EFFECT OF PRANDTL NUMBER ON ENTROPY GENERATION OF NANOFLUIDS OVER A STRETCHING SURFACE

Toorajsamimibehbahan*

Mostafa Pakdel*

Navid samimi behbahan*

Abstract— In this study the entropy generation for CUO nanofluid flow over a stretching sheet investigated numerically. Effect of the thermal expansion coefficient and free convection parameter on the velocity and temperature profile are investigated. Also the influence of thermal expansion coefficient, heat source/sink parameter, nanoparticle volume fraction of aluminum oxide (CUO) and free convection parameter, Prandtl number, Reynolds number on the entropy generation number has been investigated. By increasing of thermal expansion coefficient, the volume fraction of nanoparticles and buoyancy parameter, the entropy generation number is reduced, and with increasing Prandtl number and Reynolds number, the entropy generation number increased, And by changing heat source/sink parameter from positive to negative values the entropy generation number increases.

Keywords— Natural convection; nanofluid; temperature patterns; cavity; numerical

* Sama technical and vocational training college, Islamic Azad University, Behbahan Branch, Behbahan, Iran

1. Introduction

As there are some materials which can't be determined their behavior by applying Newtonian relations, and due to rising of industrial process importance so, a new stage is developing to perform widespread evaluation of fluid dynamics. The MHD flow of viscoelastic fluids on a moving surface has extensive applications in industrial processes, among them we can refer to production of synthetic sheets, aerodynamic extrusion of plastic sheets, cooling of metallic plates, etc.

Crane [1] considered a stretching flat sheet, so that the velocity of laminar boundary flow over it varies by distance. Chen and Char [2] investigated the problem from physical standpoint. There is another situation, so that the difference temperature between wall and fluid is high. The effects of thermal buoyancy force over a Newtonian or non-Newtonian flow and heat transfer over a stretching sheet had been investigated by many researchers [3-10]. Rajgopal et al [11] and Chang [12] investigated the viscoelastic flow which is flowing over a stretching sheet.

All the researches that have been mentioned above, don't consider the thermodynamic standpoint of stretching sheet, although they helped us to consider the thermodynamic viewpoint of this problems. Entropy generation has a big correlation with thermodynamic irreversibility which can be seen in every heat transfer phenomena. Heat transfer and viscous dissipation are the important source of entropy generation [17,18].

2. Problem definition and mathematical formulation

The steady laminar two-dimensional flow of an incompressible viscous fluid over a stretching sheet isconsidered. The x –axis is taken in the main flow direction along the plate, while the y-axis is assumed on vertical position to the plate direction.As shown in Fig.1. The stretching sheet velocity and temperature are according to the following formula. Where a and b are constant.

$$U_w(x) = bx \tag{2.1}$$

$$T_w(x) = a(\frac{x}{l}) \tag{2.2}$$

The nanofluid contains water, as base fluid, and Cuo,as nanoparticle. Nanofluid can slip over the sheet sothe velocity between nanofluid and sheet on the sheet is nonzero (Us $\neq$ 0). We have thermal equilibrium between base fluid and nanoparticle. The chemical reaction, viscous dissipation and radioactive heat transfer are neglected in this problem. With this assumption the governing equation, given by:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{2.3}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial y^2}\right)+g\frac{(\rho\beta)_{nf}}{\rho_{nf}}(T-T_\infty) \tag{2.4}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{Q}{(\rho C_p)_{nf}}(T-T_\infty) \tag{2.5}$$

The appropriate boundary conditions under consideration are written as

$$at\ \ y=0\ :\ u=U(x)-U_s,\ \upsilon=0,\ T=T_w(x) \tag{2.6-a}$$

$$as\ \ y\to\infty\ :\ u\to U_\infty,\ T\to T_\infty \tag{2.6-b}$$

Here u and v are the velocity components along the x and y directions. Where, U_s is the nanofluid velocity slip at the sheet, l is characteristic length,T_W is the temperature of nanofliud away from sheet, the physical interpretation of the infinite distance is where the sheet have no influence on the nanofluid, $\mu_{nf}$is the effective dynamic viscosity (see Eq. 2.7), $(\rho\beta)_{nf}$ is the thermal expansion coefficient (see Eq. 2.8), ρ_{nf} is the density of the nanofluid (see Eq. 2.9), α_{nf} is the thermal diffusivity coefficient of the nanofluid(see Eq. 2.10) and $(\rho c_p)_{nf}$ is the nanofluid heat capacitance(see Eq. 2.11).

$$(\mu)_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{2.7}$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s \tag{2.8}$$

$$(\rho)_{nf}=(1-\phi)\rho_f+\phi\rho_s \tag{2.9}$$

$$(\alpha)_{nf}=\frac{k_{nf}}{(\rho c)_{nf}} \tag{2.10}$$

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s \tag{2.11}$$

Here φ is nanoparticle fraction and k_{nf} is nanofluid thermal conductivity and given by fallowing equation

$$k_{nf}=k_f\left\{\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right\} \tag{2.12}$$

2.1 Similarity transformation

The fallowing dimensionless function (f (η) and Θ (η)) and similarity variable (η) are used for similarity solution.

$$u = bxf'(\eta), \quad v = -\sqrt{b\nu_f}\, f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad (2.13-a)$$

$$\eta = \sqrt{\frac{b}{\nu_f}}\, y \qquad (2.13-b)$$

Where f' (η) is an ordinary derivative of function f (η) with respect to η. F and Θ are dimensionless stream and temperature function, respectively.

After use of above considerations the momentum equation (2.4) and energy equation (2.5) convert to the fallowing equation, respectively

$$f'''(\eta) = (1-\phi)^{2.5}\left[1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right] * \left[f'(\eta)^2 - f.f''(\eta) - \lambda\frac{1-\phi+\phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)}{1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)}\theta(\eta)\right] \qquad (2.14)$$

$$\theta''(\eta) = \Pr\frac{k_f}{k_{nf}}\left[f'(\eta).\theta(\eta) - f(\eta).\theta'(\eta) - \frac{\beta_1}{1-\phi+\phi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}\theta(\eta)\right] \qquad (2.15)$$

The appropriate boundary conditions in dimensionless form are written as

$$at\ \eta = 0 \quad : \ f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1,\ f'(0) = 1 + \beta f''(0) \qquad (2.16\text{-a})$$

$$as\ \eta \to \infty \quad : \ f'(\infty) = 0,\ \theta(\infty) = 0 \qquad (2.16\text{-b})$$

Where Pr is Prandtl number and β_1 is the heat sink or source parameter and λ is the buoyancy or free convection parameter.Pr and $\beta_1$and λare according to the fallowing formula

$$\beta_1 = \frac{Q}{(\rho C_p)_f b}, \quad \Pr = \frac{\nu_f}{\alpha_f}, \quad \lambda = \frac{g(\rho\beta)_f}{\rho_f b^2 l} \qquad (2.17)$$

3. Entropy generation analyses

According to the Bejan the volumetric rate of entropy generation is given by:

$$S_G = \frac{k}{T_\infty}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{\mu}{T_\infty}\left(2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right\} + \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^2\right) \qquad (2.18)$$

Here we have two kind of entropy generation source. The term containing k represents the entropy generation due to heat transfer and the term containing μ represents the entropy generation due to the viscous effects of the fluid. For simplification assumed that the flow is hydro dynamically developed ($\partial V/\partial x = 0$). So the Eq.18 simplified to

$$S_G = \frac{k}{T_\infty}\left[(\frac{\partial T}{\partial x})^2 + (\frac{\partial T}{\partial y})^2 \right] + \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.19)$$

Now define dimensionless number for entropy generation rate N_s

$$N_s = \frac{S_G}{S_{G0}} \qquad (2.20)$$

Where S_{G0} is a characteristic entropy generation rate

$$S_{G0} = \frac{k(\Delta T)^2}{l^2 T_\infty^2} \qquad (2.21)$$

Using Eqs. (2.19), (2.13-a) and (2.13-b) the dimensionless local entropy generation rate due to fluid flow and heat transfer written

$$\mathrm{Re}_l = \frac{u_l l}{\nu} \qquad (2.23\text{-a})$$

$$Br = \frac{\mu u_w^2}{k\Delta T} \qquad (2.23\text{-b})$$

$$\Omega = \frac{\Delta T}{T_\infty} \qquad (2.23\text{-c})$$

4. Results and discussion

The flow and heat transfer in a nanofluid (CUO-water)over a stretching sheet has been solved analytically and analytic expressions of the velocity and temperature have been used to compute the entropy generation.Fig. 1 and2 depicts the entropy generation number (N_s) for several values of the slip factor (β) and Prandtl num

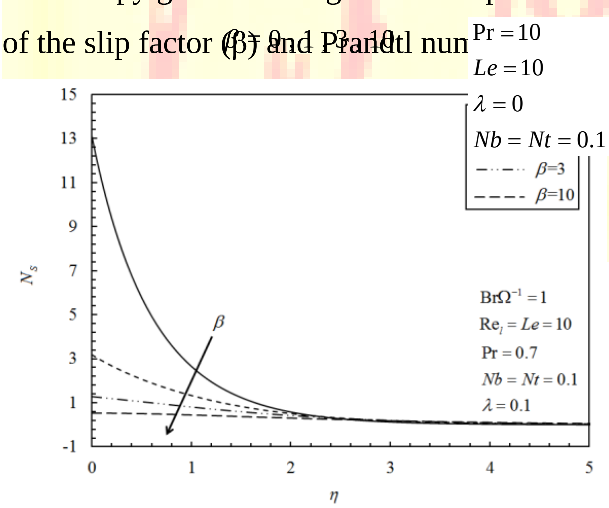

Fig.1. The Effect of β and Pr on entropy generation

number for Le =10, λ =0.1, and Nb=Nt=0.1 for a)

Pr=0.7

It is observed that for Pr=0.7, the entropy generation decreases with any increase in β: while for P>5, the entropy generation first increases for some values of η and then decreases for all slip factors. Moreover, for all values of the Prandtl numbers, an increase in the slip factor leads to a decrease in the entropy generation number near the surface.

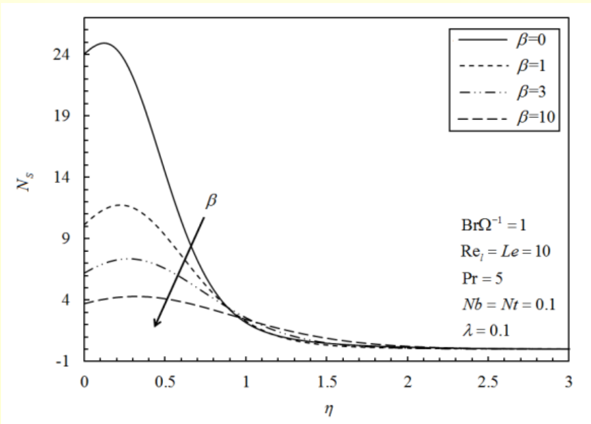

Fig.2. The Effect of β and Pr on entropy generation number for Le =10, λ =0.1, and Nb=Nt=0.1 for c)

Pr=5

5. Conclusion

In this paper second law of thermodynamics for CUO-water nanofluids on stretching sheet are investigated.Entropygeneration number equation by using of velocity and temperature equations are obtained numerically. The influence of Prandtl number on the entropy generation number has been investigated.

References

[1] L.J. Crane, Flow past a stretching sheet, Z. Angew. Math.Phys. 21 (1970) 645– 647.

[2] C.K. Chen, M.I. Char, Heat transfer of a continuous stretching surface withsuction or blowing, J. Math. Anal.Appl. vol. 135 (1988) 568e580.

[3]K. Vajravelu, Convection heat transfer at a stretching sheet with suction or blowing, J. Math. Anal.Appl. vol. 188 (1994) 1002e1011.

[4] S.K. Khan, E. Sanjayanand, Viscoelastic boundary layer flow and heat transfer over an exponential stretching sheet, Int. J. Heat Mass Transfer 48 (2005) 1534–1542.

[5] R. Cortell, A note on flow and heat transfer of a viscoelastic fluid over a stretching sheet, Int. J. Non-Linear Mech. 41 (2006) 78–85.

[6] M.S. Abel, P.G. Siddheshwar, M.M. Nandeppanavar, Heat transfer in a viscoelastic boundary layer low over a stretching sheet with viscous dissipation and non-uniform heat source, Int. J. Heat Mass Transfer 50 (2007) 960–966.

[7] R. Cortell, effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet, Phys. Lett. A 372 (2008) 631–636.

[8]R. Cortell, Viscous flow and heat transfer over a nonlinearly stretching sheet, App. Math. Comput. 184 (2007) 864–873.

[9] M.S. Abel, N. Mahesha, Heat transfer in MHD viscoelastic fluid over a stretching sheet with vriable thermal conductivity, non-uniform heat source and radiation, Appl. Math. Mod. 32 (2008) 1965–1983.

[10] O.D. Makinde, A. Aziz, Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition, Int. J. Therm. Sci. 50 (2011) 1326-1332.

[11]K.R. Rajagopal, T.Y. Na, A.S. Gupta, Flow of a viscoelastic sheet over a stretching sheet, Rheol.Acta. 23 (1984) 213–221.

[12] W.D. Chang, The non-uniqueness of the flow of viscoelastic fluid over a stretching sheet, Q. Appl. Math. 47 (1989) 365–366.

[13] S. Choi, Enhancing thermal conductivity of fluids with nanoparticle. D.A. Siginer, H.P. Wang (Eds.), Developments and Applications of Non-Newtonian Flows, MD vol. 231 and FED vol. 66.ASME, 1995, pp. 99e105.

[14] W.A. Khan, I. Pop, Boundary-layer flow of a nanofluid past a stretching sheet, Int. J. Heat Mass Transf. 53 (2010) 2477e2483.

[15] A.V. Kuznetsov, D.A. Nield, Natural convective boundary-layer flow of a nanofluid past a vertical plate, Int. J. Therm. Sci. 49 (2010) 243e247.

[16]P.S. Datti, K.V. Prasad, M.S. Abel, A. Joshi, MHD viscoelastic fluid flow over a non-isothermal stretching sheet, Int. J. Eng. Sci. 42 (2004) 935–946.

[17]A. Bejan, Second-law analysis in heat transfer and thermal design, Adv. Heat Transfer 15 (1982) 1–58.

[18] A. Bejan, Entropy Generation Minimization, CRC Press, Boca Raton, New York, 1996.