PREDICTION OF INTER-LAMINAR STRESSES IN COMPOSITE LAMINATES USING MIXED FINITE ELEMENT MODEL

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Abstract:

The introduction of composite materials has brought out a radical change in material applications in various fields. Its high strength to weight ratio, long fatigue life and other superior material properties have paved way for its use in highly weight sensitive applications such as aerospace & automotive structures. In all these applications, the level of precision and accuracy is very significant and hence a good understanding of the behavior of composite materials is essential. Delamination is one of the critical problems faced by composite laminates. It involves separation of composite laminae, especially at the free edge caused due to low strength along the ply interface and high local interlaminar stresses. High interlaminar shear stress on the free edge of angle-ply laminates can cause edge delamination, whereas the same effect will be produced by interlaminar normal stress in cross-ply laminates. Thus it is essential to accurately predict the interlaminar stresses to understand the failure mechanism involved in the delamination of laminated composites. In the present study, an 18-noded three-dimensional mixed finite element model is developed, having three transverse displacement and three interlaminar stresses as degrees of freedom per node, using minimum potential energy principle. The transverse stress quantities are invoked from the assumed displacement fields by using fundamental elasticity relations. This ensures the satisfaction of elasticity equations throughout an elastic continuum, which is lacking in numerical methods based on other mixed variational principles. The study would be extended for free and forced vibration analysis of the composite laminate.

Key Words: Metal matrix composites, Delamination, Interlaminar stress, Mixed finite element model

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Introduction:

It has been found that displacement-based equivalent single layer theories are not suitable for their analysis. Layer-wise mixed theories are required to predict interfacial transverse stresses. Earlier researchers have used two-dimensional mixed finite element models, where displacement continuity conditions at the interface are treated as the constraints and interlaminar stresses are introduced as Lagrange's multiplier. Three-dimensional mixed finite element models have also been proposed using global-local laminate variational model as well as Reissner's variational principle. In the models developed using Reissner's variational principle, the stress fields are assumed independent of the displacement fields. Hence the fundamental elasticity relation cannot be satisfied exactly.

Delamination is a failure mechanism that involves separation of composite laminae especially at the free edge, caused due to low strength along the ply interface and high local interlaminar stresses. Studies have shown that high interlaminar shear stresses on the free edge of angle ply laminates can cause delamination [1], as can high interlaminar normal stress in cross ply laminates [2]. The initiation and growth of this failure mode is due to out of plane stresses operating against the inherent weakness of the matrix. Any type of loading, mechanical, thermal or hygrothermal may evoke interlaminar stresses. These stresses between layers are caused by the mismatch in material properties of bonded adjacent layers. It has been found that the interlaminar stresses grow very rapidly at stress free boundaries. This phenomenon has been found to occur only within a very local region near the geometric boundaries.

Behaviour of composite laminates can be characterized by complex three-dimensional stress states, evidencing high interlaminar stresses caused by inherent anisotropy and mismatch of material properties of such structural members. Elasticity solutions on layered plates [3-5] indicate that interlaminar continuity of transverse normal and shear stresses as well as the layerwise continuous displacement field through the thickness of the laminated plates are the essential requirements for their analysis.

Analysis of angle-ply laminate becomes more critical and computationally involved due to the presence of extension-shear coupling. From the available literature on the analysis of angle-ply laminates [6-12], it can be emphasized that displacement-based equivalent single layer theories (ESL) are not suitable for their analysis. In order to address the behaviour of an angle-ply laminate that involves high interfacial transverse stresses with high stress gradients, layerwise mixed theories seems to be most appropriate.

A mixed layerwise finite element model with displacement and transverse stress components as primary variables can very well satisfy the requirements of transverse stress continuity in addition to the continuity of displacement fields through the thickness of the laminated composite structure. Transverse stress components are evaluated directly through such a mixed FE model. Thus, integration of equilibrium equations can be avoided which involves differentiation of in-plane stresses and displacement fields thereby introducing further approximation in the calculation of transverse stresses. Wu and Lin [13], for example, presented a two-dimensional mixed finite-element scheme based on a local higher-order displacement model for the analysis of sandwich structure, where displacement continuity conditions at the interface between layers were regarded as the constraints and the interlaminar stresses were introduced as the Lagrange multiplier. Shi and Chen [14] developed a three-dimensional mixed FE model based on the global-local laminate variational model. The model proposed a mixed use of a hybrid stress element within a high precision stress solution region in the thickness direction of the laminate and a conventional displacement finite element in the remainder. Carrera [15] developed a mixed plate element as an Extension to the C^0 Reissner-Mindlin plate element by using Reissner's mixed variational principle, in which the zig-zag variation of the in-plane displacement fields through the thickness was ensured by including additional terms in the standard Reissner-Mindlin plate model, whereas the transverse displacement field was kept unchanged. Stress degrees of freedom were introduced by assuming transverse shear stress fields. As further development, Carrera [15, 16] later also introduced the transverse normal stress field into the FE model. Because of the fact that in any mixed finite-element model developed by using Reissner's variational principle, the stress fields are assumed independent of the displacement fields, the fundamental elasticity relation cannot be satisfied exactly.

Desai *et al*. proposed an 18-node three-dimensional mixed finite-element [17] by using the minimum potential energy principle. The transverse stress quantities (τ_{xz}, τ_{yz} and σ_z, where z is the thickness direction) have been invoked from the assumed displacement fields by using fundamental elasticity relations. This ensures the satisfaction of elasticity equations through an elastic continuum, which is lacking in numerical models based on other mixed variational principles.

Because the transverse stress components are the nodal degrees of freedom in this model, their computation does not require the integration of equilibrium equations which otherwise reduce the accuracy in determination of these stresses. Moreover, it can appropriately model a composite laminated structural member of any number of lay-ups of different materials as it satisfies exactly, the requirements of through thickness continuity of transverse stress and displacement fields.

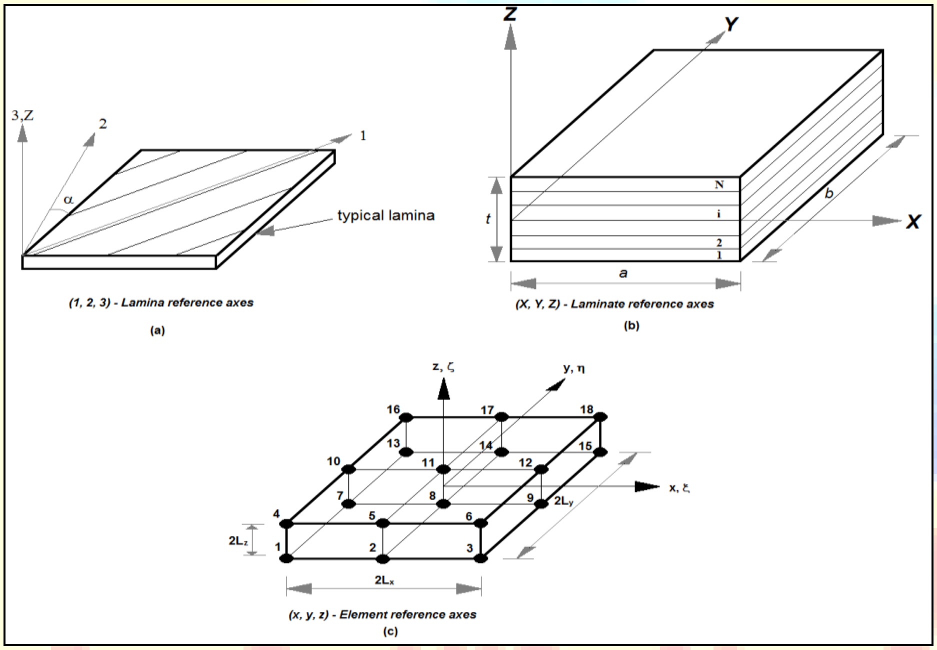

Figure 1. Geometry of: (a) ith lamina; (b) laminated plate; and (c) 18-node mixed finite element with positive set of reference axes

Methodology:

An 18-node three dimensional mixed finite-element model shown in Figure 1(c) has been developed by considering the displacement fields $u(x,y,z), v(x,y,z) and\ w(x,y,z)$ having quadratic variation along the plane of the plate and cubic variation in the transverse direction. The general displacement fields are expressed as:

$u_k = \sum_{i=1}^{n} N_i u_{k_i}$ where N_i represents the shape functions. ----------------------(1)

Since the displacement fields are assumed to have quadratic variation along the plane of the plate, we can use the standard Lagrangian interpolation functions for a nine noded element to get the multipliers along ξ and η directions. Hence the displacement fields can be expressed as

$$u_k(x,y,z)=\sum_{i=1}^{3}\sum_{j=1}^{3} g_i h_j a_{0ijk} + z\sum_{i=1}^{3}\sum_{j=1}^{3} g_i h_j a_{1ijk} + z^2\sum_{i=1}^{3}\sum_{j=1}^{3} g_i h_j a_{2ijk} + z^3\sum_{i=1}^{3}\sum_{j=1}^{3} g_i h_j a_{3ijk}, \quad k=1,2,3 \qquad \text{------------(2)}$$

where

$$g_1=\frac{\xi}{2}(\xi-1), \quad g_2=1-\xi^2, \quad g_3=\frac{\xi}{2}(1+\xi), \quad \xi=x/L_x \qquad (3.2)$$

$$h_1=\frac{\eta}{2}(\eta-1), \quad h_2=1-\eta^2, \quad h_3=\frac{\eta}{2}(1+\eta), \quad \eta=y/L_y \qquad (3.3)$$

are the Lagrangian interpolation functions for a nine noded element and

$u_1=u;\ u_2=v;\ u_3=w$

Considering cubic variation in the thickness direction, the displacement field can be expressed as

$$u=a_1+a_2z+a_3z^2+a_4z^3 \qquad \text{------------------------(3)}$$

Each lamina in the laminate has been considered to be in a three dimensional state of stress so that the constitutive relation for a typical ith lamina with reference to the fibre-matrix coordinate axes (1, 2, 3) can be shown to be

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{12}\\ \tau_{13}\\ \tau_{23}\end{Bmatrix}^i=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{12}&C_{22}&C_{23}&0&0&0\\ C_{13}&C_{23}&C_{33}&0&0&0\\ 0&0&0&C_{44}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{66}\end{bmatrix}^i\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{12}\\ \gamma_{13}\\ \gamma_{23}\end{Bmatrix}^i \qquad \text{----------------------(4)}$$

where $(\sigma_1,\sigma_2,\sigma_3,\tau_{12},\tau_{13},\tau_{23})$ are the stresses and $(\varepsilon_1,\varepsilon_2,\varepsilon_3,\gamma_{12},\gamma_{13},\gamma_{23})$ are the linear strain components referred to the lamina coordinates (1, 2, 3) and the C_{mn}s (m,n = 1,....,6) are the elastic constants of the ith lamina.

The earlier models refers to a single transformation of the stress-strain matrix for getting material properties along global coordinates, since the element coordinates are assumed to be parallel to global coordinates and the fiber is oriented at an angle with the global x axis. The present formulation would be more suited for practical cases where the element itself will be oriented at an angle to the global coordinates. Hence a second transformation matrix will also be involved. The transformed stress-strain matrix would be defined as

$$[D] = [T]_2[T]_1[C][T]_1^{-1}[T]_2^{-1} \qquad \text{--------------(5)}$$

where $[T]_1$ represents the first transformation matrix, for transforming material properties from the fiber direction to the element coordinates, and $[T]_2$ represents the second transformation matrix for transforming material properties from element coordinates to global coordinates.

The stress-strain relations for the ith lamina can be written in the laminate coordinates X, Y, Z as

$$\{\sigma\} = [D]\{\varepsilon\} \qquad \text{--------------------------(6)}$$

Here $\{\sigma\} = [\sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{xy} \quad \tau_{xz} \quad \tau_{yz}]^T$ and $\{\varepsilon\} = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}]^T$ are the stress and strain vectors with respect to the laminate reference axes (Fig 1(b))

Finite Element Formulation:

The transverse stresses can be obtained from the constitutive equation (6) and strain-displacement relations as

$$\begin{Bmatrix} \sigma_z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \qquad \text{----------(7)}$$

which can be rewritten and split as follows

$$\begin{Bmatrix} \sigma_z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} D_{31} & D_{32} & D_{34} & D_{33} & D_{35} & D_{36} \\ D_{51} & D_{52} & D_{54} & D_{53} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{64} & D_{63} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varepsilon_z \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

$$\begin{Bmatrix}\sigma_z\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} = [Dx]\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} + [Dz]\begin{Bmatrix}\varepsilon_z\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}$$

$$[Dx] = \begin{bmatrix}D_{31} & D_{32} & D_{34}\\ D_{51} & D_{52} & D_{54}\\ D_{61} & D_{62} & D_{64}\end{bmatrix}, [Dz] = \begin{bmatrix}D_{33} & D_{35} & D_{36}\\ D_{53} & D_{55} & D_{56}\\ D_{63} & D_{65} & D_{66}\end{bmatrix}$$

$$\begin{Bmatrix}\varepsilon_z\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial w}{\partial z}\\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\\ \frac{\partial y}{\partial z} + \frac{\partial w}{\partial y}\end{Bmatrix} = [Dzi]\begin{Bmatrix}\sigma_z\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} - [Dzx]\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

-----------(8)

where $[Dzi] = [Dz]^{-1};\ [Dzx] = [Dz]^{-1}[Dx]$

from equation(8), the following equations may be obtained:

$$\frac{\partial u}{\partial z} = Dzi_{21}\sigma_z + Dzi_{22}\tau_{xz} + Dzi_{23}\tau_{yz} - Dzx_{21}\varepsilon_x - Dzx_{22}\varepsilon_y - Dzx_{23}\gamma_{xy} - \frac{\partial w}{\partial x}$$

$$\frac{\partial v}{\partial z} = Dzi_{31}\sigma_z + Dzi_{32}\tau_{xz} + Dzi_{33}\tau_{yz} - Dzx_{31}\varepsilon_x - Dzx_{32}\varepsilon_y - Dzx_{33}\gamma_{xy} - \frac{\partial w}{\partial y}$$

$$\frac{\partial w}{\partial z} = Dzi_{11}\sigma_z + Dzi_{12}\tau_{xz} + Dzi_{13}\tau_{yz} - Dzx_{11}\varepsilon_x - Dzx_{12}\varepsilon_y - Dzx_{13}\gamma_{xy}$$

--------(9)

The displacement fields $u(x, y, z)$, $v(x, y, z)$ and $w(x, y, z)$ expressed in equation (1), can now be further expressed in terms of nodal variables by using equations (9) as:

$$u_k(x, y, z) = \sum_{n=1}^{18} g_i h_j (f_q u_{kn} + f_p \hat{u}_{kn})$$

---------------(10)

where 'n' is the node number in the 3D element, shown in Figure 1(c). u_{kn} (k = 1, 2, 3 and n = 1, 2,...., 18) are the nodal displacement variables whereas $\hat{u}_{kn}$ contains the nodal transverse stress variables.

Finally equation (10) yields the displacement fields $u(x, y, z)$, $v(x, y, z)$ and $w(x, y, z)$ in terms of the nodal degrees of freedom as

$\{u\} = [N]\{q\}$ where

$\{u\} = [u \quad v \quad w]^T \quad [N] = [N_1 \quad N_2 \quad \cdots \quad N_n \quad \ldots \quad N_{18}] \quad \{q\} = [q_1^T \quad q_2^T \quad \ldots \quad q_n^T \quad \ldots \quad q_{18}^T]^T$

$$\{q_n\} = [u_n \quad v_n \quad w_n \quad (\tau_{xz})_n \quad (\tau_{yz})_n \quad (\sigma_z)_n]^T \qquad \text{---------(11)}$$

The total potential energy Π of the laminate can be obtained from

$$\Pi = \frac{1}{2}\int_V \{\varepsilon\}^T\{\sigma\}dV - \int_V \{q\}^T\{p_b\}dV - \int_\Sigma \{q\}^T\{p_t\}\,ds \qquad \text{--------------(12)}$$

where $\{p_b\}$ is the body force vector per unit volume and $\{p_t\}$ is traction load vector acting on any surface of the laminated plate. Here 'Σ' is a surface of the element subjected to traction forces. The strain vector $\{\varepsilon\}$ and the stress vector $\{\sigma\}$ can be expressed as

$\{\varepsilon\} = [B]\{q\}$ and $[B] = [L][N]$ where

$$[L] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix}^T$$

$\{\sigma\} = [D][B]\{q\}$ and $[B] = [B_1 \quad B_2 \quad \ldots \quad B_n \quad \ldots \quad B_{18}]$

Minimization of the total potential energy functional, equation (12), yields the element property matrix $[K]^e$ and the element influence vector $\{f\}^e$ as

$$[K]^e = \int_{-L_x}^{L_x}\int_{-L_y}^{L_y}\int_{-L_z}^{L_z} [B]^T[D][B]dxdydz$$

$$[f]^e = \int_{-L_x}^{L_x}\int_{-L_y}^{L_y}\int_{-L_z}^{L_z} [N]^T\{p_b\}dxdydz + \iint_\Sigma [N]^T\{p_t\}ds \qquad \text{-----------(13)}$$

Numerical integration:

Gaussian quadrature is used to integrate elemental stiffness matrix and force vector. The numerical integration has been performed by considering three Gaussian points along the 'x' and 'y' directions; and five Gauss points along the 'z' direction of the element. Thus, in all, 45 Gauss points have been considered per element in numerical integration. Gaussian quadrature for 3 and 5 point integration rules are given in Table 1

The numerical integration procedure to obtain stiffness matrix is given below

$$[K]^e = \int_{-L_x}^{L_x}\int_{-L_y}^{L_y}\int_{-L_z}^{L_z}[B]^T[D][B]dxdydz$$

$$[K]^e = \int_{-1}^{1}\int_{1}^{-1}\int_{-1}^{1}[B]^T[D][B]L_xL_yL_z\,d\xi\,d\eta\,d\zeta$$

$$[K]^e = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{5}[B]^T[D][B]W_1(i)W_2(j)W_3(k)L_xL_yL_z$$ ------------(14)

Table 1. Sample points and weights for Gaussian quadrature

No. of points	Sample points	Weights
3	-0.7745966692	0.5555555556
	0	0.8888888889
	0.7745966692	0.5555555556
5	-0.9061798459	0.2369268851
	-0.5384693101	0.4786286705
	0	0.5688888889
	0.5384693101	0.4786286705
	0.9061798459	0.2369268851

The global equation can be obtained in the following form after assembly:

$$[K]\{Q\} = \{F\}$$ ------------------(15)

where $[K]$, $\{Q\}$ and $\{F\}$ are, respectively, the global property matrix, the global degrees of freedom vector and the global influence vector.

The source code:

A computer program incorporating the present three-dimensional mixed formulation has been developed in FORTRAN-90 for the analysis of symmetric/unsymmetric composite laminates. The code for the present formulation is given in Annexure.

Validation:

The intended purpose of the following set of problems is to ascertain the accuracy of the developed elements in various applications and thus evaluating the element prior to actual use. Though models with regular geometry rarely exist in reality, due to the limitation in data extraction for an 18- noded element, only models with regular geometry, i.e. models which have a cuboidal shape or models which can be divided into a set of cuboids alone have been considered. The results are tabulated by normalizing the values obtained from the FEM solution with the theoretical results available in literatures. For the validation problem, a laminated plate undergoing cylindrical bending is considered. The loading is sinusoidal, that is $q = q_0 \sin(\pi x/L)$.

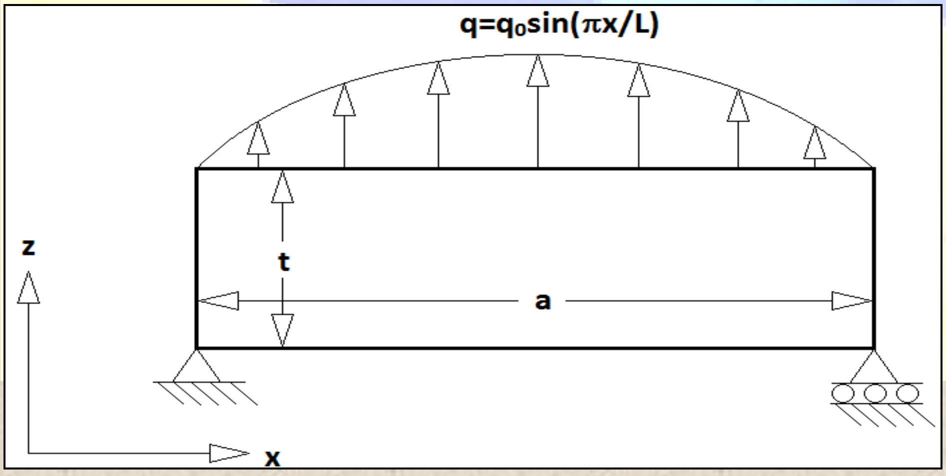

Figure 2. Geometry of the model

Consider a plate with two cross-ply laminates ($0°/90°$) of equal thickness with transversely isotropic materials. The geometry of the model is as shown in Figure 2. The orientation of the fiber in the bottom layer is parallel to the length of the strip (say x-direction) and in the top layer the orientation is transverse to the length. The material properties are same as those used by Pagano [6] for a transversely isotropic high-modulus graphite/epoxy composite. These are expressed in Table 2.

Table 2. Material properties and boundary conditions

Material properties:

$E_1 = 25 \times 10^6 psi$	$E_2 = E_3 = 10^6 psi$
$G_{12} = G_{13} = 0.5 \times 10^6 psi$	$G_{23} = 0.2 \times 10^6 psi$
$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$	

Boundary conditions:

Surface/edges	BC on displacement field	BC on stress field
Edge $X = 0; Z = 0$	$u = v = w = 0$	–
Edge $X = a; Z = 0$	$v = w = 0$	–
Faces $X = 0$ and $X = a$	$w = 0$	–
Faces $Y = 0$ and $Y = b$	$v = 0$	–
Top face $Z = t/2$	–	$\sigma_z = \hat{q}(X, Y)$

Results and discussion:

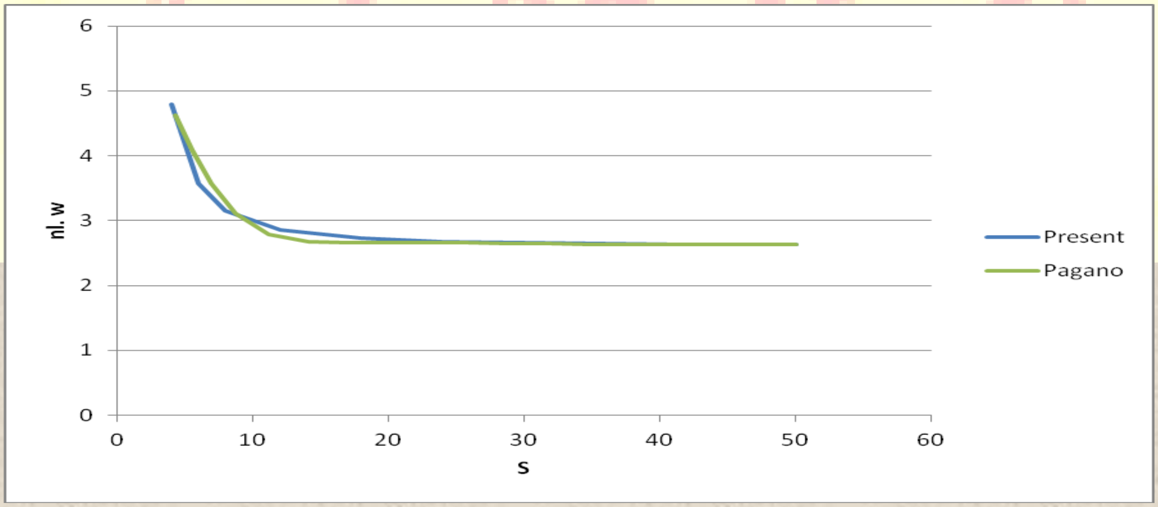

Figure 3.Normalized transverse deflection corresponding to different aspect ratios (L/t) due to cylindrical bending of a two laminae ($0^\circ/90^\circ$) plate

Plane-strain conditions are imposed by modelling the geometry with the unit width and applying boundary conditions as shown in Table 2. The plot of the normalized transverse displacement $\overline{w}$ for aspect ratios (L/t) between 4 and 50 are shown in Figure 3. The close agreement between the current solution and those of Pagano's elasticity solution is evident.

Conclusions:

The evaluation of interlaminar stresses to high level of accuracy is very essential while considering designs of weight sensitive applications like aerospace and automotive structures. The ESL theories have been found to unsuitable for such analysis since it causes discontinuity of interlaminar stresses at the interface of any two successive laminae. Hence mixed formulations are very essential for the continuity of stresses through the thickness of a laminated plate. Though the continuity of stresses is satisfied, most of the mixed formulations do not satisfy the fundamental elasticity equations exactly. The limitation of the earlier models is that the fiber orientation of laminae used in those model should always be with respect to the global axes, which is not the case with most of the practical cases. The model proposed overcomes this limitation by modifying the formulation, so as to include the analysis of models with complex geometries also.

A source code has been developed in FORTRAN for this formulation. The element is validated with standard test results for a typical problem.

Scope of future work:

Since the element is a non-standard one, currently no software provides the display and meshing for this element. This limitation prevents the analysis of complex models. Hence only simple models with regular geometry can be used for validations and the case studies. This problem can be overcome by developing a suitable software for meshing of complex elements.

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