GOAL PROGRAMMING MODEL WITH THE DEPLOYMENT OF ATMS MACHINES RANDOM DEMAND

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Abstract

We all have experienced the discomfort of waiting in a queue. Unfortunately, this phenomenon is becoming increasingly common in urban societies with increasing population. One of the problems of ATM machines is the long queue in front of these machines which results in customer dissatisfaction. Therefore, banks should attempt to solve this huge problem, so that they could prevent distrust to electronic banking and its tools including ATMs. So, nonlinear queuing model was suggested by Passandideh and Niaki (2010) with two objectives of minimizing customer waiting time and percentage of idle time for ATM. In the present research, alinear approximation of this model is investigated to obtain a simpler and more accurate solution. This linear model has been examined with 13 numerical examples, solved by GAMS software. It was found that the process time is far less than the previous non-linear model. Based on this model and using these examples a goal programming model was introduced. Solving this linear model (by software GAMS) resulted in more optimized solutions than the previous linear model which showed improvements compared to Passandideh-Niaki model.

Keywords: Queuing theory, Automated Teller Machine (ATM), Linear-Fractional Programming (LFP), Goal Programming (GP)

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Introduction

Facility layout and location problems have been the subject of analysis since the seventeenth century (Francis et al. 1992). Even though these problems have received considerable attention over the years, it was not until the emergence of the interest in operations research and management science that the subject received renewed attention in a number of disciplines. Currently, there exists a strong interdisciplinary interest in facility layout and location problems. Mathematicians, operation researchers, architects, computer scientists, economists, engineers from several disciplines, management scientists, technical geographer, transportation system designers, regional scientists, and urban planners have discovered a commonality of interest in a concern for the layout and location of the facilities. Each brings different interpretations and different solutions to the problem. One of the objectives of the facility layout and location problem is to find the locations of the facilities in a system such that the sum of system operating costs is minimized. For example, Li et al. (1999) developed a dynamic programming model to find the location of web proxies with minimum cost. The stochastic queue median (SQM) of Berman et al. (1985) considers a mobile server such as an emergency response unit, in which in response to each demand call (e.g., patients), the available sever (e.g., ambulance) travels to the demand location to provide services. The flow-capturing model introduced by Hodgson (1990) is another closely related subject. Locating gas stations, convenience stores, and billboards are some applications of the flow-capturing model (Berman et al. 1995; Hodgson and Berman 1997), in which sometimes the server may be congested (Berman 1995). As an example, Shavandi and Mahlooji (2006) presented a fuzzy location-allocation model for congested systems. They utilized fuzzy theory to develop a queuing maximal covering location-allocation model which they called the fuzzy queuing maximal covering location-allocation model. Our model is a classical M/M/1 queuing system. In locating the facilities, we take both the customer waiting time and facility (automatic teller machine) idle time percentage into account. An automatic teller machine is a communicating device which allows the customers of a financial institution to access financial interactions with no need to human force or bank employees.

Problem definition and assumptions

Consider a facility location and server allocation problem in which the demand is stochastic and the servers are immobile with limited service capacity. In other words, there exists a service system in which the customers with uncertain demands travel to a facility with a permanent location to receive service. The automated teller machines (ATMs) and internet mirror sites are examples of the system under consideration. For these systems, the objective is to determine the optimal allocations of some business centers (customers) to the machines. The demand of each business center follows a Poisson process. Furthermore, the time of service in each ATM is assumed to follow an exponential distribution. In the assignment process of the business centers to ATMs, two objectives may exist; (1) minimizing the expected total time of the business center representatives that travel to the machines plus their waiting time at the ATM, and (2) the minimization of the ATM idle times.

Problem modeling

The parameters and the variables of the model are:

m: Number of customer nodes.

n: Number of potential facility node

t_{ij} : The travelling time from customer i to facility node j, $i = 1, \ldots, m$ and $j = 1, \ldots, n$.

$T = [t_{ij}]$: The travelling time matrix.

λi : Demand rate of service requests from customer node i, $i = 1, \ldots, m$.

μ: The common service rate of each server.

Y_j : Demand rate at open facility j, $j = 1, \ldots, n$.

w_j : Expected waiting time of customers assigned to facility node j, $j = 1, \ldots, n$.

$\pi_{0\,j}$: The probability of the server being idle at open

Facility (idle probability) j, $j = 1, \dots, n$.

z_1 : Sum of traveling and waiting time.

z_2 : Average idle probabilities for all facilities.

$$y_j = \begin{cases} 1 & : \text{if the facility is opened at node } j, \\ 0 & : \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & : \text{if customer i is assigned to facility } j \;, \\ 0 & : \text{otherwise} \end{cases}$$

Then, the aggregate travelling time of the customers per unit time (T1) is obtained by:

$$1)\; T_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_i t_{ij} x_{ij}$$

Since an open facility behaves like an M/M/1 queue (Gross and Harris 1998), the expected waiting time at an open facility site j is $W_j = \frac{1}{\mu - Y_j}$ where $Y_j = \sum_{i=1}^{m} \lambda_i x_{ij}$. Hence, the average waiting time of customers per unit time (T_2) is:

$$2)\; T_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_i W_j x_{ij} = \sum_{j=1}^{n} \frac{Y_j}{\mu - Y_j}$$

Thus, the first objective is the sum of traveling and waiting time ($z_1 = T_1 + T_2$) that must be minimized.

According to the characteristics of an M/M/1 queue, the idle probability at open facility j is $\Pi_{0j} = \frac{1-Y_j}{\mu}$. Hence, we need to minimize z_2 as the average of $\pi_{0\,j}$ s for all j. This will be the second objective of the model. In short, the mathematical programming model of the problem at hand becomes:

$$3)\ Min\ Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_i t_{ij} x_{ij} + \sum_{j=1}^{n} \frac{Y_j}{\mu - Y_j}$$

$$MinZ_2 = \left(\frac{\sum_{j=1}^{n} (1 - (\frac{Y_j}{\mu})}{\sum_{j=1}^{n} y_j} \right)$$

s.t:

$$\sum_{j=1}^{n} y_j \le K$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad , i = 1, \dots, m$$

$$x_{ij} \le y_j \quad , i = 1, \dots, m \ , \quad j = 1, \dots, n$$

$$\sum_{i=1}^{m} \lambda_i x_{ij} \ \le \mu \quad , j = 1, \dots, n$$

$$Y_j = \sum_{i=1}^{m} \lambda_i x_{ij} \quad , j = 1, \dots, n$$

$$y_j \in \{ 0,1 \} \ , \quad x_{ij} \in \{ 0,1 \} \ \ , \mathrm{i} = 1, \dots, \mathrm{m} \quad , \mathrm{j} = 1, \dots, \mathrm{n}$$

The first constraint of model (3) sets an upper limit for the maximum number of open facilities. The second and the third constraints ensure that each customer demand is satisfied by only one open facility. The fourth constraint guarantees the input to each server to be less than its capacity. Finally, the fifth constraint is the input of each server (Pasandideh-Niaki, 2010).

So, there are two objectives in the model described above and it needs interpretations with optimizing multiple objective decision making. On the other hand, this model is Linear-Fractional Programming (Changing Variables). To transform a nonlinear model to a linear one, we can use the features of fractional models.

Linear-Fractional Programming (LFP)

Linear-fractional programming model is a model which objective function is composed of a division of two first order equations with linear restrictions. Therefore, the model discussed here is a fractional programming model. (Mehregan, Data Envelopment Analysis, 2008).

In other words, the objective function in programming is a ratio of two linear (non-linear) functions. Transforming this model to linear programming needs two times of changing variables as follows.

Linear Models

The model can be changed by Changing Variables the main issue in the general expressed as follows :

$$4)\ MinZ_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_i t_{ij} x_{ij} + \frac{1}{\mu}\sum_{j=1}^{n} Y_j \quad : \quad \left|\frac{Y_j}{\mu}\right| \le 1$$

$$MinZ_2 = \frac{1}{t_0}\sum_{j=1}^{n}\left(1 - \left(\frac{Y_j}{\mu}\right)\right)$$

s.t:

$$\sum_{j=1}^{n} y_j = t_0$$

$$\sum_{i=1}^{m} \lambda_i x_{ij} \le \mu \quad , j = 1, \dots, n$$

$$Y_j = \sum_{i=1}^{m} \lambda_i x_{ij} \quad , j = 1, \dots, n$$

$$y_j \in \{ 0,1 \} \quad , \qquad x_{ij} \in \{ 0,1 \} \quad , \mathrm{i} = 1, \dots, \mathrm{m} \quad , \mathrm{j} = 1, \dots, \mathrm{n}$$

And it can be seen that the model has transferred to a linear equation.

Examples of Solving Linear Models

After linearization, some numerical examples will be solved using *GAMS* software which is the first software in operational research for solving linear and non-linear problems. The results for 13 models are presented in table 1:

Table1. Solving linear equation for different numerical values

model	m	n	μ	K	t_0	Z_1^*	Z_2^*	time for solution(s)
1	3	4	5	2	1,2	1.4	1.8	4.290
2	14	20	20	8	1,2,...,8	4.35	2.45	2.265
3	30	20	17	5	1,2,...,5	14.2941	3.8	2.462
4	30	20	35	12	1,2,...,12	10.2	1.6333	2.585
5	40	30	35	20	1,2,...,20	9.2857	1.4714	2.230
6	45	35	35	15	1,2,...,15	9.3429	2.2914	7.987
7	50	40	52	7	1,2,...,7	16.1346	5.6484	2.464
8	55	45	60	6	1,2,...,6	26.1	7.4250	2.621
9	60	55	70	6	1,2,...,6	23.08857	9.0976	2.669
10	65	55	60	8	1,2,...,8	21.1333	6.8104	2.496
11	100	65	120	8	1,2,...,8	48.0667	8.0750	2.277
12	150	65	150	15	1,2,...,15	32.1	4.3009	3.292
13	150	100	150	7	1,2,...,7	136.0467	16.5856	6.303

Goal Programming

Goal programming, a method for multi-objective decision making, has been presented by Charnes and Cooper in 60s and developed by Ignizo and Lee. Goal programming is the first technique for multi-objective function which has been exclusively accepted in industry and services (Mehregan, 2007) . Like other problems, goal programming modeling can be formulated in different forms of linear, non-linear or integers.

If the management wants to achieve the goal and not less than that, variable d^- is ommited in corresponding restriction and $P_k d^+$ comes in the objective function. And if the management wants to achieve the goal and not more than that, d^+ is ommited in corresponding restriction and $P_k d^-$ comes in the objective function. The latter is the case for the present research and therefore, the objective function and goal restriction are:

Goal restriction: $\sum_{j=1}^{n} C_{ij} X_j + d_i^- - d_i^+ = b_i$, Objective function: $P_k d^+$

Therefore, in this model the possibility of not achieving the goal is minimized while exeeding the goal is not allowed. (Minimum d_i^-, while $d_i^+ = 0$).

In the last section, approximate optimized values of Z_1^*, Z_2^*were found using the linearized model. Since these values are approximate, so they should be made exact. One method is goal programming and let them (Z_1^*, Z_2^*) change in a certain range using goal programming and in that range, meeting the constraints be more acceptable. The goal is minimizing $Z_{1,} Z_2$ and in the last section we found Z_1^*, Z_2^*. $Z_1 \le Z_1^*$ and $Z_2 \le Z_2^*$ because:

1. Find less values of $Z_{1,} Z_2$, so that a better minimum is found.
2. This model is aimedat minimizing the objective functions.

Base on these explanations, the restriction and objective function are:

Objective function : $Minz = P_1(d_1^-), P_2(d_2^-)$

First goal: $Z_1 + d_1^- = Z^*{}_1$

Second goal: $Z_2 + d_2^- = Z^*{}_2$

So, the goal model would be:

5) $\mathrm{Minz} = \mathrm{P_1}(\mathrm{d_1^+} + \mathrm{d_1^-}), \mathrm{P_2}(\mathrm{d_2^+} + \mathrm{d_2^-})$

s.t:

$\mathrm{Z_1} - \mathrm{d_1^+} + \mathrm{d_1^-} = \mathrm{Z^*{}_1}$

$\mathrm{Z_2} - \mathrm{d_2^+} + \mathrm{d_2^-} = \mathrm{Z^*{}_2}$

$$\sum_{j=1}^{n} y_j = t_0$$

$$\sum_{i=1}^{m} \lambda_i x_{ij} \leq \mu \quad , j = 1, \dots , n$$

$$Y_j = \sum_{i=1}^{m} \lambda_i x_{ij} \quad , j = 1, \dots , n$$

$$d_1^+ . d_1^- = 0$$

$$d_2^+ . d_2^- = 0$$

$$y_j \in \{ 0,1 \} \quad , \qquad x_{ij} \in \{ 0,1 \} \quad , i = 1, \dots , m \quad , j = 1, \dots , n$$

Solving the goal model for 13 models presented in the last example

After programming and modeling the equation above with GAMS, the responses are:

Table 2. A summary of calculations for goal programming

model	Z_1^*	Z_2^*	d_1^-	d_2^-	Z_1	Z_2	Result
1	1.4	1.8	.4000	0	1	1.8	further opitmized response
2	4.35	2.45	.3500	0	4	2.45	further opitmized response
3	14.2941	3.8	0	0	14.2941	3.8	unchanged response
4	10.2	1.6333	0	0	10.2	1.6333	unchanged response
5	9.2857	1.4714	0	0	9.2857	1.4714	unchanged response
6	9.3429	2.2914	0	0	9.3429	2.2914	unchanged response
7	16.1346	5.6484	.1346	0	16	5.6484	further opitmized

							response
8	26.1	7.4250	.1000	0	26	7.4250	further opitmized response
9	23.0857	9.0976	0	0	23.0857	9.0976	unchanged response
10	21.1313	6.8104	0	0	21.1313	6.8104	further opitmized response
11	48.0667	8.0750	.0667	0	48	8.0750	further opitmized response
12	32.1	4.3009	.1000	0	32	4.3009	further opitmized response
13	136.0467	16.5856	.0470	0	135.9997	16.5856	further opitmized response

Conclusion

It can be easily inferred from Table4 that the time needed for solving linear models with GAMS is much less than the models solved with Lingo and Genetics softwares (less than one minute in all models).

Table 4. comparing linear and non-linear solutions

Model	time needed for solving non-linear models (min)		time needed for solving linear models (second)
	Genetics	Lingo	
1	0	0	4.290
2	.4	1.80	2.265
3	.53	3.13	2.462

4	.74	10.3	2.585
5	1.21	31.3	2.230
6	1.75	41.2	7.987
7	2.52	54.5	2.464
8	2.75	63.5	2.621
9	3.02	72.1	2.669
10	3.41	95.6	2.496
11	5.12	515	2.277
12	40.06	-	3.292
13	not performed	not performed	6.303

After solving the linearized model in section 5, goal programming was applied to obtain better results compared to linearized model. A summary of these calculations has been presented in Table2. Analyzing this table can answer the main query of this reseach. The non-linear Pasandideh-Niaki model is less accurate than the linear model (because it is solved non-linearly) and also, it has a longer process time compared to the method presented in the present research work. On the other hand, the solutions after goal programming in the present model are more optimized or at least not worse due to features of goal programming. Therefore, it can be concluded that the solutions of goal programming are more optimized than non-linear Pasandideh-Niaki model and also, less time is needed for processing and solving the method.

Sugesstions

It is suggested that models should be developed to find approximate solution with acceptable accuracy without linear approximation, so that there would be no need to linearization and approximation of the model.

The next suggestion is that this model (algorithm) can be changed so that the applications become more and With its simplicity and ease of use allow it to design a software banks managers and Credit financial institutions to achieve their goals in minimizing customer waiting time and Unemployment and the percentage of ATMs Machines.

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