CONTACT PATTERN AND TRANSMISSION ERROR STRESS ANALYSIS ON HYPOID GEAR BY USING FINITE ELEMENT SOLUTION APPROACH

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Abstract

A computationally efficient load distribution model is proposed for face milled hypoid gears produced by Formate and generate processes.Tooth surfaces are defined directly from the cutter parameters and machine settings. A novel methodology based on the ease-off topography is used to determine the unloaded contact patterns. The proposed ease-off methodology finds the instantaneous contact curves through a surface of roll angles, allowing an accurate unloaded tooth contact analysis in a robust and accurate manner. Rayleigh-Ritz based shell models of teeth of the gear and pinion are developed to define the tooth compliances due to bending and shear effects efficiently in a semi-analytical manner. Base rotation and contact deformation effects are also included in the compliance formulations. With this, loaded contact patterns and transmission error of both face-milled and face-hobbed spiral bevel and hypoid gears are computed by enforcing the compatibility and equilibrium conditions of the gear mesh. The proposed model requires significantly less computational effort than finite elements (FE) based models, making its use possible for extensive parameter sensitivity and design optimization studies. Comparisons to the predictions of a FE hypoid gear contact model are also provided to demonstrate the accuracy of the model under various load and misalignment conditions.

The proposed ease-off formulation is generalized next to include various types of tooth surface deviations in the tooth contact analysis

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I. Introduction

1.1 **Motivation and background**

Hypoid gears are widely used in many power trains to transfer power between two non-intersecting crossed axes. Their most common and highest-volume applications can be found in front and rear axles of rear-wheel-drive or all-wheel-drive vehicles [1]. Figure 1 shows a sample of hypoid gear application for the rear axle. A rear axle has three primary functions: (i) transmit power from the drive train axis to the wheel axle, that is usually perpendicular to the drive train axis with an offset, (ii) provide the capability to the vehicle to turn corners without any slippage at its wheels through its differential, and (ii) provide the final stage of speed reduction (torque increase) that is typically of the order of three to four

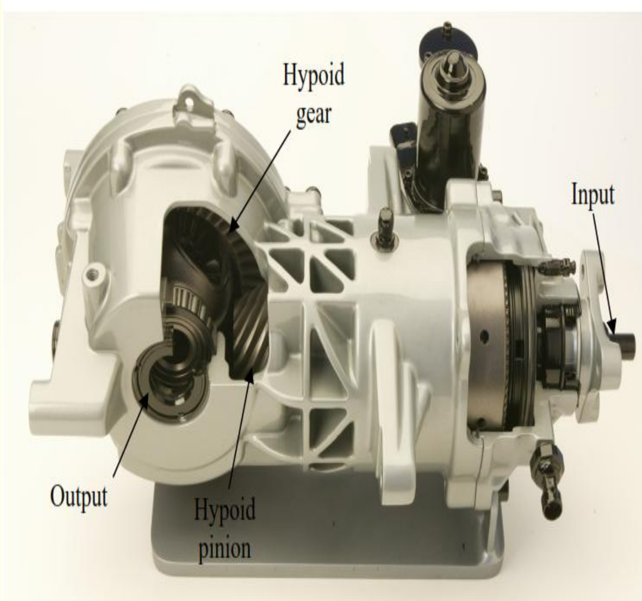


Figure 1 : A cut-away of an 'auxiliary' axle (Rear Drive Module) used in midsize passenger cars and SUV's (Courtesy of American Axle & Manufacturing Inc.).

A pair of hypoid gears is commonly used to deliver this third final drive function. In the arrangement shown in Figure , the smaller of the hypoid gears, called the pinion, is at the end of the drive shaft and is in mesh with the larger hypoid gear (called the gear).Hypoid gears can be considered as one of the most general cases of gearing basedon their geometry, such that other gear types can be obtained from it by assigning certainvalues to some of the geometric parameters [2-4]. The main function of the hypoid gear pair in a rear axle is to transmit power between two axes that are at a shaft angle (usually90^o) [1,5] and at a certain amount of shaft off-set *da* as shown in Figure . A higher level of power transmission through such a kinematic configuration is possible through use of a hypoid gear pair, which can provide a better balance amongst all primary design requirements such as strength, noise and power density. The trade-off between these performance characteristics while satisfying the kinematic constraintsresults in the hypoid tooth form that is rather complex geometrically

Modeling Methodology

The overall methodology used to develop the hypoid load distribution model is illustrated in the flowchart of Gear blank dimensions, cutter geometry,machine settings, assembly dimensions and misalignments, torque and speed are all included as input parameters for the load distribution model. These parameters are commonly put together by hypoid gear manufacturers in a standard form that is called a *special analysis file*. The pinion and gear cutter surfaces are first constructed and used to define the extended pinion and gear surfaces (including surface coordinates, normals andcurvatures) by applying fundamental equation of meshing between a gear blank and its respective cutter surfaces. These extended tooth surfaces are then trimmed in 3D spaceso that they are contained by the blanks, and transformed to a global coordinate systemwhere any misalignments in the directions of shaft offset (ΔE), pinion axis (ΔP), gear axis(ΔG) as well as the shaft angle error ($\Delta\Sigma$) can be applied. Next, ease-off and surface ofroll angle are constructed and an UTCA model is developed by bringing the toothsurfaces together andan unloaded contact pattern is defined by choosing a separation tolerance between the tooth surfaces

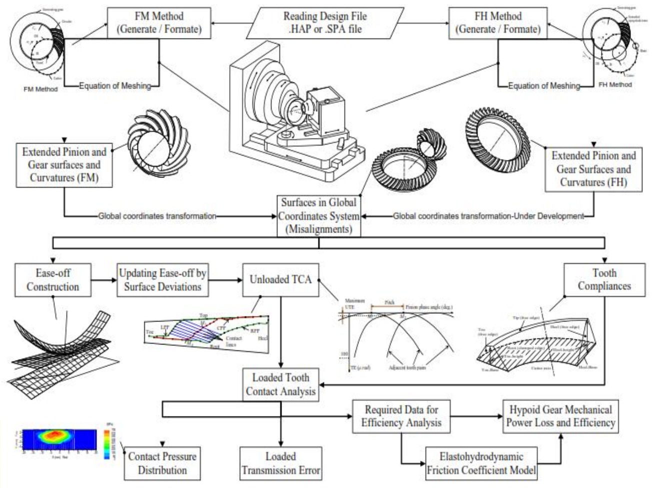
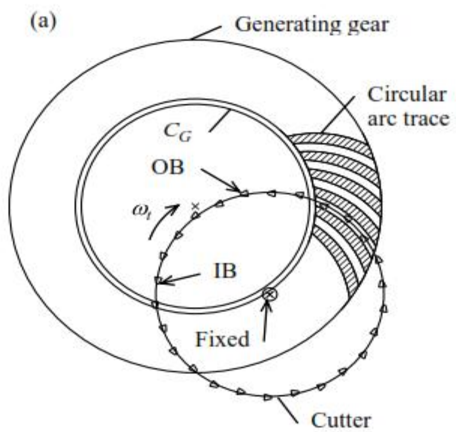


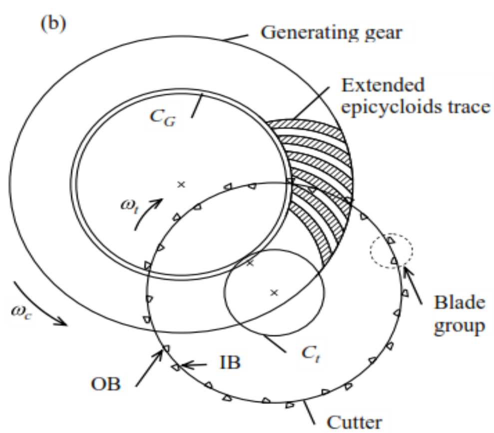
Figure : Flowchart of overall hypoid gear loaded tooth contact analysis method.

Definition of Tooth Surface Geometry

The concept of the generating gear is a key to the basic understanding of hypoidgears because this hypothetical gear can be treated as cutting tool for both the pinion andthe gear [2.5]. In a FM cutter head, blades are arranged around the cutter head axis on anequal radius for inside and outside blades (IB and OB, respectively) to form a conicalshape due to cutter axis rotation. The inside blades cut convex side of a tooth slot whilethe outside blades cut concave side of the same slot, as shown in Figure(a) . Facehobbingcutter heads (such as PENTAC® or TRI-AC®) like the one shown in Figureroll while cutting such that each set of IB-OB blades (called blade group) will passthrough a different tooth slot. The cutting process can be considered as rolling of twogears together, except the teeth of one of the gears are replaced by blade group of the cutter head. By rolling the cutter head and the gear blank together while advancing the cutter head into the blank, the gear is cut by the continuous indexing method. While theaxis of the generating gear for FM process is fixed, it is located on the centre of a circle *Ct* for FH process that rolls on the generating gear circle *CG* , as shown in Figure .(b)

Therefore, the edges of a blade in FH process traces extended epicycloids since theyusually lie on a radius that is larger than the radius of rolling circle *Ct* .





Cutting Tool Geometry and the Relative Motion

The cutting edge is divided into four different sections as the edge (or tip radius), toprem, profile and flankrem that are all shown in Figure . The edge and flankrem are usually circular arcs while toprem is usually a straight line at a slight angle from the profile section. Most of the cutting is done by the profile section of the blade that is usually a straight line or a circular arc. For a typical FM cutter, $\tau = \mu = \delta b = 0$ Referring to Figure , an arbitrary point A on cutting edge is at position r =r(s) relative to the local coordinate system X_b fixed to the cutter head (with its origin at reference point M) where s is the distance of point A to point M along the blade edge. With this, the unit tangent vector is t = dr/ds and if the cutting edge is a line, it can be

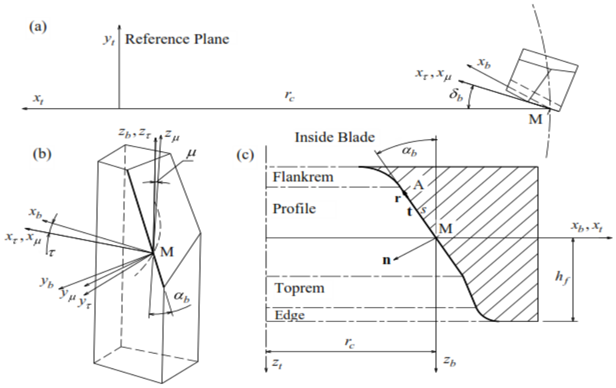


Figure : (a) Cutter head, (b) blade and (c) cutting edge geometry.

Contact Pattern and Transmission Error

Any value η , of any point on the Q surface is the pinion roll angle. As shown in Figure for a specific pinion roll angle η_i intersection of the plane z=- η_i and theQ surface defines x and y coordinates of all points on the projection plane that have the

same roll angle, stating theoretically that they lie on the same contact line/curve. SinceQ ist a plane, this intersection for hypoid gears is usually a curve rather than a straight line as assumed by most studies. The instantaneous contact curve $C(\eta_i)$ shown in Figure is determined by projecting the intersection curve first on projection plane and then projecting this projected curve once more on the ease-off surface. The minimum distance from $C(\eta_i)$ to the projection plane at point $H(\eta_i)$ is the instantaneous unloaded transmission error $TE(\eta_i)$.Moreover, moving in both directions from point $H(\eta_i)$ along $C(\eta_i)$ within a preset separation distance δ , gives the unloaded contact line length $L(\eta_i)$.Repeating this procedure for every pinion roll angle η increment, unloaded transmission error curve TE(η J)and the unloaded tooth contact pattern are computed. Here the contact curves are between real pinion and conjugate gear.

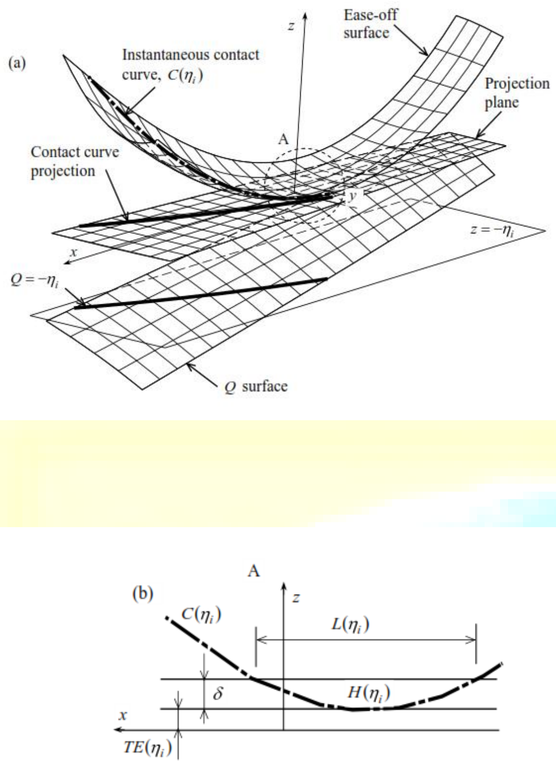


Figure : Unloaded TCA computation procedure: (a) gear projection plane, ease-off and Q surfaces, and (b) instantaneous contact curve, contact line and unloaded transmission error.

Figure : Unloaded TCA computation procedure: (a) gear projection plane, ease-off and *Q* surfaces, and (b) instantaneous contact curve, contact line and unloaded transmission error.

Example Hypoid Unloaded Tooth Contact Analysis

listed in Table 2.1 for An example hypoid gear pair whose basic parameters are the drive-side contact (concave side of pinion and convex side of gear) is considered to demonstrate the capabilities of the proposed hypoid gear geometry computation and unloaded tooth contact pattern models. This is a FM gear set representative of automotive rear axle gear sets.The predicted unloaded transmission error (UTE) curves computed by the modelfor three adjacent tooth pairs i - 1, i and i +1are shown in Figure . Here UTE is plotted against the mesh cycles (pinion roll angle) where each of the individual UTE curves corresponds to a single tooth pair in mesh. Individual curves for two adjacent tooth pairs are one mesh cycle apart. At the intersection point of the two adjacent UTE curves , transition from one tooth pair to adjacent tooth pair occurs. The transmission error value of the intersection point is the maximum UTE, which is attempted to be minimized for unloaded tooth contact pattern optimization procedures. The corresponding predicted unloaded tooth contact pattern is shown below

Parameter	**Pinion**	**Gear**
Number of teeth	11	41
Hand of spiral	Left	Right
Mean spiral angle (deg)	40.5	28.5
Shaft angle	90	90
Shaft offset	20	20
Outer cone distance	115.0	111.0
Generation type	Generate	Formate
Cutting method	FM	FM

Table 2.1 : Basic drive side geometry and working parameters of the example hypoid gear pair

Conclusion and Contributions

Computation of the contact pressure distributions is essential to every hypoid gear analysis intended to predict required functional parameters of the hypoid gear pair, including the transmission error, contact stresses, root bending stresses, fatigue life and mechanical power losses. The hypoid gear literature lacked a model to compute the load distribution accurately and efficiently without resorting to computationally demanding FE methods. The main potential reasons for that was the absence of a general and reliable formulation to define the geometry of FH and FM hypoid tooth surfaces from cutter parameters, machine motions and settings. This void, combined with the numerical difficulties in matching the tooth surfaces using the conventional methods and lack of a semi-analytical tooth compliance formulation for hypoid gears, has hampered in design, analysis and optimization of hypoid gears. This research study fills some of this void. The model proposed in this study to simulate the contacts of FM and FH hypoid gear pairs under both unloaded and loaded condition provides major enhancements to the current state of hypoid analysis. Specifically:

(i) The method using ease-off topography to compute the unloaded contact conditions is novel. It is superior to the conventional method in various aspects, including its numerical stability and computational efficiency. This method and its surface of roll angle concept is also general such that it can be applied to the other gear types.

(ii) Application of mounting errors and in clusion of global and local deviations are rather straightforward with the proposed model while these have typically been difficult or impossible tasks when the conventional methods were used

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