SOLUTION OF BURGER'S EQUATION BY HOMOTOPY PERTURBATION TRANSFORM METHOD

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ABSTRACT

In this paper, Homotopy Perturbation Transform Method (HPTM) is employed to approximate the solution of the Burgers equation which is a one-dimensional non-linear partial differential equation in fluid dynamics. This method is a combined form of the Laplace Transform method with the Homotopy Perturbation method. The nonlinear terms can be easily handled by the use of He's Polynomials. The explicit solutions obtained were compared with the exact solutions. The method finds the solution without any discretization or restrictive assumptions and free from round-off errors and therefore reduce the numerical computation to a great extent. The results reveal that the HPTM is very effective, convenient and quite accurate to systems of partial differential equations. It is predicted that the HPTM can be found widely applicable in engineering.

Keywords: Burgers equation, Nonlinear partial differential equation, Homotopy-Perturbation Transform Method (HPTM), He's Polynomial, Laplace Transform, Inverse Laplace Transform

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INTRODUCTION

The one –dimensional Burger's equation[8] given by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}-\varepsilon\frac{\partial^2 u}{\partial x^2}=0\,,\,0<x<1\,,\,t>0 \qquad (1)$$

Was introduced by Bateman and later by Burger. The equation is the one-dimensional quasi-linear parabolic partial differential equation which appears as a mathematical model of many physical events, such as gas dynamic, turbulence and shock wave theory. In recent years, many researchers have used various numerical methods specifically based on finite-difference, finite element boundary element technique and spectral method to solve the Burger's equation.The distinctive feature of equation (1) is that it is the simplest mathematical formulation of the competition between nonlinear advection and the viscous diffusion. It contain the simplest form of nonlinear advection term $u\,u_x$ and dissipation term $\varepsilon\,u_{xx}$ where $\varepsilon = 1/\mathrm{Re}$ (ε is kinematics viscosity and Re is Reynolds number) for simulating the physical phenomena of wave motion and thus determines the behaviour of the solution. It is well known that the exact solution of Burger's equation can be computed only for restricted values of kinematics viscosity. The mathematical properties of Eq.(1) have been studied by Cole (1951) [3]. Here a approximate solution of Burger's equation is solved by using Homotopy Perturbation Transform Method for a suitable initial and boundary conditions. Many numerical solutions for Eq. 1 have been adopted over the years. Finite element techniques have been employed frequently. For example, Varoglu and Finn (1980) presented an iso-parametric space Btime finite-element approach for solving Burgers equation, utilizing the hyperbolic differential equation associated with Burgers equation. Another approach which has been used by Caldwell et at (1981) is the finite-element method such that by altering the size of the element at each stage using information from the previous steps. Caldwell etal. (1980) give an indication of how complementary variational principles can be applied to Burgers equation. Later, Saunders et al. (1984) have demonstrated how a variational-iterative scheme based on complementary variational principles can be applied to non-linear partial differential equations and the test problem chosen is the steady-state version of Burgers equation Ozis and Ozdes (1996) [7] applied a direct variational method to generate limited form of the solution of Burgers equation. Ozis etal. (2003) applied a simple finite-element approach with linear elements to Burgers equation reduced by Hopf-Cole transformation. Aksan and Ozdes

(2004) [1] have reduced Burgers equation to the system of non-linear ordinary differential equations by discretization in time and solved each non-linear ordinary differential equation by Galerkin method in each time step.

Here, the reduced Burgers equation is solved by homotopy perturbation transform method and compared with exact solution. It is well-known that the HPTM converge very fast to the results. Moreover, contrary to the conservative methods which require the initial and boundary conditions, the HPTM provide an analytical solution by using only the initial conditions. The boundary conditions can be used only to justify the obtained result. A comparison will be made to show that the method is equally able to arrive at exact solutions of Burgers' equation.

BASIC IDEAS OF HOMOTOPY PERTURBATION TRANSFORM METHOD

To illustrate the basic idea of this method [6,4],we consider a general nonlinear non-homogeneous partial differential equation with initial conditions of the form

$$\begin{aligned} &Du(x,t)+Ru(x,t)+Nu(x,t)=g(x,t) \\ &u(x,0)=h(x), \quad u_t(x,0)=f(x) \end{aligned} \qquad (2)$$

Where D is the second order linear differential operator $D=\frac{\partial^2}{\partial t^2}$, R is the linear differential operator of less order than D, N represent the general non-linear differential operator and $g(x,t)$ is the source term. Taking the Laplace transform (denoted by L) on both side of Eq(1):

$$L[Du(x,t)]+L[Ru(x,t)]+L[Nu(x,t)]=L[g(x,t)] \qquad (3)$$

Using the differentiation property of the Laplace transform,[5] we have

$$L[u(x,t)]=\frac{h(x)}{s}+\frac{f(x)}{s^2}-\frac{1}{s^2}L[Ru(x,t)]+\frac{1}{s^2}L[g(x,t)]-\frac{1}{s^2}L[Nu(x,t)] \qquad (4)$$

Operating with the Laplace inverse on both side of Eq.(4) gives

$$u(x,t)=G(x,t)-L^{-1}\left[\frac{1}{s^2}L\left[Ru(x,t)+Nu(x,t)\right]\right] \qquad (5)$$

Where $G(x,t)$ represent the term arising from the source term and the prescribed initial conditions. Now, we apply the homotopy perturbation method [9,10]

$$u(x,t)=\sum_{n=0}^{\infty}p^n u_n(x,t) \tag{6}$$

And the nonlinear term can be decomposed as

$$Nu(x,t)=\sum_{n=0}^{\infty}p^n H_n(u) \tag{7}$$

For some He's polynomial H_n that are given by

$$H_n(u_0......u_n)=\frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N(\sum_{i=0}^{\infty}(p^i u_i)\right]_{p=0},\ n=0,1,2,3,.....$$

Substituting Eqs. (7) and (6) in Eq. (5) we get

$$\sum_{n=0}^{\infty}p^n u_n(x,t)=G(x,t)-p\left(L^{-1}\left[\frac{1}{s^2}L\left[R\sum_{n=0}^{\infty}p^n u_n(x,t)+\sum_{n=0}^{\infty}p^n H_n(u)\right]\right]\right) \tag{8}$$

This is the coupling of the Laplace transform and the homotopy perturbation method using He's polynomial. Comparing the coefficient of like powers of p, the following approximations are obtained [4,7]

$$p^0: u_0(x,t)=-\frac{1}{s^2}L\left[Ru_0(x,t)+H_0(u)\right]$$

$$p^1: u_1(x,t)=-\frac{1}{s^2}L\left[Ru_1(x,t)+H_0(u)\right]$$

$$p^2: u_2(x,t)=-\frac{1}{s^2}L\left[Ru_1(x,t)+H_1(u)\right] \tag{9}$$

$$p^3: u_3(x,t)=-\frac{1}{s^2}L\left[Ru_2(x,t)+H_2(u)\right]$$

The best approximations for the solutions are

$$u=\lim_{p\to 1}u_n=u_0+u_1+u_2+....$$

STATEMENT OF THE PROBLEM-1

Consider the Burger's equation (1) with the initial condition and homogeneous boundary condition

$$u(x,0)=\sin\pi x,\ 0<x<1 \tag{10}$$

$$u(0,t)=u(1,t)=0\ ,\ t>0 \qquad (11)$$

The exact solution[3] of the Burger's equation (1) with initial condition (10) and boundary conditions (11) is obtained as

$$u(x,t)=2\pi\varepsilon\frac{\sum_{n=1}^{\infty}a_n e^{-n^2\pi^2\varepsilon t}n\sin(n\pi x)}{a_0+\sum_{n=1}^{\infty}a_n e^{-n^2\pi^2\varepsilon t}n\cos(n\pi x)} \qquad (12)$$

Where a_0 and a_n (n=1,2,….) are Fourier coefficients and they are defined by the following equations, respectively

$$a_0=\int_0^1 \exp\left\{-(2\pi\varepsilon)^{-1}\left[1-\cos(\pi x)\right]\right\}dx \qquad (13)$$

$$a_n=2\int_0^1 \exp\left\{-(2\pi\varepsilon)^{-1}\left[1-\cos(\pi x)\right]\right\}\cos(n\pi x)\,dx \qquad (n=1,2,3......) \qquad (14)$$

Solution Procedure

Here solution is obtained by applying Homotopy Perturbation Transform Method on equation (1)

Taking Laplace Transform on both sides subject to the initial condition ,we get

$$L\left[\frac{\partial u}{\partial t}\right]=L\left[\varepsilon\frac{\partial^2 u}{\partial x^2}\right]-L\left[u\frac{\partial u}{\partial x}\right] \qquad (15)$$

This can be written as

$$\left[su(x,s)-u(x,0)\right]=\varepsilon L\left[\frac{\partial^2 u}{\partial x^2}\right]-L\left[u\frac{\partial u}{\partial x}\right] \qquad (16)$$

On applying the above specified initial condition we get

$$su(x,s)-\sin\pi x=\varepsilon L\left[\frac{\partial^2 u}{\partial x^2}\right]-L\left[u\frac{\partial u}{\partial x}\right] \qquad (17)$$

$$u(x,s)=\frac{1}{s}(\sin\pi x)+\varepsilon\frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right]-\frac{1}{s}L\left[u\frac{\partial u}{\partial x}\right] \qquad (18)$$

Taking Inverse Laplace Transform on both side we get

$$L^{-1}\left[u(x,t)\right]=L^{-1}\left[\frac{1}{s}(\sin\pi x)\right]+L^{-1}\left[\varepsilon\frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right]\right]-L^{-1}\left[\frac{1}{s}L\left[u\frac{\partial u}{\partial x}\right]\right] \qquad (19)$$

$$u(x,t)=(\sin\pi x)\,L^{-1}\left[\frac{1}{s}\right]+L^{-1}\left[\varepsilon\frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right]-\frac{1}{s}L\left[u\frac{\partial u}{\partial x}\right]\right] \qquad (20)$$

Now we apply the homotopy perturbation method in the form

$$u(x,t)=\sum_{n=0}^{\infty}p^n\left(u_n\left(x,t\right)\right) \qquad (21)$$

Equation (20), now reduces to

$$\sum_{n=0}^{\infty}p^n\left(u_n\left(x,t\right)\right)=\sin\pi x[1]+p\,L^{-1}\left[\varepsilon\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}p^n\left(u_n\left(x,t\right)\right)\right)_{xx}\right]\right]$$
$$-p\,L^{-1}\left[\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}p^n\left(u_n\left(x,t\right)\right)\right)\left(\sum_{n=0}^{\infty}p^n\left(u_n\left(x,t\right)\right)\right)_{x}\right]\right] \qquad (22)$$

This can be written in expanded form as

$$u_0+pu_1+p^2u_2+p^3u_3+....=\left(\sin\pi x\right)+pL^{-1}\left[\varepsilon\frac{1}{s}L\left[\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_2}{\partial x^2}+\frac{\partial^2 u_3}{\partial x^2}+....\right)\right]\right]$$
$$-pL^{-1}\left[\frac{1}{s}L\left[\left(u_0+pu_1+p^2u_2+p^3u_3+....\right)\left(\frac{\partial u_0}{\partial x}+\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial x}+\frac{\partial u_3}{\partial x}+...\right)\right]\right] \qquad (23)$$

Comparing the coefficient of various power of p, we get

$$p^0: u_0(x,t)=\sin\pi x$$

$$p^1: u_0(x,t)=L^{-1}\left[\varepsilon\frac{1}{s}L\left[\left(\frac{\partial^2 u_0}{\partial x^2}\right)\right]\right]-L^{-1}\left[\frac{1}{s}L\left[\left(u_0\frac{\partial u_0}{\partial x}\right)\right]\right]$$

$$p^2: u_2\left(x,t\right)=L^{-1}\left[\varepsilon\frac{1}{s}L\left[\left(\frac{\partial^2 u_1}{\partial x^2}\right)\right]\right]-L^{-1}\left[\frac{1}{s}L\left[\left(u_1\frac{\partial u_0}{\partial x}+u_0\frac{\partial u_1}{\partial x}\right)\right]\right]$$

$$p^3: u_3\left(x,t\right)=L^{-1}\left[\varepsilon\frac{1}{s}L\left[\left(\frac{\partial^2 u_2}{\partial x^2}\right)\right]\right]-L^{-1}\left[\frac{1}{s}L\left[\left(u_2\frac{\partial u_0}{\partial x}+u_1\frac{\partial u_1}{\partial x}+u_0\frac{\partial u_2}{\partial x}\right)\right]\right] \qquad (24)$$

Proceeding in a similar manner, we get

$$p^4: u_4\left(x,t\right)=L^{-1}\left[\varepsilon\frac{1}{s}L\left[\left(\frac{\partial^2 u_3}{\partial x^2}\right)\right]\right]-L^{-1}\left[\frac{1}{s}L\left[\left(u_3\frac{\partial u_0}{\partial x}+u_2\frac{\partial u_1}{\partial x}+u_1\frac{\partial u_2}{\partial x}+u_0\frac{\partial u_3}{\partial x}\right)\right]\right]$$

$$p^5: u_5\left(x,t\right)=L^{-1}\left[\varepsilon\frac{1}{s}L\left[\left(\frac{\partial^2 u_4}{\partial x^2}\right)\right]\right]-L^{-1}\left[\frac{1}{s}L\left[\left(u_4\frac{\partial u_0}{\partial x}+u_3\frac{\partial u_1}{\partial x}+u_2\frac{\partial u_2}{\partial x}+u_1\frac{\partial u_3}{\partial x}+u_0\frac{\partial u_4}{\partial x}\right)\right]\right]$$

and so on

Then the solution $u(x,t)$ in series form is given by

$$u(x,t)=\sin(\pi x)+\left(-\varepsilon\pi^2\sin(\pi x)-\pi\cos(\pi x)\sin(\pi x)\right)t$$
$$+\left(\frac{1}{2}\right)\left(\pi^2\sin(\pi x)\begin{pmatrix}\pi^2\varepsilon^2+6\pi\varepsilon\cos(\pi x)\\ +2\cos(\pi x)^2-\sin(\pi x)^2\end{pmatrix}\right)t^2 \tag{25}$$
$$+\left(-\frac{1}{6}\right)\left(\pi^3\sin(\pi x)\begin{pmatrix}\pi^3\varepsilon^3+28\pi^2\varepsilon^2\cos(\pi x)+36\pi\varepsilon\cos(\pi x)^2\\ +6\cos(\pi x)^3-15\pi\varepsilon\sin(\pi x)^2-\\ 10\cos(\pi x)\sin(\pi x)^2\end{pmatrix}\right)t^3$$

STATEMENT OF PROBLEM-2

Let us consider one-dimensional Burger's equation of the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}-v\frac{\partial^2 u}{\partial x^2}=0 \qquad 0<x<1 \tag{26}$$

with initial and boundary condition as

$$u(x,0)=\frac{2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)}{1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x} \tag{27}$$

$$u(0,t)=u(1,t)=0 \tag{28}$$

The analytical solution takes the form [Cole, 1951][3].

$$u(x,t)=\frac{2\pi v\left(A_1\exp\left(-v\pi^2 t\right)\sin\pi x+2A_2\exp\left(-4v\pi^2 t\right)\sin 2\pi x\right)}{A_0+A_1\exp\left(-v\pi^2 t\right)\cos\pi x+A_2\exp\left(-4v\pi^2 t\right)\cos 2\pi x} \tag{29}$$

With constant A_0, A_1, A_2 and v be given as follows:

$$A_0=v=1 \quad A_1=\frac{1}{4} \quad A_2=\frac{1}{2}$$

Solution Procedure

Applying the same procedure again on equation (26) subject to the initial condition we get

$$L\left[\frac{\partial u}{\partial t}\right]=L\left[v\frac{\partial^2 u}{\partial x^2}\right]-L\left[u\frac{\partial u}{\partial x}\right] \tag{30}$$

This can be written on applying the above specified initial condition as

$$u(x,s)=\frac{1}{s}\left(\frac{2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)}{1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x}\right)+v\frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right]-\frac{1}{s}L\left[u\frac{\partial u}{\partial x}\right] \tag{31}$$

Taking Inverse Laplace Transform on both side , we get

$$L^{-1}[u(x,s)]=L^{-1}\left[\frac{1}{s}\left(\frac{2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)}{1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x}\right)\right]+vL^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right]\right]-L^{-1}\left[\frac{1}{s}L\left[u\frac{\partial u}{\partial x}\right]\right] \tag{32}$$

$$u(x,t)=\left(\frac{2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)}{1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x}\right)L^{-1}\left[\frac{1}{s}\right]+vL^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right]\right]-L^{-1}\left[\frac{1}{s}L\left[u\frac{\partial u}{\partial x}\right]\right] \tag{33}$$

Now on applying the homotopy perturbation method in the form

$$u(x,t)=\sum_{n=0}^{\infty}p^n\left(u_n(x,t)\right)$$

Equation (33) can be reduces to

$$\sum_{n=0}^{\infty}p^n\left(u_n(x,t)\right)=\left(\frac{2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)}{1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x}\right)[1]+pL^{-1}\left[v\frac{1}{s}L\left[\left(\sum_{n=0}^{\infty}p^n\left(u_n(x,t)\right)\right)_{xx}\right]\right]- \\ pL^{-1}\left[\frac{1}{s}L\left[\left(\left(\sum_{n=0}^{\infty}p^n\left(u_n(x,t)\right)\right)\right)\left(\sum_{n=0}^{\infty}p^n\left(u_n(x,t)\right)\right)_x\right]\right] \tag{34}$$

On expansion of equation (34) and comparing the coefficient of various powers of p , we get

$$p^0: u_0(x,t)=\frac{2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)}{1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x} \tag{35}$$

$$p^1 : u_0(x,t) = L^{-1}\left[v\frac{1}{s}L\left[\left(\frac{\partial^2 u_0}{\partial x^2}\right)\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\left(u_0\frac{\partial u_0}{\partial x}\right)\right]\right]$$

$$p^2 : u_2\left(x,t\right) = L^{-1}\left[v\frac{1}{s}L\left[\left(\frac{\partial^2 u_1}{\partial x^2}\right)\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\left(u_1\frac{\partial u_0}{\partial x} + u_0\frac{\partial u_1}{\partial x}\right)\right]\right]$$

$$p^3 : u_3\left(x,t\right) = L^{-1}\left[v\frac{1}{s}L\left[\left(\frac{\partial^2 u_2}{\partial x^2}\right)\right]\right] - L^{-1}\left[\frac{1}{s}L\left[\left(u_2\frac{\partial u_0}{\partial x} + u_1\frac{\partial u_1}{\partial x} + u_0\frac{\partial u_2}{\partial x}\right)\right]\right]$$

Further values can be obtained in similar manner

Then the solution $u(x,t)$ in series form is given by

$$u(x,t)=\frac{2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)}{1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x}+$$

$$\left(-\left(2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)\left(\frac{2\pi\left(\frac{1}{4}\pi\cos\pi x+2\pi\cos 2\pi x\right)}{1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x}-\frac{2\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)\left(-\frac{1}{4}\pi\sin\pi x-\pi\sin 2\pi x\right)}{\left(1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x\right)^2}\right)\right)\right.$$

$$\left./\left(1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x\right)+v\left(-\frac{2\pi\left(-\frac{1}{4}\pi^2\cos\pi x-2\pi^2\cos 2\pi x\right)\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)}{\left(1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x\right)^2}-\frac{4\pi\left(\frac{1}{4}\pi\cos\pi x+2\pi\cos 2\pi x\right)\left(-\frac{1}{4}\pi\sin\pi x-\pi\sin 2\pi x\right)}{\left(1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x\right)^2}+\frac{4\pi\left(\frac{1}{4}\sin\pi x+\sin 2\pi x\right)\left(-\frac{1}{4}\pi\sin\pi x-\pi\sin 2\pi x\right)^2}{\left(1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x\right)^3}+\frac{2\pi\left(-\frac{1}{4}\pi^2\sin\pi x-4\pi^2\sin 2\pi x\right)}{\left(1+\frac{1}{4}\cos\pi x+\frac{1}{2}\cos 2\pi x\right)}\right)\right) \quad (36)$$

RESULTS AND DISCUSSION

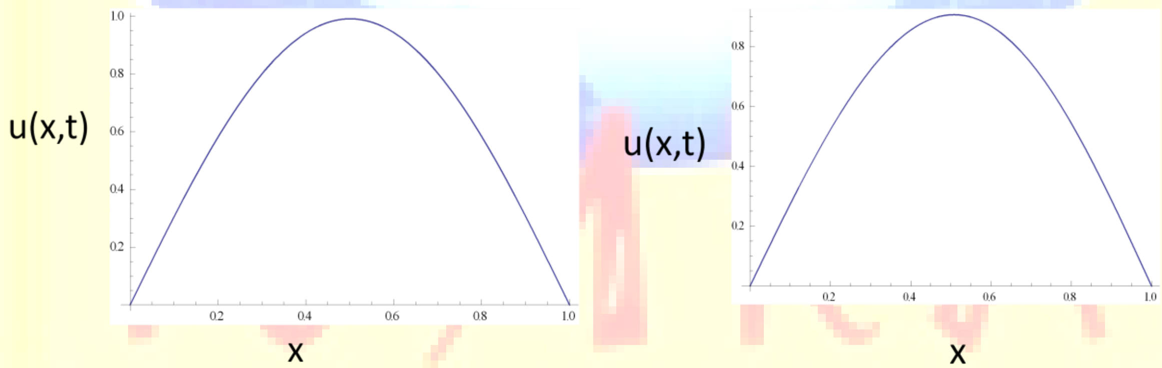
Here an approximate solution is obtained of Eqn. (1) and (26) and compared with the exact solution to accentuate the accuracy of the present method for moderate size viscosity values, results are summarized in Table 1-2 which shows that solution are in good agreement with exact solution.

$\varepsilon = 0.05$					$\varepsilon = 1$				
t=0.001			t=0.01		t=0.001			t=0.01	
x	**Exact**	**HPTM**	**Exact**	**HPTM**	**x**	**Exact**	**HPTM**	**Exact**	**HPTM**
0.1	0.30795	0.307945	0.29865	0.298646	0.1	0.30509	0.305088	0.27324	0.273253
0.2	0.58601	0.586006	0.57044	0.570442	0.2	0.58057	0.580565	0.52156	0.521581
0.3	0.80713	0.807126	0.79034	0.790338	0.3	0.77762	0.799621	0.72185	0.721858
0.4	0.94966	0.949662	0.93696	0.936963	0.4	0.94082	0.940817	0.85459	0.854583
0.5	0.99950	0.999502	0.99460	0.994596	0.5	0.99018	0.990174	0.90571	0.905702
0.6	0.95151	0.951506	0.95513	0.955133	0.6	0.94261	0.942609	0.86833	0.868327
0.7	0.81011	0.81011	0.81976	0.819756	0.7	0.80252	0.802522	0.74410	0.744098
0.8	0.58899	0.58899	0.59988	0.599883	0.8	0.58347	0.583466	0.54382	0.543824
0.9	0.30979	0.309789	0.31686	0.316853	0.9	0.30688	0.306881	0.28700	0.287002

Table 1: Comparison of HPTM solution of (Problem-1) with exact solution for $\varepsilon = 0.05$ **and** $\varepsilon = 1$ **at different time levels**

$\varepsilon=1$				
	t=0.001		t=0.01	
x	**Exact**	**HPTM**	**Exact**	**HPTM**
0.1	2.48162	2.48162	1.96798	1.96794
0.2	4.93668	4.93668	3.77756	3.77738
0.3	7.02391	7.02391	5.03188	5.03601
0.4	7.31402	7.31402	4.81818	4.89473
0.5	2.99481	2.99481	2.14628	2.24919
0.6	-3.87346	-3.87346	-1.72645	-1.88938
0.7	-6.35443	-6.35443	-3.76949	-3.78399
0.8	-5.09368	-5.09368	-3.46417	-3.46367
0.9	-2.66127	-2.66127	-1.93801	-1.93802

Table 1: Comparison of HPTM solution of (Problem-2) with exact solution for $\varepsilon=0.05$ **and** $\varepsilon=1$ **at different time levels**



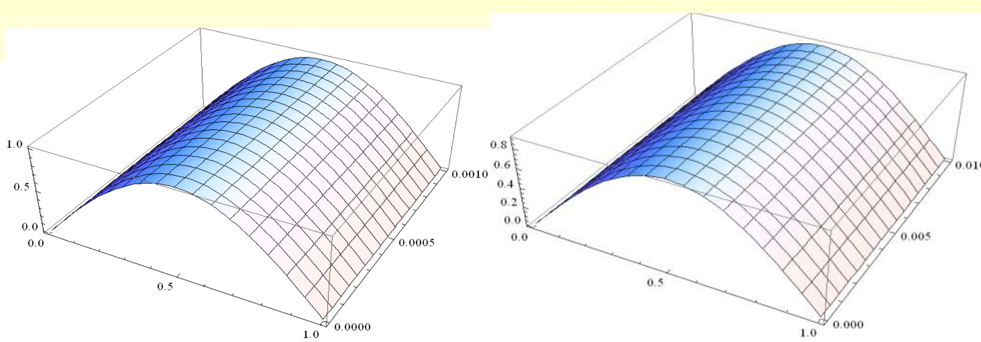


Fig 1: Graph of (Problem-1) of $u(x,t)$ vs x at time t=0.001 and t=0.01 with $\varepsilon = 1$

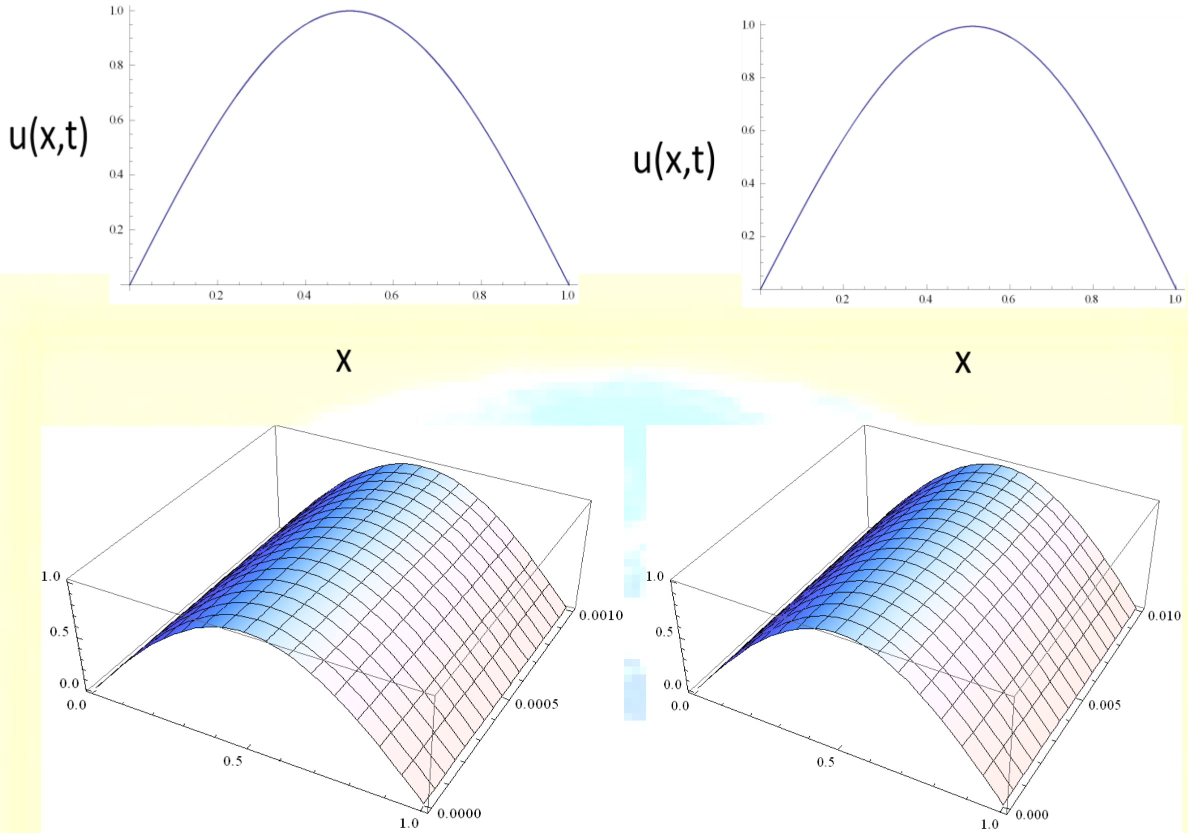
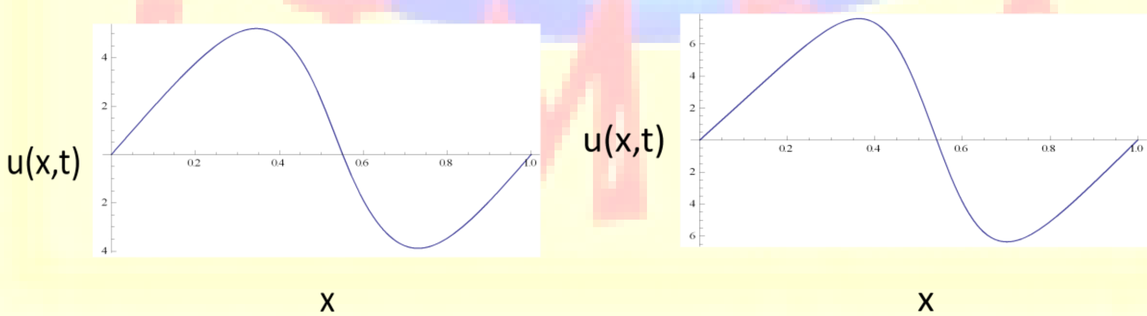


Fig 2: Graph of (Problem -1) of $u(x,t)$ vs x at time t=0.001 and t=0.01 with $\varepsilon = 0.05$



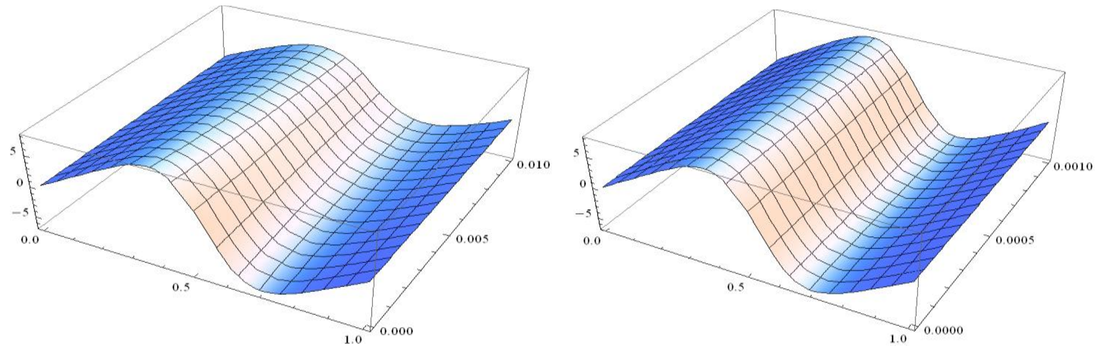


Fig 3: Graph of (Problem-2) of $u(x,t)$ vs x for t= 0.01 and t=0.001 with $\varepsilon=1$

CONCLUSION:

In this paper, solution of Burgers equation is obtained by applying homotopy perturbation transform method with specific initial conditions. The results shows that this method are powerful mathematical tools for solving Burgers equation and very effective and quite accurate in solving partial differential equation. Its small size of computation in comparison with the computational size required in other numerical methods and its rapid convergence shows that method is reliable and introduces a significant improvement in solving partial differential equation over existing methods. Hence HPTM can be considered as a nice refinement in existing numerical technique and might find wide application in different field of sciences.

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