also due image segmentation being an ill-posed problem, which exhibits multiple solutions for the different possible values of the number of classes not known *a priori*.

Hence, in the absence of a *unique* ground-truth segmentation, the clustering quality measure has to quantify the agreement of an automatic segmentation (i.e., given by an algorithm) with the variation in a set of available manual segmentations representing, in fact, a very small sample of the set of all possible perceptually consistent interpretations of an image [30]. The authors [28] address this concern by soft nonuniform weighting of pixel pairs as a means of accounting for this variability in the ground truthset. More formally, let us consider a set of manually segmented (ground truth) image $\{S_1^{gt}, S_2^{gt}, \dots, S_L^{gt}\}$ corresponding to an image of size N . Let S^{test} be the segmentation to be compared with the manually labeled set $\{l_i^{S_k^{gt}}\}$ and designates the set of region labels associated with the segmentation maps S_k^{gt} at pixel location x_i , the probabilistic RI is defined by

$$PRand(S^{test}, \{S_k^{gt}\}) = \frac{1}{\frac{N(N-1)}{2}} \sum_{i,j \ i < j} \times \left[p_{ij} \mathcal{I}(l_i^{S^{test}} = l_j^{S^{test}}) + (1 - p_{ij}) \mathcal{I}(l_i^{S^{test}} \neq l_j^{S^{test}}) \right] \quad (3)$$

Where a good choice for the estimator p_{ij} of (the probability of the pixel and having the same label across $\{S_k^{gt}\}$) is simply given by the empirical proportion

$$p_{ij} = \frac{1}{L} \sum_{k=0}^{k=L} \delta(l_i^{S_k^{gt}}, l_j^{S_k^{gt}}) \quad (4)$$

Where δ is the delta Kronecker function. In this way, the PRI measure is simply the mean of the Rand index computed between each pair $(S^{test}, S_k^{gt}) (k = 0, \dots, L)$ [16]

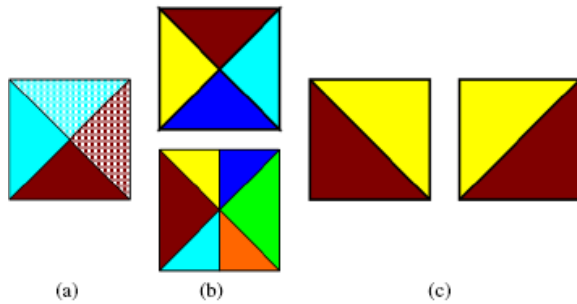


Figure 4: Synthetic example of permissible refinements: (a) Input image, (b) Segmentations for testing, and (c) ground truth set

Consider an example fig 1 here a) act as the input image and b) says that two possible results are generated by segmentation algorithms and the result c) shows that images are ground truth hand labeled that are generated by people. The hand segmenters can divide the image based on color and texture properties. The two hand segmentation result are showed because either any one of the hand segmented result shows the edges of the images clearly [26].

The PR does not allow refinement or coarsening that is not inspired by one of the human segmentations, hence the PR index gives low (low similarity, high error) scores of 0.3731 and 0.4420, respectively [26].

As a consequence, the PRI measure will favor (i.e., give a high score to) a resulting acceptable segmentation map which is consistent with most of the segmentation results given by human experts. More precisely, the resulting segmentation could result in a compromise or a consensus, in terms of level of details and contour accuracy exhibited by each ground-truth segmentations. Fig. 5 gives a fusion map example, using a set of manually generated segmentations exhibiting a high variation, in terms of level of details. Let us add that this probabilistic metric is not degenerate; all the bad segmentations will give a low score without exception [30].

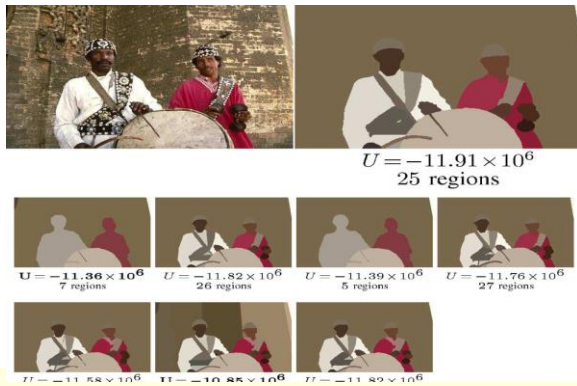


Fig. 5. First row and from left to right; a natural image from the Berkeley database (no. 229036) and the resulting fusion map using the set of 7 input hand-labeled ground-truth segmentations of the Berkeley database [1] with their number of regions

VI FUSION RESULT

The initial segmentation maps which will be fused by K-means [31] clustering technique, applied on an input image expressed by different color spaces and different number of classes. As simple cues (i.e., as input multidimensional feature descriptor), we used the set of values of the re-quantized color histogram, with equidistant binning, estimated around the pixel

to be classified. In our application, this local histogram is equally re-quantized, for each of the three color channels, in a $N_b = q_b^3$ bin descriptor, computed on an overlapping squared fixed-size ($N_w = 7$) neighborhood centered around the pixel to be classified

EXPERIMENTAL RESULT

i) **Algorithm:** The final result is obtained by maximizing its similarity to the k-means results which are taken to be ground truth segmentations

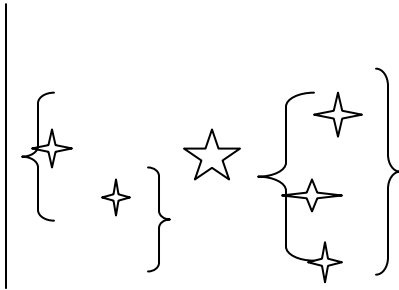


fig.6

The 4 point star represents the clustering points and the 5 point star assume as the centroid.

Now we can see the steps involved in the algorithm

- i) Initially assume a Random cluster
- ii) Then assign the centroid between the clustering points
- iii) Take each cluster and see which clustering is closest to the centroid
- iv) Among that clustering points again assume the centroid and once again see which one is the closest clustering point.

K-means clustering based segmentation of the image for varying parameters is carried out. The feature vector of the clustering is taken to be the local histogram of each pixel. The local histogram is calculated by binning the pixels in a 7X7 window centered at the pixel. The color image intensities are requantized to one dimension using a particular bin size. Euclidean distance is taken as the basis of the clustering. A brief description of k-means clustering is as follows.

An initial set of cluster centroids is assumed and every data point is assigned to the cluster with the closest centroid. The centroids are calculated and reassignment is done. This procedure is repeated until the reassignment does not change the label of any data point. The local histogram is taken as the data point. The segmentation is done for various values of the aforementioned parameters.

Penalized Rand Estimator is taken as the objective function. So the segmentation process consists of the following steps

- 1) Run k-means algorithm on the image for varying parameters for image color space, number of clusters and number of bins.
- 2) Evaluate penalized rand estimator for each result and select the best one.
- 3) Use ICM optimization procedure to refine the best result into the final result.

VII STOCHASTIC ICM OPTIMIZATION TECHNIQUE

The best one is taken as the starting point of an Iterative Conditional Modes (ICM) optimization procedure. ICM is a greedy algorithm whose every step the PRand is maximized. The proposed method is a multiresolution strategy. The optimization is performed for a low resolution version of the image and the result is extended to the full resolution by duplication. Then another ICM procedure is performed at the full resolution to get the final result.

Heuristics based ICM optimization technique is used The ICM optimization procedure is used in the existing system to optimize U as described in the paper. Here pixel by pixel we check the value of U for every possible segmentation class and choose the segmentation that minimizes U. This is a greedy algorithm and can get stuck at local minima. In order to improve the result we apply a simulated annealing style ICM optimization.

Images	U (ICM (Existing system))	U (Simulated Annealing based ICM)
Bird.jpg	0.8871	6.1882
12074.jpg	0.1659	0.1753
12003.jpg	5.2687	2.2601
8143.jpg	7.2575	7.2701
2092.jpg	0.2005	0.2000
15088.jpg	2.1846	0.1845
15004.jpg	5.3490	5.3478
8049.jpg	5.2112	5.2195

Table1

Stochastic ICM procedure:

Set the Temperature at 10

For each pixel,

Calculate U for every possible values of class for the pixel

Take the minimum and second minimum of the U values.

Accept the second minimum with a probability which is calculated as

$$\text{prob} = \exp(-(\text{mval2}-\text{mval1})/\text{mval2})/T;$$

Where mval1 and mval2 are the minimum and second minimum respectively and T is the Temperature. Decrease the Temperature. If there is no sufficient improvement terminate the iteration. From the results in the table it can be seen that there is a marginal improvement in the optimization by stochastic ICM.

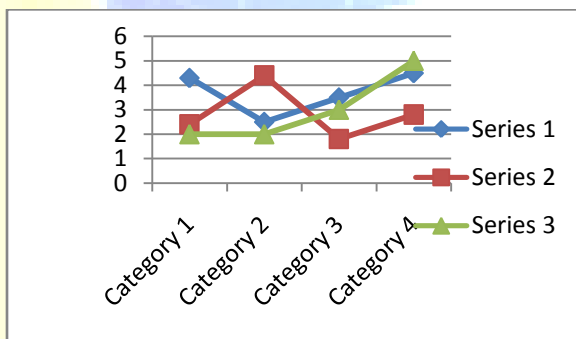


fig7

This graph represents the existing system ICM optimization technique and the stochastic ICM optimization technique.

VIII. CONCLUSION

In this paper, we have presented a new and efficient segmentation strategy based on a Stochastic ICM Optimization procedure. The goal is to combine several quickly estimated segmentation maps in order to achieve a more reliable and accurate segmentation

result. This fusion is achieved in the penalized maximum PRI sense which has a perceptual meaning. This fusion framework remains simple to implement, perfectible, by increasing the number of segmentation to be fused, and general enough to be applied to various digital image and computer vision applications.

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REFERENCES

- [1]Max Mignotte,” *A Label Field Fusion Bayesian Model and Its Penalized Maximum Rand Estimator for Image Segmentation*”, IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 19, NO. 6, JUNE 2010
- [2].Bhandarkar.S.M,Muizhang,”Image Segmentation using evolutionary computation” *Evolutionary computation*IEEE Transactions,vol-3,pp 1-21,1999
- [3].S. Banks, *Signal Processing, Image Processing and Pattern Recognition*. Englewood Cliffs, NJ: Prentice-Hall, 1990.
- [4]J. Besag, “On the statistical analysis of dirty pictures,” *J. Royal Stat.Soc.*, vol. B-48, pp. 259–302, 1986.
- [5]D. Comaniciu and P. Meer, “Mean shift: A robust approach toward feature space analysis,” *IEEE Trans. Pattern Anal. Mach. Intell.*, vol.24, no. 5, pp. 603–619, May2002.
- [6] Q. Luo and T. Khoshgoftaar, “Unsupervised multiscale color image segmentation based on mdl principle,” *IEEE Trans. Image Process.*, vol. 15, no. 9, pp. 2755–2761, Sep. 2006.
- [7]J. Shi and J. Malik, “Normalized cuts and image segmentation,” *IEEE Trans. Patt. Anal. Mach. Intell.*, vol. 22, no. 8, pp. 888–905, Aug. 2000.
- [8] P. Felzenszwalb and D. Huttenlocher, “Efficient graph-based image segmentation,” *Int. J. Comput. Vision*, vol. 59, no. 2, pp. 167–181, Sep.2004.

- [9] P. Arbelaez and L. Cohen, "A metric approach to vector-valued image segmentation," *Int. J. Comput. Vision, Special Issue Geometrical, Variational, Level Sets Methods Comput. Vision*, vol. 69, no. 1, pp.119–126, Aug. 2006.
- [10] P. Arbelaez, "Boundary extraction in natural images using ultrametric contour maps," in *Proc. 5th IEEE Workshop Perceptual Organization Comput. Vision (POCV)*, New York, Jun. 2006, pp. 182–189.
- [11] S. Zhu and A. Yuille, "Region competition: Unifying snakes, region growing, and Bayes/MDL for multiband image segmentation," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 18, no. 9, pp. 884-900, Sep.
- [12] M. Mignotte, C. Collet, P. Pérez, and P. Bouthemy, "Sonar image segmentation using a hierarchical MRF model," *IEEE Trans. Image Process.*, vol. 9, no. 7, pp. 1216–1231, Jul. 2000.
- [13] M. Mignotte, C. Collet, P. Pérez, and P. Bouthemy, "Three-class Markovian segmentation of high resolution sonar images," *Comput. Vision Image Understanding*, vol. 76, no. 3, pp. 191–204, Dec. 1999.
- [14] F. Destrempe, J.-F. Angers, and M. Mignotte, "Fusion of hidden Markov Random Field models and its Bayesian estimation," *IEEE Trans. Image Process.*, vol. 15, no. 10, pp. 2920–2935, Oct. 2006.
- [15] Z. Kato, T. C. Pong, and G. Q. Song, "Unsupervised segmentation of color textured images using a multi-layer MRF model," in *Proc. Int. Conf. Image Process. (ICIP)*, Barcelona, Spain, Sep. 2003, pp.961–964.996.
- [16] M. Mignotte, "Segmentation by fusion of histogram-based k-means clusters in different color spaces," *IEEE Trans. Image Process.*, vol.17, no. 5, pp. 780–787, May 2008.
- [17] Empirical Evaluation Methods in Computer Vision, H.I. Christensen and P.J. Phillips, eds. World Scientific Publishing, July 2002.
- [18] D. Martin, "An Empirical Approach to Grouping and Segmentation," PhD dissertation, Univ. of California, Berkeley, 2002.
- [19] J. Freixenet, X. Munoz, D. Raba, J. Marti, and X. Cuff, "YetAnother Survey on Image Segmentation: Region and Boundary Information Integration," *Proc. European Conf. Computer Vision*, pp. 408-422, 2002.

- [20] Q. Huang and B. Dom, "Quantitative Methods of Evaluating Image Segmentation," Proc. IEEE Int'l Conf. Image Processing, pp. 53-56, 1995.
- [21] C. Fowlkes, D. Martin, and J. Malik, "Learning Affinity Functions for Image Segmentation," Proc. IEEE Conf. Computer Vision and Pattern Recognition, vol. 2, pp. 54-61, 2003.
- [22] M. Meila, "Comparing Clusterings by the Variation of Information," Proc. Conf. Learning Theory, 2003.
- [23] M. Meila, "Comparing Clusterings: An Axiomatic View," Proc. 22nd Int'l Conf. Machine Learning, pp. 577-584, 2005.
- [24] M.R. Everingham, H. Muller, and B. Thomas, "Evaluating Image Segmentation Algorithms Using the Pareto Front," Proc. European Conf. Computer Vision, vol. 4, pp. 34-48, 2002.
- [25] W. M. Rand, "Objective criteria for the evaluation of clustering methods," *J. Amer. Stat. Assoc.*, vol. 66, no. 336, pp. 846-850, 1971.
- [26] R. Unnikrishnan and M. Hebert, "Measures of Similarity," Proc. IEEE Workshop Computer Vision Applications, 2005.
- [27] A. Dempster, N. Laird, and D. Rubin, "Maximum Likelihood From Incomplete Data via the EM Algorithm," *J. Royal Statistical Soc., Series B*, pp. 1-38, 1977.
- [28] R. Unnikrishnan, C. Pantofaru, and M. Hebert, "A measure for objective evaluation of image segmentation algorithms," in *Proc. IEEE Int. Conf. Comput. Vision Pattern Recog. (CVPR), Workshop Empirical Evaluation Methods Comput. Vision*, San Diego, CA, Jun. 2005, vol. 3, pp. 34-41.
- [29] D. Martin, C. Fowlkes, D. Tal, and J. Malik, "A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics," in *Proc. 8th Int. Conf. Comput. Vision (ICCV)*, Vancouver, BC, Canada, Jul. 2001, vol. 2, pp. 416-423.
- [30] R. Unnikrishnan, C. Pantofaru, and M. Hebert, "Toward objective evaluation of image segmentation algorithms," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 29, no. 6, pp. 929-944, Jun. 2007.
- [31] S. P. Lloyd, "Least squares quantization in PCM," *IEEE Trans. Inf. Theory*, vol. 28, no. 2, pp. 129-136, Mar. 1982.