BIANCHI TYPE-V MODELS WITH DECAYING COSMOLOGICAL TERM-Λ

UTTAM KUMAR DWIVEDI*

ABSTRACT

In this paper, we investigate anisotropic Bianchi type-V cosmological model in the presence of perfect fluid and decaying cosmological term - Λ (t). The Einstein's field equations have been solved by assuming the decay law for Λ (Overduin and Cooperstock, 1998) as $\Lambda = \frac{\beta \ddot{R}}{R} + \frac{1}{R^2}$ and w = 1, where w is the equation of state parameters and R is the average scale factor of the universe. Physical and kinematical parameters of the model have also been discussed.

KEY WORDS: Bianchi V; stiff fluid; variable cosmological term.

* Govt. Engineering College, Rewa (MP) – 486 001 India.

One of the most important and outstanding problems in cosmology is the cosmological constant problem. Some of the recent discussion on the cosmological constant problem and consequence on cosmology with a time varying Λ are investigated by Ratra and Peebles (1988), Dolgov (1983, 1997), Dolgov and Sazhin (1990), Sahni and Starobinsky (2000). Several ansats have been proposed in which the Λ term decays with time (See Ref Gasperini (1987), Berman (1990), Chen and Wu (1990), Abdussattar and Vishwakarma (1996), Pradhan *et al.* (2001, 2005).

Stiff fluid cosmological models create more interest in the study because for these models, the speed of light is equal to speed of sound and its governing equations have the same characteristics as those of gravitational field (Zeldovich,1970), Barrow (1986) has discussed the relevance of stiff equation of state e = p to the matter content of the universe in the early state of evolution of universe. Wesson (1978) has investigated an exact solution of Einstein field equation with stiff equation of state.

The astronomical observation have revealed that on large scale the universe is isotropic and homogeneous in its present state of evolution. But it might not be the same in past. Therefore, the models with anisotropic background taht approach to isotropy at late times, are most suitable for describing the entire evolution of the universe. The spatially homogeneous and anisotropic Bianchi-V space time provides such a frame work. In the literature Bianchi type-V cosmological models with decaying Λ have been studied by Pradhan (2001), Carvalho *et al.* (1992), Schutzhold (2002), Vishwakarma (2000) to name only a few. The cosmological consequences of this decay law are very attractive. This law provides reasonable solutions to the cosmological puzzles presently known.

In this paper we study Bianchi type-V model, with decaying cosmological constants. This work is organized as follows : The model and field equation are given in section-2. The field equation are solved in section-3. The physical behaviour of the solutions is discussed detail in last section.

Metric and Field Equations:

We consider the Bianchi type-V space-time given by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2x}\{B^2(t)\,dy^2 + C^2(t)dz^2\} \qquad \ldots (1)$$

We assume the cosmic matter consisting of perfect fluid represented by the energy-momentum tensor

$$T_{ij}=(\rho+p)v_i\,v_j+pg_{ij} \qquad \text{... (2)}$$

satisfying linear equation of state

$$p=w\rho\,, \qquad 0\le w\le 1 \qquad \text{... (3)}$$

where p is the isotropic pressure, ρ is the energy density and v^i the flow vector fluid satisfying $v_i v^i$ = -1. The Einstein field equations (in gravitational units 8 π G = C = 1) with time-dependent cosmological term Λ (t) are

$$R_{ij}-\frac{1}{2}Rg_{ij}=-T_{ij}+\Lambda(t)g_{ij} \qquad \text{... (4)}$$

For the line-element (1), the field equations (4) in comoving system of coordinates lead to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{1}{A^2}=\Lambda-p \qquad \text{... (5)}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}-\frac{1}{A^2}=\Lambda-p \qquad \text{... (6)}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=\Lambda-p \qquad \text{... (7)}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{3}{A^2}=\Lambda+p \qquad \text{... (8)}$$

$$\frac{2A}{A}=\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \qquad \text{... (9)}$$

Vanishing divergence of Einstein tensor $R_{ij}-\frac{1}{2}R\,g_{ij}$ gives rise to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\dot{\Lambda}=0 \qquad \text{... (10)}$$

We define average scale factor R for Bianchi V universe as

$$R^3=ABC \qquad \text{... (11)}$$

We define generalized Hubble parameter H and generalized deceleration parameter q as

$$H=\frac{\dot{R}}{R}=\frac{1}{3}(H_1+H_2+H_3) \qquad \text{... (12)}$$

and $$q=-\frac{\dot{H}}{H^2}-1 \qquad \text{... (13)}$$

where $H_1=\dot{A}/A, H_2=\dot{B}/B, H_3=\dot{C}/C$ are directional Hubble's factors along x, y and z directions respectively.

We introduce volume expansion θ and shear scalar σ as usual

$$\theta = v^i_{ji} \qquad \text{... (14)}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\,\sigma^{ij} \qquad \text{... (15)}$$

σ_{ij} being shear tensor.

For the Bianchi type V metric, we have

$$\theta = 3\dot{R}/R \qquad \text{... (16)}$$

and $\sigma = K/R^3$ ... (17)

where K is an integration constant.

Equations (5) – (8) can be written in terms of H, σ and q as

$$p - \Lambda = (2q-1)H^2 - \sigma^2 + 1/R^2 \qquad \text{... (18)}$$

$$\rho + \Lambda = 3H^2 - \sigma^2 - 3/R^2 \qquad \text{... (19)}$$

Solution of the field equations :

From equations (18) and (19) with the help of (3), (12) and (13) we obtain

$$\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} - \frac{2}{R^2} = \frac{1}{2}(1-w)\,\rho + \Lambda \qquad \text{... (20)}$$

Thus, we have one equation with three unknowns R, ρ and Λ. We require two more conditions to close the system. We assume that w = 1 and take the decay law for Λ as

$$\Lambda = \beta\frac{\ddot{R}}{R} + \frac{1}{R^2} \qquad \text{... (21)}$$

where β is a constant.

Equation (2) becomes

$$(1-\beta)\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} - \frac{3}{R^2} = 0 \qquad \text{... (22)}$$

For β = 1, we get

$$R = \sqrt{\frac{3}{2}}t + t_0 \qquad \text{... (23)}$$

where to is an integration constant.

For this solution metric (1) assumes the following form

$$ds^2 = dt^2 + (at+t_0)^2[dx^2 + e^{2x - \frac{k}{a(at+t_0)^2}}\{e^{\frac{2k}{a(at+t_0)^2}}dy^2 + dz^2\}] \qquad \text{... (24)}$$

where $a = \sqrt{3/2}$.

Discussion :

Matter density ρ and cosmological term Λ for the model (24) are given by

$$\rho = \frac{(t\sqrt{3/2} + t_0)^4 - 2k^2}{2(t\sqrt{3/2} + t_0)^6}$$

$$\Lambda = \frac{1}{(t\sqrt{3/2} + t_0)^2}$$

Scale factor R, expansion scalar θ and shear σ are obtained as

$$R = \sqrt{3/2}t + t_0$$

$$\theta = \frac{3\sqrt{6}}{\sqrt{6}t + 2t_0}$$

$$\sigma = \frac{k}{(\sqrt{3/2}t + t_0)^3}$$

The model has point singularity at $t = -t_0\sqrt{2/3}$. We observe that ρ, p, θ, σ all infinite at initial singularity and become zero at t = ∞. Therefore, all the physical parameter decreases as t increases. Since σ / θ → 0 as t → ∞ the model approaches isotropy for large values of t. The deceleration parameter q = 0 throughout in the evolution of the model. Therefore, rate of the expansion of the model is constant.

References :

Overduin J.M. and Cooperstock, F.I. 1998, Evolution of the with scale factor a variable cosmological term *Phys. Rev. D* **58**, 043506.

Ratra, B.,Peebles, P.J.E. 1988, Cosmological consequences of a rolling homogeneous scalar field *Phys. Rev. D.* **37**, 3409.

Dolgov, A. D. 1983, In The Very Early Universe (eds. G.W. Gibbons, S.W. Hawking, S.T.C. Siklos), Cambridge University Press, Cambridge.

Dolgov, A. D., Sazhin, M.V., Ya. B. Zeldovich, 1990, Basics of Modern Cosmology, Editions Frontiers.

Dolgov, A.D. 1997, Higher spin fields and the problem of the cosmological constant *Phys. Rev. D.*. **55**, 5881.

Sahni, V., Starobinsky, A. 2000, *The case for a positive cosmological Λ-term Int. J. Mod. Phys. D.* **9**, 373.

Gasperini, M. 1987, A thermal approach to the cosmological constant problem *Phys. lett. B.* **194**, 347.

Berman, M.S. 1990, Static Universe in a Modified Brans-Dicke Cosmology, *Int. J. of Theor. Phys., 29, 567.*

Berman, M.S. 1991, Cosmological Models with Variable Cosmological Term. *Phys.l Rev. D*, ***43***, *1075-8.*

Chen, W., Wu, Y.S. 1990, Implications of a cosmological constant varying as R^{-2} *Phys. Rev. D.*, **41**, 695.

Abdussattar, Vishwakarma, R.G. 1996, A model of the universe with decaying vaccum energy, *Pramana – J. Phys.*, **47**, 41.

Pradhan, A., Kumar, A. 2001, LRS Bianchi I cosmological models with perfect fluid, *Int. J. Mod. Phys. D.*, 10, **291**.

Carvalho, J.C., Lima, J.A.S., Waga, I.1992, Cosmological consequences of a time-dependent Λ term, *Phys. Rev. D.* **46**, 2404.

Schutzhold, R. 2002, Small cosmological constant from the QCD trace anomaly *Phys. Rev. Lett.* **89**, 081302.

Vishwakarma, R.G. 2000,A study of angular size-redshift relation for models in which Λ decays as the energy density *Clas. Quantum. Grav.* **17**, 3833.

Zel'dovich, Ya. B. 1972, A hypothesis, unifying the structure and the entropy of the Universe, *Mon. Not. R. Astron. Soc.* **160**.

Barrow, J.D. 1986,The Deflationary Universe: An Instability of the De Sitter Universe, *Phys. Lett. B.* **180**, 335.

Wesson, P.S. 1978, An exact solution to Einstein's equations with a stiff equation of state, *J. Math. Phys.* **19**, 2283.