MATHEMATICAL MODELLING OF FACILITY LOCATION PROBLEMS WITH LINK DISRUPTIONS AND NETWORK DESIGN: A CASE STUDY

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Abstract

The unexpected disruption failures in transportation network systems are one of the most typical problems in the real practical environments. Proposing a robust designed facility location for these situations is one of the most effective ways to hedge against them. This paper studies the combined budget constrained facility location/network design problem with respect to transportation link disruptions and proposes a mixed integer linear programming formulation to model it. Regarding to the probability of link disruptions, the objective function of the proposed model minimizes the total expected transportation costs. Moreover, a practical case study is presented in detail to illustrate the application of the proposed mathematical model. Finally, a sensitivity analysis is done to provide an insight into the behavior of the proposed model in response to changes of key parameters of the problem.

Keywords: Facility location, Network design, Reliability, Link disruption, health care, case study.

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1. Introduction

Reducing the total costs and also improve the performance is one of the strategic and practical problem for key players in supply chain, including manufacturers, distributors and retailers in order to stay in today competitive market world. However, improving and optimizing of each part of the supply chain can help to these companies to manage several risks in their systems. As a considerable point, facility location and allocation topic is one of the fundamental strategic decisions so that can be affected on different costs of an integrated system such as initial investment and transportation costs. So proposing an effective and near reality modeling can display a more clear description of the problem. This leads to find more practical solution and subsequently reduce its several related costs.

In this paper, we develop a new integrated approach to facility location problems with respect to the subjects of budget constraint, network design and system reliability as it can be named reliable budget constrained facility location/network design problem (RBFLNDP) in order to improve the efficiency of transportation network systems. The motivation of this research is to consider simultaneously three practical factors (budget constraint, network design and system reliability) to develop the mathematical modeling of facility location problems, which has not been considered until now based on the authors' best knowledge. Moreover, there are numerous practical instances of facility location problems in which simultaneously considering of budget constraint, network design and system reliability can lead to more practical and realistic mathematical modeling of the problem. Health care service centers and emergency services are the most obvious and practical paradigms that in which simultaneously considering of network design and system reliability in locating facilities and assigning demands plays a critical role in order to improve the efficiency, practicality and also reliability of system. Therefore, study of facility location problems with such constitution can practically improve solving of the mentioned problems in supply chain systems, including industrial factories and service centers, by obtaining more effective and accurate solutions. As an another significant point, with respect to the huge investment for facility location and network design, the attention to the failures of system based on several disruptions in facility locating and network design has been increased recently [1-3].

The rest of this paper is organized as follows. Section 2 reviews the literature and describes the existing research gap. Section 3 describes the problem and also represents the model formulation, and a case study that exactly shows the application of the model formulation is demonstrated and solved by the model in Section 4. Sensitivity analysis of the model parameters is reported in section 5. Finally, the conclusion is presented in Section 6.

2. Literature Review and Research Gap

In order to place our contribution in the right perspective, at the first, we briefly review two main streams of literature that may be of interest for comparison: the literature on facility location problem regarding to network design and also on facility location problem with respect to system reliability. Then, the research gap is described according to literature review.

As we know, facility location considers the optimization of the predefined objective functions, such as minimizing the operational cost or maximizing the area covering, in locating facilities and allocating customers to them. As a general view, the facility location problems can be classified according to different objective functions such as the *P*-median and *P*-center problems [4], the uncapacitated facility location problems [5], the maximum covering location problems [6] and the set covering location problems [7]. On the other hand, in the network design, the basic problem is to optimally construct a network that enables some kind of flow, and possibly that satisfies some additional constraints. The nodes usually are given and the network is constructed from a set of potential links.

All of the aforementioned classical models locate facilities on a predetermined network. However, the topology of the underlying network may profoundly impact upon the optimal facility locations and can have many applications in industries and services. Some studies obviously illustrate its undeniable effect on improvement of the objective function value [8-10]. In other words, simultaneously considering the facility location and network design, the proposed problem can be described in more realistic formulation and modeling.

In the literature review, Daskin et al. in 1993 introduced the first initial model of facility location/network design problem (FLNDP) [11]. They presented some preliminary results which showed the effect of network design topic in mathematical modeling of facility location problems and their optimal solution. Later, Melkote [8] in his doctoral thesis developed three models for the FLNDP including uncapacitated FLNDP (UFLNDP), the capacitated FLNDP (CFLNDP), and the maximum covering location/network design problem (MCLNDP). The results of the thesis were published in [9-10]. Drezner and Wesolowsky [11] proposed a new network design problem with potential links, each of which could be either constructed at a given cost or not. Moreover, each transportation link could be constructed as either a one-way or two-way link. They developed four basic problems subject to two objective functions; finally, they solved the problems by a descent algorithm, a simulated annealing (SA), a tabu search (TS), and a genetic algorithm (GA) as main solution procedures. In another doctoral thesis, Cocking [12-13] expanded some efficient approaches to solve the static budget constrained FLNDP. Some useful algorithms were developed to find good upper bounds and good lower bounds on the optimal solution. Simple greedy heuristics, a local search heuristic, metaheuristics including SA and variable neighborhood search (VNS), as well as a custom heuristic based on the problem-specific structure of FLNDP were the main heuristics and metaheuristics that were proposed in Cocking's doctoral thesis. Besides, a branch-and-cut algorithm using heuristic solutions as upper bounds, and cutting planes to improve the lower bound of the problem were developed. The method reduced the number of nodes which were needed to approach optimality. Recently, Bigotte et al. [14] studied the FLNDP so that in which, the multiple levels of urban centers and multiple levels of network links were considered simultaneously for developing of a mixed integer mathematical model. In fact, in order to improve accessibility of all kinds of facilities, the best transfers of urban centers and network links to a new level of hierarchy are determined. Jabalameli and Mortezaei [15] proposed an extension of the CFLNDP in which the maximum amount of demands can be carried by a link is limited. They presented a bi-objective mixed integer programming formulation of the problem and developed a hybrid algorithm to solve the resulted problem. Contreras and Fernandez [16] reviewed the relevant modeling aspects, alternative formulations and several algorithmic strategies for the FLNDP. In fact, they studied general network design problems in which, design decisions to locate facilities and to select links on an underlying network are combined with operational allocation and routing decisions to satisfy demands. Contreras et al. [17] presented a combined FLNDP to minimize the maximum customer-facility travel time. They developed and compered two mixed integer programming formulations by generalizing the model of the classical *P*-center problem so that the models simultaneously consider the location of facilities and the design of its underlying network.

Another significant subject that can affect facility location and allocation is reliability. Most of the studies assume that all parts of the considered system are always available and unfailable. However these assumptions can affect on the flexibility of their designs and significantly reduce the efficiency of them especially when some disruptions are occurred. The terrorist attacks of 9/11, the catastrophic devastation caused by Hurricane Katrina [18-20] and the Japan's tragic earthquake and the following tsunami in 2011 [21] are the most obvious examples. these recent events show higher risks happening from disruptions and change the aspects of the modern business world. Although most of the designers and experts believe that the existing international supply chains are strong and reliable, in reality, many are fragile and easily disrupted when the unexpected events happen. Paying huge fines by Boeing Company in compensation for postponing the delivery of the Dreamliner 787 is an evident example [22]. Several potential threats can lead to disruptions in supply chain systems, e.g., natural disasters such as avalanche, hurricane, volcano, heavy rain or snow, industrial accidents; operational eventualities such as equipment failures or supplier discontinuities; power outages; labor strikes; and terrorism. Although these disruption events may only lead to short-term facility contingencies, they can also cause not only serious operational consequences, such as higher transportation costs, order delays, inventory shortages, loss of market shares, and so on, but also extended negative financial impacts. An empirical study by [23] has illustrated that over the time period of 1989–2000, the abnormal stock returns of firms that have been affected by disruptions were nearly 40%. Evidence has also emphasized that these firms had a hard time recovering from the negative effects of disruptions and that their equity risk significantly increased around the announcement date. Similar findings are described by [24].

It is mentioned that in a supply chain system, when a facility failure occurs, as an efficient solution, customers may have to be reassigned from their original facilities to the other available facilities; likewise, when a link disrupts, extra transportation costs may be paid for transferring by disrupted link because of several difficulties or, as an another solution, demands may be reassigned to the alternative links. In both mentioned conditions, the transportation costs certainly increased.

In the traditional locational analysis literature, Drezner [25] was one of the first researchers that proposed the mathematical models for facility location with unreliable suppliers. He studied the unreliable P-median and (P,q) center location problems, in which a facility has a given probability of becoming inactive. In the following research way, Snyder and Daskin [26-28] proposed an implicit formulation of the stochastic P-median and fixed charge problems based on level assignments, in which the candidate sites are subject to random disruptions with equal probability. Shen et al. [29] and Berman et al. [30] relaxed the assumption of uniform failure probabilities, formulated the stochastic fixed-charged facility location problem as a nonlinear mixed integer program, and expanded several heuristic solution algorithms. Berman et al. [30] concentrated on an asymptotic property of the problem and verified that the solution to the stochastic *P*-median problem coincides with the deterministic problem as the failure probabilities approach zero. They also presented some efficient heuristics with bounds on the worst-case performance. Lim et al. [31] suggested a reliability continuum approximation (CA) approach for facility location problems with uniform customer density. For simplification, a specific form of failure-proof facility was supposed to exist; a customer is always reassigned to a failure-proof facility after its nearest regular facility failed, regardless of other regular facilities. Hanley and Church [32] developed a new facility location–interdiction covering model for finding a robust alignment of facilities that has a suitable efficiency in the worst situations of facility loss. They formulated the problem as two mathematical models. At the first model, all possible interdiction patterns are considered and a standard MIP formulation is proposed. In the second model, the optimal interdiction pattern is implicitly defined in terms of the chosen facility location layout and more compact bi-level programming formulation is developed. Peng et al. [33] studied the effect of considering of reliability topic on logistic networks design with facility disruptions and illustrated that applying a reliable network design are often possible with negligible increases in total location and allocation costs depends on decision makers opinion. They considered the commodity production/delivery system without respect to open/close decisions on the arcs of supply chain system and by applying the *p*-robustness criterion (which bounds the cost in disruption scenarios), they simultaneously minimize the nominal cost (the cost when no disruptions occur) and reduce the disruption risk. Recently, Liberatore et al. [34] introduced the problem of optimizing fortification plans in median distribution systems in the face of disruptions that involve large areas. They developed an effective exact solution algorithm to solve it optimally. Also, they showed empirically that ignoring correlation effects in a system can lead to suboptimal protection plans that result in an unnecessary increase in the system cost when disruptions take place. Moreover, Jabbarzadeh et al. [35] studied a supply chain design problem with the risk of disruptions at facilities and formulated the problem as a mixed-integer nonlinear program which maximizes the total profit for the whole system. The proposed model simultaneously determines the number and locations of facilities, the subset of customers to serve, the assignment of customers to facilities, and the cycle-order quantities at facilities.

Although literature on facility location problems is abundant, there are not any studies in which facility location problems was considered regarding to budget constraint, network design and system reliability topics simultaneously.

As a result, there is an obvious research gap in this area to manage the practical facility location problems and proposing a new mathematical model formulation, which can obtain optimal facility location and link construction under some special conditions such as system reliability, can lead decision makers to more accurate solutions for the considered problem. In other words, an integrated comprehensive model provides an enough effective and confidant approach to be applied by different decision makers especially facility location planners for locating of several facilities and constructing of potential links in order to improve the efficiency and responsibility of supply chain.

3. Problem definition and model formulation

3.1. Definition

Suppose that in a geographical region, a set of demand nodes exists and a set of roads as transportation links that contains existing and new candidate links is defined to construct a transportation network on the mentioned region. Likewise, a set of facilities exists in the region and a number of new facilities are determined to locate in the region. As a reminder point, the location cost of existing facilities and construction cost of existing links are zero. Because of the geographical situation of the region and its mountainous paths, all of the transportation links (containing existing and new links) are not reliable. In fact, due to some unexpected events such as heavy rain or snow, avalanche, hurricane and volcano, they occasionally disrupt and become very difficult to use. Accordingly, the assigned demands of disrupted links may be transferred by having some delays or installing some spare equipment, such as car tire chain, and transferring slowly, or even finding some further alternative links. However, in the link disruption conditions, all of the possible solutions lead to increase the delivery time or the traveled distances by demand nodes and raise the transportation costs as extra mobilizing cost, delay cost or shortage cost. If the increase in transportation costs is considered as failure cost, then a part of objective function can be defined as “failure cost” and the total nominal cost (including the investment cost and the transportation cost of nominal (normal) conditions) together with the failure cost (transportation cost of disruption conditions) can be simultaneously considered and optimized. In the other words, it is clearly desired to locate a set of new facilities and construct new candidate links so that the total expected operational costs, including transportation costs in nominal and disruption conditions, are minimized. The problem is to determine: (1) the optimum locations of new facilities regarding to network design and disruption probability of transportation links, (2) the new transportation links that should be constructed in the proposed network, (3) the amount of demands of nodes that should be transported by the transportation links, and, (4) The fraction of every demand that should be supplied by new and exciting facilities in the nominal and disruption conditions.

Assumptions

The assumptions for RFLNDP can be described as follows:

1. Each node of network illustrates a demand point.
2. The facilities and network links (transportation roads) are uncapacitated.
3. New facilities can only be located on the nodes of the network and may not be located on the links of network.
4. At most only one new facility can be located on each node.
5. The general structure of the network is planned based on a *customer-to-server* system, which means that the demands themselves travel to the relevant facilities in order to be served.
6. All travel costs are symmetric.
7. All network links are directed.
8. There is an investment budget constraint for facility location and link construction/improvement.
9. All of the facilities (including existing and new facilities) are reliable.
10. Based on the geographical situation, the transportation links sometimes will disrupt with a specified probability.
11. The disruption of each link leads to increase its transportation cost as a constant coefficient of nominal transportation cost.
12. It is may be happened that several links simultaneously have disruptions and not be available at a

time.

13. Locating new facilities and allocating demand nodes are considered so that the facilities location costs and link construction costs as well as the expected transportation costs (in the nominal and disruption conditions of transportation links) are simultaneously optimized, subject to if any link disrupts, then, by paying extra costs for transferring of related demands, the resulting cost are known as the link disruption costs and added to the objective function.

Notifications

Parameters:

P number of new facilities to open, ($P \geq 2$)

N set of demand nodes in the network

M set of transportation links in the network (including existing and new candidate links)

d_i demand at node $i \in N$

f_i fixed cost of locating a facility at node $i \in N$

IB investment budget constraint for facility location and link construction/improvement

q_{ij} probability that the transportation link (i, j) will disrupt ($0 \leq q_{ij} \leq 1$)

u_{ij} increasing coefficient of transportation cost of link (i, j) in disruption conditions

c_{ij} construction/improvement cost of the link (i, j)

t_{ij}^0 transportation cost of a unit flow on link (i, j)

t_{ij}^l transportation cost of a unit flow of demand node l on link $(i, j) = t_{ij}^0\, d_l$

We assume all parameters are integer-valued except all kinds of costs. As an important point, it is mentioned that $t_{ij}^0(t_{ij}^l)$ presents a link-specific transportation cost, not an origin-destination transportation cost and we have to utilize link-specific transportation cost as an initial parameter of RFLNDP model because in RFLNDP, unlike RFLP, the network is not known in advance. Hence, we cannot calculate origin-destination transportation costs.

Variables:

Z_i = 1 if a facility is located at node i, 0 otherwise

X_{ij} = 1 if link (i, j) is constructed, 0 otherwise

Y_{ij}^{l} = fraction of demand of node l that flows on link $(i, j) \in M$

$Y_{ij}^{i} = X_{ij} \quad (i, j) \in M$

W_i^{l} = fraction of demand of node l that is served by a facility at node $i \in N$

$W_i^{i} = Z_i \quad i \in N$

3.2. Model formulation

Using these notations and assumptions, the mathematical formulation of the RBFLNDP is shown below:

(RBFLNDP) Minimize

$$\sum_{(i,j)\in M}(1-q_{ij})t_{ij}^{i}X_{ij}+\sum_{(i,j)\in M}q_{ij}u_{ij}t_{ij}^{i}X_{ij}+\sum_{(i,j)\in M}(1-q_{ij})t_{ij}^{l}Y_{ij}^{l}+\sum_{(i,j)\in M}q_{ij}u_{ij}t_{ij}^{l}Y_{ij}^{l} \quad (1)$$

Subject to:

$$\sum_{i\in N} f_i Z_i+\sum_{(i,j)\in M} c_{ij}X_{ij}\le IB \quad (2)$$

$$Z_i+\sum_{j\in N}X_{ij}=1 \qquad \forall i\in N \quad (3)$$

$$X_{li}+\sum_{j\in N:j\ne l}Y_{ji}^{l}=\sum_{j\in N}Y_{ij}^{l}+W_i^{l} \qquad \forall i,l\in N:i\ne l,\ \forall(l,i)\in M \quad (4)$$

$$\sum_{j\in N:j\ne l}Y_{ji}^{l}=\sum_{j\in N}Y_{ij}^{l}+W_i^{l} \qquad \forall i,l\in N:i\ne l,\ \forall(l,i)\notin M \quad (5)$$

$$Z_l+\sum_{i\in N:i\ne l}W_i^{l}=1 \qquad \forall l\in N \quad (6)$$

$$Y_{ij}^{l}\le X_{ij} \qquad \forall(i,j)\in M,\quad \forall l\in N:i\ne l \quad (7)$$

$$W_i^{l}\le Z_i \qquad \forall i,l\in N:i\ne l \quad (8)$$

$$X_{ij}+X_{ji}\le 1 \qquad \forall(i,j)\in M \quad (9)$$

$$\sum_{i\in N}Z_i=P \quad (10)$$

$$Y_{ij}^{l}\ge 0 \qquad \forall(i,j)\in M,\quad l\in N:l\ne i \quad (11)$$

$$W_i^{l}\ge 0 \qquad \forall i,l\in N \quad (12)$$

$$Z_i \in \{0,1\} \qquad \forall i \in N \tag{13}$$

$$X_{ij} \in \{0,1\} \qquad \forall (i,j) \in M \tag{14}$$

In this formulation, the objective function (1) minimizes the expected value of transportation cost (in nominal and disruption conditions). Constraint (2) emphasizes on investment budget constraint at facility location and link construction. In general observation, constraints (3-6) consider the rational conditions of the transportation flow between demand nodes and facilities. Specifically, Constraints (3) ensure that demand at i is either served by a facility at i or by transporting on some links out of i. Constraints (4) and (5) state conservation of flow for transshipped demand. Constraints (6) impose that the demand of node l must find a destination, whether it is estimated by node l itself (z_{lk}) or by the other nodes i (W_i^l). Constraints (7) and (8) guarantee that potential links and facilities are not used if they are not constructed. Constraints (9) emphasize that on any given link, an optimal solution flow will be in only one direction; Therefore, both links (i , j) and (j , i) cannot be constructed. Constraint (10) restricts the total number of newly located facilities to the predetermined facilities of P. Constraints (11) and (12) force the flow variables to be non-negative; while, Constraints (13) and (14) enforce the binary restriction on the facility location and link construction decision variables. As it mentioned, according to the single assignment property, every demand of node is completely assigned to the closest single facility. That is, nothing is gained by "splitting up" a demand and sending parts of it to different facilities. Therefore, the fractions of demands, which served a single facility, are integer-valued, while W_i^l and Y_{ij}^l are integral [8].

4. Describing an Application of the Proposed Model in a Case Study

The application of the proposed mathematical model is described as a practical case study, the goal of which is to improve accessibility to health care centers (facilities) for the urban residence centers (demand nodes) in a province of Iran named Yazd.

Yazd, with 131575 km^2 of area, is known as the fourth largest province of Iran. With respect to its geographical position, Yazd is one of the leading provinces in the field of medical and health care services. Besides, its inexpensive health care services for patients and proximity to deprived southern provinces have dramatically increased its demand for health care services. Also, Yazd consists of 19 urban residence centers (towns) with total population of 983252.

Reliable data were collected, as far as possible, for the problem. There are two available health care service centers scattered throughout the district, including hospitals and large health centers at urban centers. Also other residence centers (19-2=17 residence centers that don't have any health care service centers) are known as potential nodes to open new health care service centers (new facilities).

According to the current conditions, roads in Yazd province are classified into three categories in term of quality: high, medium and low. In fact, depending on the type of the roads, constructing/improving costs vary; as a result, low and medium quality roads can be upgraded to high quality roads with lower constructing costs. Furthermore, because of some technical and environmental conditions, such as sand storms, staff strikes, and the occurrence of natural disasters such as earthquakes, the roads are not reliable in all the times and some of the roads occasionally are not available.

Fig. 1 shows the residence centers and the transportation road network of Yazd province, as well as the existing health care service centers in cities of the province. It can be seen that there are 50 existing and 45 potential links or roads which have three different qualities and picture with various thicknesses in the graph of Fig. 1.

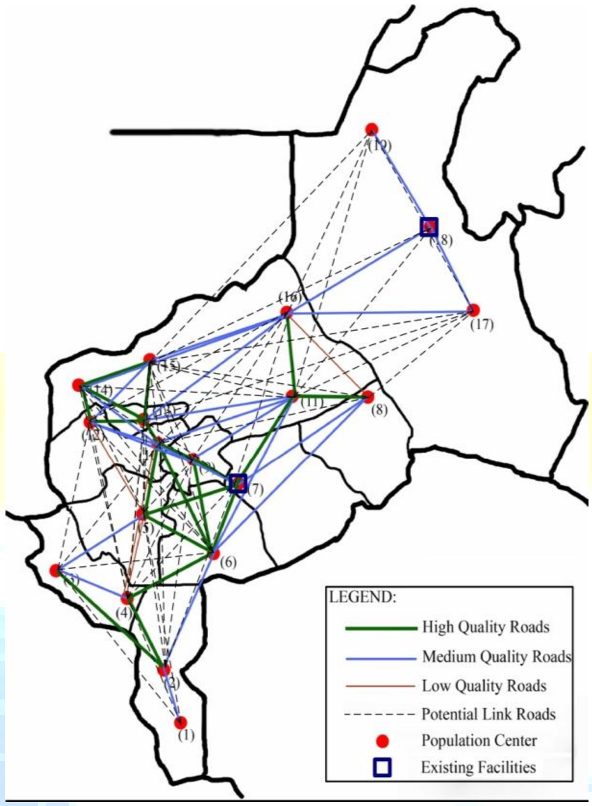

Fig. 1: The map of different transportation roads of Yazd province

The transportation cost for each client in kilometer is randomly calculated subject to a discrete uniform distribution in [0.10, 0.15]. The construction cost of new roads and improvement cost of the existing roads are calculated per kilometer and are between [100000, 400000] as many of transportation cost according to their qualities. Each residence center is a client node with a demand equals its population. The fixed cost of opening facility depends on node demands and varies between 2584010MU (Monetary Units) and 10221186MU. Also, the maximum level of investment budget constraint for facility locating and road constructing/improving is determined 35000000000MU. Because of the limitation of paper volume, other complementary information contains distances among different towns, transportation cost per unit flow, construction cost of new roads, and improvement cost of existing roads; the rest of complementary information which is about different towns of Yazd province has not been mentioned here.

It is worth mentioning that the ministry of health and medical education and ministry of road and transportation are responsible for investment in health care centers and road network construction or improvement, respectively; besides, they should provide a comprehensive plan to improve the quality of health services in each province. According to the medical equipment limitations and other technical reasons, the ministry of health and medical education have also determined that just one new health care center can be located in Yazd and, because of two existing health care centers, the value of P is 3 (as a reminder point, the facility location cost (f_i) of existing health care centers is zero, i.e. $f_7 = f_8 = 0$).

In order to improve the physical access to the health care centers in Yazd, the main goals in the considered case study are to determine:

(1) the optimum locations of new health care centers,
(2) the transportation links that should be constructed/improved in the proposed network regarding to probability of occasional unavailability of some roads,
(3) the amount of demands of residence centers that should be transported by transportation links, and
(4) the fraction of each demand that should be supplied by new and exciting health care centers.

According to the mentioned conditions, it is evident that the case study can be exactly investigated as a reliable budget constrained facility location/network design problem (RBFLNDP). As a result, the proposed model is a suitable mathematical modeling for the case study. As a propositional option to decrease total costs, the model suggests that new facilities can be established in the nodes in which no facility has located. Also, constructing new roads or improving existing roads as it is shown in Fig. 1 is suggested as the other propositional options to reduce the total costs.

The case was modeled by the model and coded in GAMS and solved by CPLEX solver. The results are presented in Fig. 2 which visually illustrates the obtained optimal solution. As Fig. 2 shows, the value of Z_6 , Z_7 and Z_{18} are determined to 1. This means that the optimum location of the new facility is node 6. Also, with respect to the main structure of the case study, constructing new roads is not necessary and only the quality of roads between nodes 1 and 2, nodes 17 and 18, and nodes 18 and 19 should be improved from medium to high. ($X_{1,2}=1$; $X_{2,6}=1$; $X_{3,4}=1$, $X_{4,6}=1$, $X_{5,6}=1$, $X_{12,13}=1$, $X_{14,13}=1$, $X_{15,13}=1$, $X_{13,10}=1$, $X_{10,9}=1$, $X_{9,7}=1$; $X_{16,11}=1$, $X_{8,11}=1$, $X_{11,7}=1$, $X_{19,18}=1$, $X_{17,18}=1$).

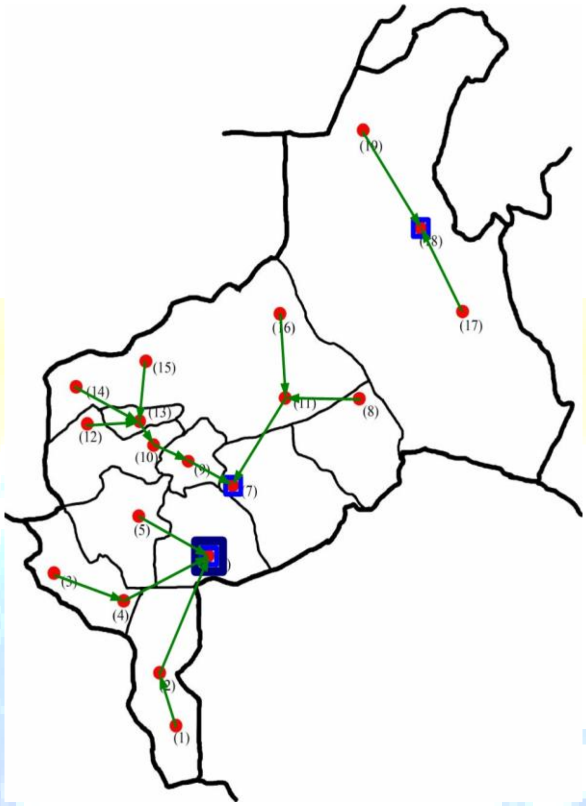

Fig. 2: The optimal solution of the case study

As Fig. 2 presents, in normal conditions for all new located and existing health care centers, the afflicted people of some towns should be transferred directly to the health care center located in the identified town. But the afflicted people of other towns should be transferred to the health care center located in the determined town via some intermediate towns. The optimal value of objective function is 38225029909.2MU and the fixed facility locating cost is 4733586000MU, a fixed road construction/improvement cost is 27720000000MU. Remember that in failure conditions of the link roads, the afflicted people of some towns should be transferred by some extra costs, leading to raise transportation costs and sometimes the further costs as "failure costs".

5. Sensitivity Analysis

In order to consider the effect of various parameters of the proposed mathematical model on the optimal solution, a relatively thorough sensitivity analysis have been conducted based on different variations in the range of the parameters of the model. Sensitivity analysis which is carried out in three different subsections is described as follows:

A) Changes of the number of new facilities

The number of facilities (*P*) is one of the significant parameters of the proposed model that can affect the optimal solution. Fig. 3 shows the changing procedure of *Z** according to the changes in values of (*P*) and the value of u_{ij}.

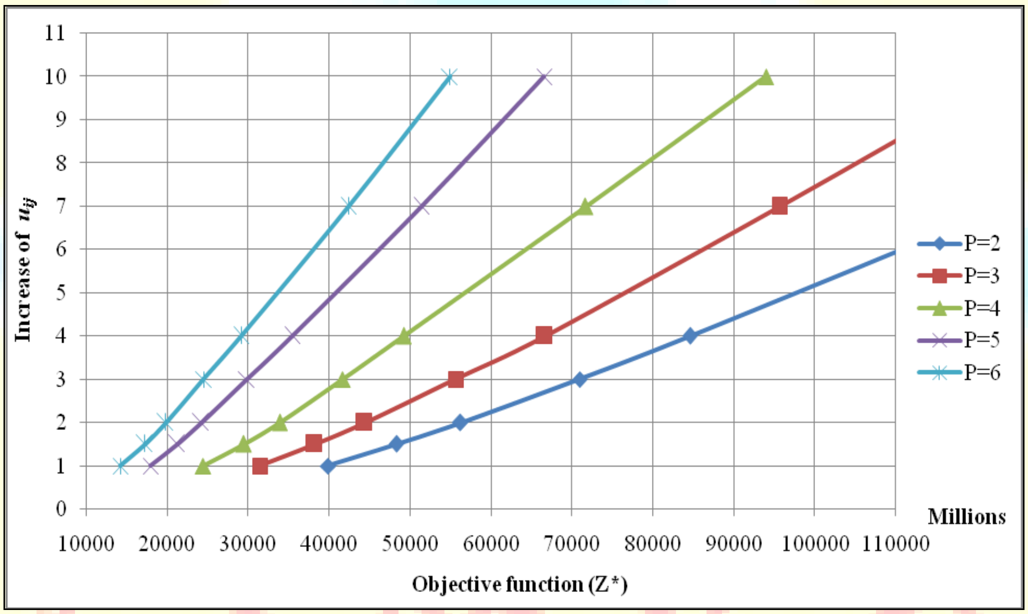

Fig. 3: The changing procedure of Z* according to the changing of the (*P*) and the u_{ij}

According to Fig. 3, one can conclude that, the value of *Z** will decrease for the initial different values of (*P*) and will decrease growing the number of facilities. This procedure is more obvious especially with growing the increasing coefficient of transportation cost of link (i , j) in disruption conditions (u_{ij}).

B) Changes of the transportation cost

Another considerable parameter of the model is the transportation cost (t_{ij}^{0}) in links of network that can have significant effect on the obtained optimal solution. Fig. 4 and Fig. 5 present the changing procedure of Z^* according to different increasing and decreasing values respectively in percentage of the transportation cost for different values of u_{ij}.

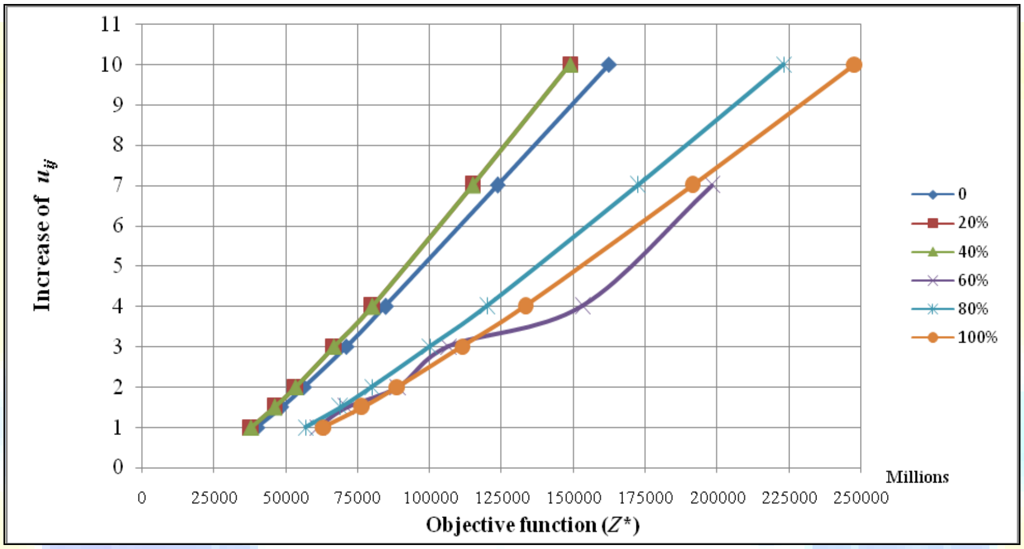

Fig. 4: The changing procedure of Z^* for different increasing in percentage of the transportation cost (t_{ij}^{0})

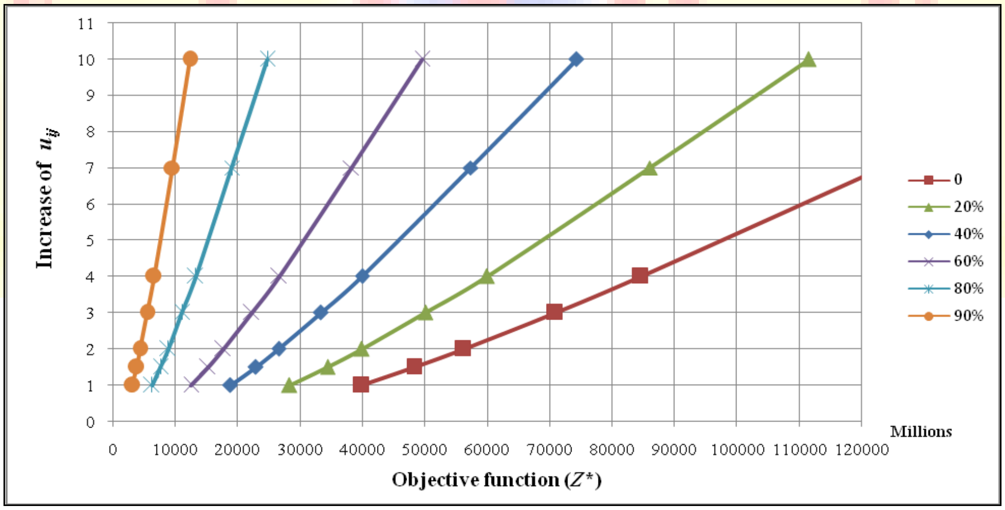

Fig.5: The changing procedure of Z^* for different decreasing in percentage of the transportation cost (t_{ij}^{0})

Fig. 4 shows that the Z^* increases when the transportation cost increases, and Fig. 5 demonstrates that the Z^* also increases when the transportation cost decreases. As Fig. 4 shows, for some increases of the transportation cost, the value of the Z^* increases and for others decreases; so we cannot conclude any specified procedure about pattern of it. However, Fig. 5 illustrates that with decreasing the transportation cost, the value of objective function (Z^*) decreases.

C) Changes of the link construction/improvement cost

The cost of link construction/improvement (c_{ij}) is another important parameter of the model that can affect the optimal solution. Fig. 6 and Fig. 7 present the changing procedure of Z^* according to different increasing and decreasing values respectively in percentage of c_{ij} for different values of u_{ij}.

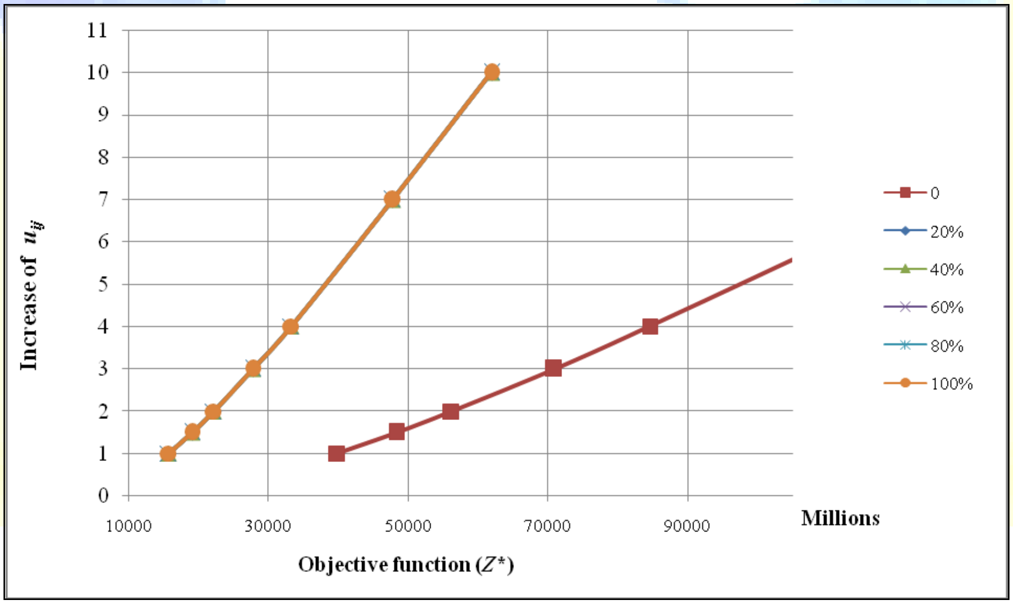

Fig. 6: The changing procedure of Z^* for different increasing in percentage of the link construction/improvement cost (c_{ij})

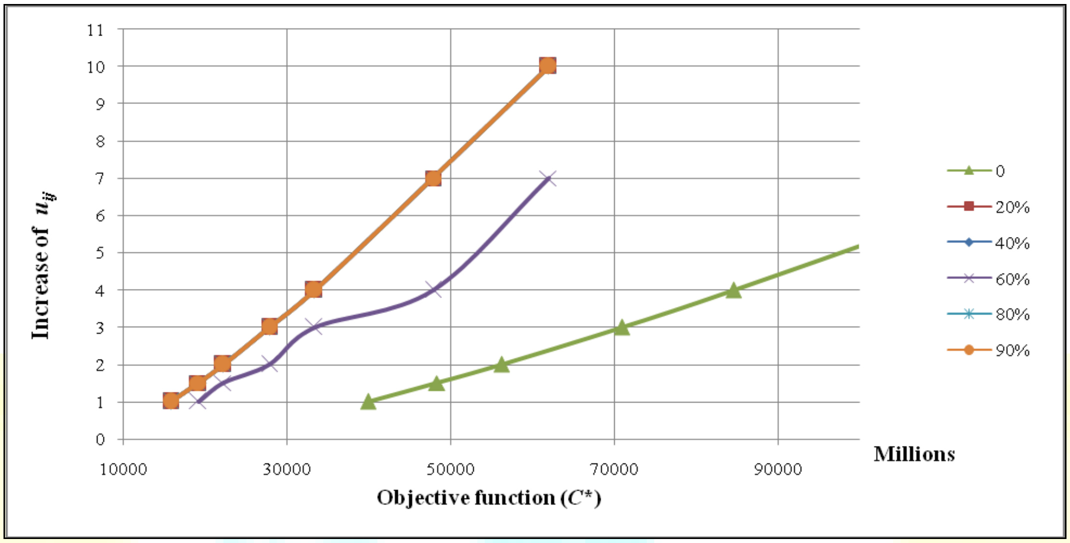

Fig. 7: the changing procedure of Z^* for different decreasing in percentage of the link construction/improvement cost (c_{ij})

In the same way, Fig. 6 and Fig. 7 demonstrate the changing pattern of Z^* for different values of u_{ij} and different percentages of increasing and decreasing in c_{ij} . According to Fig. 6 and Fig. 7, one can find out that for our case study, with increasing or decreasing of link construction/improvement cost, the value of Z^*decreases but not very sensitive and sharp; however, the effect of link construction/improvement cost (c_{ij}) on the growing of Z^* is not as much as the transportation cost (t_{ij}^0).

As a general concluding remarks on sensitivity analysis of parameters in the proposed mathematical model, the changes of the number of facilities (P) has the greatest effect on the changing procedure of the value of Z*, while the least effect on the changing procedure of the value of Z^* is related to the changes of the (c_{ij}).

6. Conclusions and future research

Considering of several practical factors in efficient designing of facility location and allocating demands to them plays a critical rule in obtaining a suitable practical solution. In this paper, a mixed integer-linear programming (MILP) model was proposed in order to both minimize the expected transportation costs in a kind of facility location problem with respect to budget constraint, network design and link disruption topics. The proposed MILP model determines the optimal locating of new facility locations, optimal constructing of transportation links and also optimal allocating demand nodes to facilities regarding to constructed links.

A practical case study was presented in detail to illustrate the application of the proposed mathematical model. The results show that the model not only can present a more accurate description of RBFLNDP but also can propose efficient feasible solutions to be used in industries and services. Moreover, a sensitivity analysis was done to provide an insight into the behavior of the model in response to changes of the key parameters. The results show that the greatest effect on the changing procedure of the value of Z^* is related to the changes of the number of facilities (P) while the changes of link construction/improvement cost (c_{ij}) has the least effect on the changing procedure of the value of Z^*.

For future researches, first, in this paper, only the RBFLNDP with uncapacitated facilities and links was studied; however, considering the RBFLNDP with capacitated system can have more practical application in real industrial and service environments. Second, one can seek and test some efficient heuristics and metaheuristics such as tabu search (TS) and particle swarm optimization (PSO) for improving the efficiency of proposed solution method.

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