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# CONTENTS

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Title

# RELIABILITY PREDICTION OF FAULT-TOLERANT MULTICOMPUTER INTERCONNECTION NETWORKS

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## Abstract:

This paper proposes a new method to identify all the maximal incomplete sub cubes present in a faulty cube taking maximum fault tolerance level i.e. number of faulty nodes is equal to the system dimension. The procedure is a distributed one, as every healthy node next to a failed one performs the same procedure independently and concurrently. Then the reliability expression for the maximal incomplete sub cube is derived. This method is well supported by an efficient algorithm which runs polynomially. The proposed method is found to be simple, general and efficient and thus is applicable to all the cube based topologies. The reliability of some important cube based topologies are evaluated and compared under the same condition.

**Key words**: Cube, Hypercube, Maximal Incomplete sub cube, discarded region, Conjugate sub cube, Reliability.

## Introduction:

A multicomputer interconnection network is used to interconnect a large number of standalone processors. Therefore a wide variety of interconnection networks have been proposed for parallel computing systems like rectangular meshes, trees, shuffle exchange networks, omega networks and binary cubes [1, 3]. A widely used topology for the interconnection of computing nodes in multiprocessor systems is the binary cubes, also known as the Boolean n-cubes. Due to attractive properties like regularity, symmetry, small diameter, strong connectivity, recursive construction and partition ability the n-cube topology has enjoyed the largest popularity. These property leads to simple routing, support for wide application spectrum and fault tolerance for interconnection system [2, 9].

The n-dimensional cube is composed of $2^n$ nodes and has n- edges per node, n-bit binary addresses are assign to the nodes to the cubes in such a way that an edge or link connects two nodes if and only if their binary addresses differ by a single bit .This Inter connection network supports large numbers of resources with small diameters. But the major drawbacks of the cube networks are the numbers of communication ports and channels per processors is the same as the logarithm of the total numbers of processors in the system. Therefore the number of communication ports and channels per processors increases by increasing the total number of processors in the system [2]. This drawback seems to be waived in the case of incomplete cubes, which shows the emulation performance as the n/w scales up in size [4, 5].

The probability of fault in a larger system is given due importance [12]. Whenever a fault arises, an n-cube may operate in a gracefully degradable manner due to the execution of parallel algorithms in smaller fault free sub cubes, which are comprises of healthy nodes. In order to maintain cube topology in the presence of faults, researchers have proposed addition of spare nodes thereby replacing the failed components with spares [6]. This results in a much larger system than what is attained by any conventional reconfiguration scheme which identifies only complete sub cube [7]. Also fault tolerance can be achieved by reconfiguring the larger system to smaller sized system after the occurrence of fault. Unlike a complete one, an incomplete cube can be of any arbitrary size, i.e. can be used to interconnect systems with any numbers of processors, making it possible to finish a given batch of jobs faster than it's complete counterpart alone by supporting simultaneous execution of multiple jobs of different sizes by assigning more nodes to execute the job cooperatively [8]. Thus reconfiguring a faulty n-cube in to a maximal incomplete cube tends to lower potential performance degradation [14].

With the increase in size, the complexity of the interconnection network increases there by corresponding increase in computational power to maintain acceptable performance under reliable conditions. For this the reliability prediction of a multicomputer network is quite essential, to be used in critical applications [10].

This paper proposes an efficient distributed procedure for locating or identifying all maximal incomplete subcubes present in a faulty n-cube. This procedure exhibit better empirically polynomial time complexity with respect to the system dimension and the number of faults. The concept of discarded regions eliminates those nodes impossible to be part of any fault free sub cube containing the given node. There by forming the maximal incomplete sub cube. This method is illustrated through a 3-dimensinal hypercube. Then a generalized reliability expression for the maximal incomplete sub cube has been derived, which is supported by an effective algorithm. The reliability of three important multicomputer interconnection networks viz. Hypercube ($HC_n$), Crossed cube ($CQ_n$) and Folded hypercube ($FH_n$) [11, 13] has been computed by using the proposed algorithm and compared under the same platform.

## Architectural Details of the Cube based topologies:

### Hypercube ($HC_n$):

An n-dimensional cube network of N processing elements (PEs) is specified by the following routing functions:

$$C_i(a_{n-1}\cdots a_1a_0)=a_{n-1}\cdots a_{i+1}\overline{a}_{i-1}\cdots a_1a_0\ ,\ for\ i=0,1,2,\cdots n-1 \quad (1)$$

In the *n* cube, each PE located at a corner is directly connected to n neighbors. The neighboring PEs differs in exactly one bit position. There are $2^n$ number of processing elements and $n.2^{n-1}$ number of links in an *n*-dimensional hypercube (Fig.1).

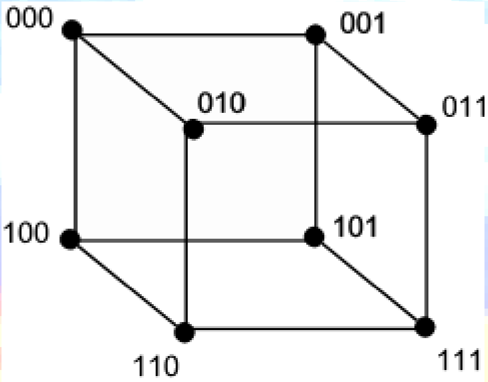


**Figure 1: Hypercube (*N=8, n=3*)**

### Crossed Cube ($CQ_n$):

The n-dimensional crossed cube ($CQ_n$) is a connected, regular graph of degree *n* with $2^n$ vertices (Fig.2). It is a labeled graph defined recursively as follows: $CQ_1$ is $K_2$, the complete graph on two vertices with labels 0 and 1. For $n>1$, $CQ_n$ contains $CQ^0_{n-1}$ and $CQ^1_{n-1}$ joined according to the following rule: the vertex $u=0u_{n-1}\ u_{n-2}\ \ldots u_1$ *from* $CQ^0_{n-1}$ and vertex $v=1v_{n-1}\ v_{n-2}\ldots v_1$ from $CQ^1_{n-1}$ are adjacent in $CQ_n$ if and only if

(i) $u_{n-2}=v_{n-2}$ *if n is even*

and

(ii) *for* $0 \leq i < [(n-1)/2]$, $u_{2i+1}\, u_{2i}\, v_{2i+1}\, v_{2i}$ (2)

It follows from the above explanation that every vertex in $CQ_n$ with a leading 0 bit has exactly one neighbor with a leading bit and vice versa. Further, in a crossed cube, for all $n \geq 1$, $(u_{n-1} \ldots u_0, v_{n-1} \ldots v_0)$ is an edge of $CQ_n$ if and only if there exists a 1 with

(i) $u_{n-1} \ldots u_l = v_{n-1} \ldots v_l$

(ii) $u_{l-1} \neq v_{l-1}$

(iii) $u_{l-2} = v_{l-2}$ *if l is even*, and

*for* $0 \leq i < [(n-1)/2]$, $u_{2i+1}\, u_{2i}\, v_{2i+1}\, v_{2i}$ (3)

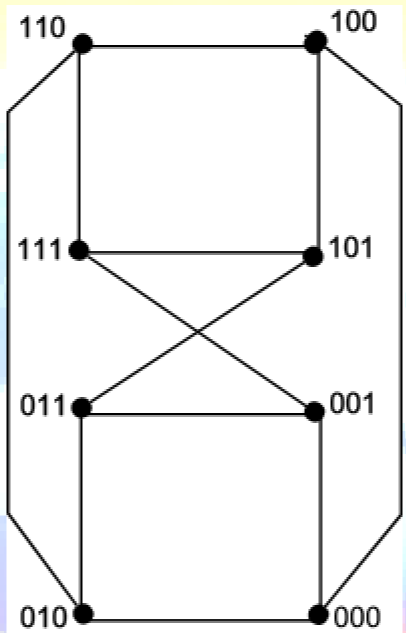


**Figure 2: Crossed Cube *(N=8, n=3)***

**<u>Folded Hypercube</u> (FHC *n* ):**

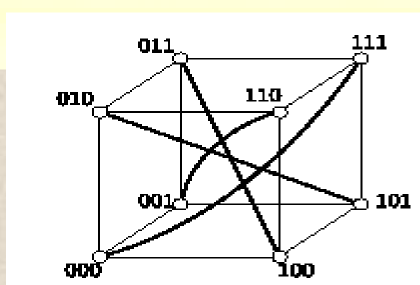


**Figure 3: Folded hypercube *(N=8, n=3)***

As a variant of the hypercube, the n-dimensional folded hypercube $FHC_n$ is obtained from the hypercube $HC_n$ by adding $2^{n-1}$ edges, called complementary edges. Each of them in between vertices, X= $(x_1, x_2, \ldots\ldots, x_n)$ and $\overline{\overline{X}} = (\bar{x}_1, \bar{x}_2, \ldots\ldots\ldots\ldots\bar{x}_n)$ where $\bar{x}_i = 1 - x_i$. The graph of $FHC_3$ is shown in fig.3. It has been shown that $FHC_3$ is ( n+1) –regular (n+1)-connected . Like $HC_n$ , $FHC_n$ is a Cayley graph and so $FHC_n$ is vertex transitive.

It has diameter of (n/2), about a half of the diameter of $HC_n$ . Thus the folded hypercube is superior and enhanced version of $HC_n$ .There are n+1 internally disjoint paths of length at most (n/2)+1, between any pair of vertices in $FHC_n$ .The deletion of less than[n/2]-2 vertices or edges does not increase the diameter of $FHC_n$ and the deletion of up to n vertices or edges increase the diameter by at most one. The above properties mean that interconnection networks modeled by $FHC_n$ are extremely robust.

## **Proposed method for finding subcube & reliability:**

### **Notation and assumptions:**

*Notation:*

$IC_n$ *n-dimensional cube interconnection network*

$I_{n-1}$ *maximal incomplete subcube*

$s$ *source node*

$d$ *destination node*

$\otimes$ *discarding operation*

* *don't care symbol*

$n$ *system dimension*

$N$ *numbers of nodes in hypercube*

$u,v,w$ *adjacent nodes of source* node

$\bar{v}$, $\bar{w}$ *antipodal nodes of v, w*

$\lambda$ *node failure rate node*

*t* *mission time*

*G* *probabilistic graph or Reliability Logic Graph*

p *probability of success*

*q* *probability of failure*

Assumptions:

1. Nodes and links failures are statistically independent of each other.
2. The identification process is uniformly distributed.
3. Repair facility is not available.

## Basic Properties:

**Definition 1:** A maximal sub cube for a given source node S, is a fault free sub cube which involve S but not present entirely in any other fault free sub cube involving S for a given faulty node.

The maximal sub cube is defined with respect to a given node which may be of different sizes for different given nodes. Care should be taken so that no faulty nodes and antipodal nodes of given nodes present in maximal sub cube.

**Definition 2:** A maximal incomplete sub cube is obtained when link is properly added to a maximal sub cube of n-1 dimension so that the destination node D is reached.

Unlike complete cube an incomplete cube can be constructed with any numbers of nodes to avoid the practical restriction of cube topology on the numbers of nodes in a system must be a power of 2. A proper incomplete sub cube in a faulty cube refers to a fault free incomplete sub cube which is not contained entirely in any of the fault free sub cube.

**Definition 3:** A discarded region in an interconnection network is the smallest sub cube comprises of a faulty node and the antipodal nodes of the (n-1) fault free adjacent nodes.

For a faulty n-cube ICn and a given source node S. It is possible to identify systematically every fault free sub cube which involves the source node S. This is expressed by set P = {Pi/Pi is a fault free sub cube in ICn and Pi involve node S}.This can be done by determining the region which never contribute to any fault free sub cube containing the node S.

Each fault results in one such regions known as discarded region which is the smallest sub cube involving both the faulty and the antipodal nodes of adjacent (n-1) nodes. A discarded region is addressed by performing ⊗ operation on the labels of the faulty node and the antipodal node where ⊗ is the bit operation defined as: it yields 0 (or 1) if the two corresponding bits are "0" (or "1"),and it is ∗ if the two corresponding bits differ.

**Definition 4:** A conjugate sub cube is either a complete sub cube or may be incomplete one, formed by taking the nodes present out side the maximal incomplete sub cube.

The conjugate sub cube must involve the faulty node and the antipodal nodes of the n-1 adjacent nodes. The maximal incomplete sub cube and its conjugate sub cube together form a complete n-cube. The conjugate sub cube at least involve one fault, otherwise a fault free $IC_n$ exists. Every fault free incomplete sub cube has one and only one conjugate sub cube.

**Theorem-1:** For an n-dimensional cube with a given maximum tolerance level 'n', the reliability expression is recursively found to be

$$R_{s-d} = {}^{n}C_1\, {}^{n-1}C_1\, {}^{n-2}C_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots {}^{2}C_1 * {}^{2^{n-1}-2}C_{n-2} * P^{n+1} * (1-P)^{2^{n-1}-n}$$

**Proof:** Given a source node 'S ' destination node 'D'. As the dimension of the graph is n, Adj(s) | =n. Choosing a node as faulty it can be carried out in ${}^{n}C_1$ ways. Out of the remaining n-1 Adj(s) nodes the path from source to destination can be taken in ${}^{2^{n-1}-2}C_{n-2}$ ways.

Now discarding the faulty node a maximal incomplete sub cube is obtained.

The given tolerance level is n i.e. n number of nodes can be faulty without disturbing a path from source 'S' to destination 'D'. So total numbers of working nodes= $2^n - n$. Taking P=Probability of success of a node, In sub cube $I_{n-1}$, choosing a node n from Adj(s) can be in ${}^{n-1}C_1$ i.e. out of n-1 nodes one will work with probability P, which contributes the term p. Now since source node and destination node are fixed, one node N ϵ $I_{n-1}$ should be chosen working otherwise the path will be destroyed. So without loss of generality the path contains 'n' working nodes with probability p and ($2^{n-1}$-n) numbers of failure nodes with probability (1-p). So the Reliability expression

$$R_{s-d} = {}^{n}C_1\, {}^{n-1}C_1\, {}^{n-2}C_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots {}^{2}C_1 * {}^{2^{n-1}-2}C_{n-2} * P^{n+1} * (1-P)^{2^{n-1}-n} \quad (4)$$

## Proposed Method:

Let $IC_n$ denotes an n-dimensional interconnection network i.e. n-dimensional cube. Each node in $IC_n$ is labeled by a n-bit string. For a given source node S, there exists a numbers of adjacent nodes, out of which at least one node is faulty. Otherwise it will destroy the regularity property of $IC_n$. The addresses of the adjacent nodes are differ in obtained from equation (1) exactly one bit. Assume 'u' be the faulty node, 'v' and 'w' be the non-faulty nodes. Where 'u', 'v', 'w' are represented as binary strings. $\bar{v}$ and $\bar{w}$ be the antipodal nodes of v and w. Taking bit operation $u \otimes \bar{v}$ and $u \otimes \bar{w}$ results n discarded regions. This leads to formation of an incomplete interconnection network $I_{n-1}^{m}$ .m numbers of nodes in fault free incomplete cube with dimension of n-1.Then the reliability of incomplete cube is calculated by using the equation

$$R_{s-d} = {}^{n}C_1\, {}^{n-1}C_1\, {}^{n-2}C_1 \ldots\ldots\ldots\ldots\ldots\, {}^{2}C_1 * {}^{2^{n-1}-2}C_{n-2} * P^{n+1} * (1-P)^{2^{n-1}-n}$$

A recursive algorithm for generating the reliability expression R for the maximal incomplete cube is provided below.

## Algoritm For Reliability Expression:

*Reliability (G, S, D, n)*

*{*

*If* $(n \geq 2)$

*{*

*Adjacent=Adj(S)*

*Choose a node N from adjacent in* ${}^{n}C_1$ *ways.*

$N' =$ Antipodal (N)

$V_i' = N' \otimes \{V \sim (S \cup D \cup N)\}$

$V' = \{V \sim (S \cup D \cup (Adj(S) \sim N)\}$

*for i=1 to* $|V'|$

*Discard* $N' \otimes V_i'$ *region*

$G' = (V', E')$

$$R = R \times {}^{n}C_1 \; p^{n+1} q^{2^{n-1}-n}$$

*Reliability* $(G', S, D, n-1)$

}

*else*

*return;*

}

*Efficiency:* The complexity of the proposed algorithm is found to be $O(n^2)$.

## Illustration:

The proposed method is illustrated through a most widely used network of n-cube topology known as Hypercube.

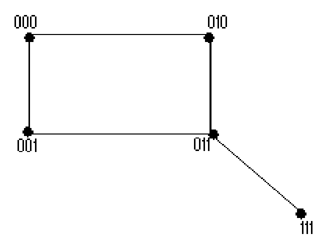


**Fig.4 (a) 100 taken as faulty**

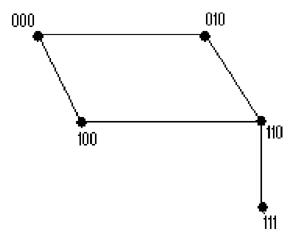


**Fig.4 (b) 001 taken as faulty**

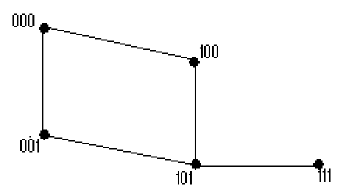


**Fig. 4(c) 010 taken as faulty**

**Fig.4: Incomplete maximal sub cubes of a 3-D hypercube**

Consider a 3-D Hyper cube $HC_3$ as shown in fig.1 having the source and destination nodes labeled as 000 and 111 respectively. Out of the three adjacent nodes of source node let cube node 001 is faulty, Then the antipodal nodes of the two other adjacent nodes are 011 and 101.A discarded region is addressed simply by performing operation $\otimes$ on the labels of the faulty node and the antipodal nodes. In fig.1(a) faulty node is 001,antipodal node of 100 is 011 and 001 ⊗ 011 = 0∗1 and 001 ⊗ 101 = ∗01.After removing this two discarded regions a maximal incomplete sub cube results which is shown in fig.4(a).The same operation can be performed by taking 100 and 010 cube nodes as faulty nodes. This results in the two other maximal incomplete sub cube as shown in fig. 4(b) & 4(c).

## Results and Discussions:

The reliability of three important multicomputer interconnection networks viz. Hypercube ($HC_n$), Crossed cube ($CQ_n$) and Folded hypercube ($FH_n$) has been computed by using the proposed algorithm. The results have been plotted against the mission time (t) in hours versus reliability under different node failure rates for each network (Figs. 5-7). Also reliability of the above multicomputer networks are compared under a common platform for the same node failure rate of $\lambda$ = 0.005 (Fig. 8).

Hypercube provides reliability of less than 50% even at mission time 100 hours and under low node failure rate such as $\lambda$=0.0025(Fig.5). Hypercube is not suitable for high node failure rate, as it gives a reliability value of 32% even for small duration of time as 100 hours. The reliability values decreases exponentially and becomes zero at a mission time of 700 hours for low node failure rate $\lambda$=0.0025. The reliability of Crossed cube is plotted against mission time for different node failure rate as shown in the Fig.6. Crossed cube provides a reliability value of 63% at mission time 100 hours under low node failure rate $\lambda$=0.0025. Above 50% of reliability can be achieved under the following mission time and node failure rate. The Folded hypercube (Fig.7) records a reliability value of 83% at mission time of 100 hours for node failure rate of $\lambda$=0.0025.The reliability value decreases to 70% for a mission time 200 hours for same node failure rate. After that it exponentially decreases and becomes nearly zero at 1000 hours. This network provides reliability value below 70% for moderate node failure rate of $\lambda$=0.005 at a mission time 100 hours.

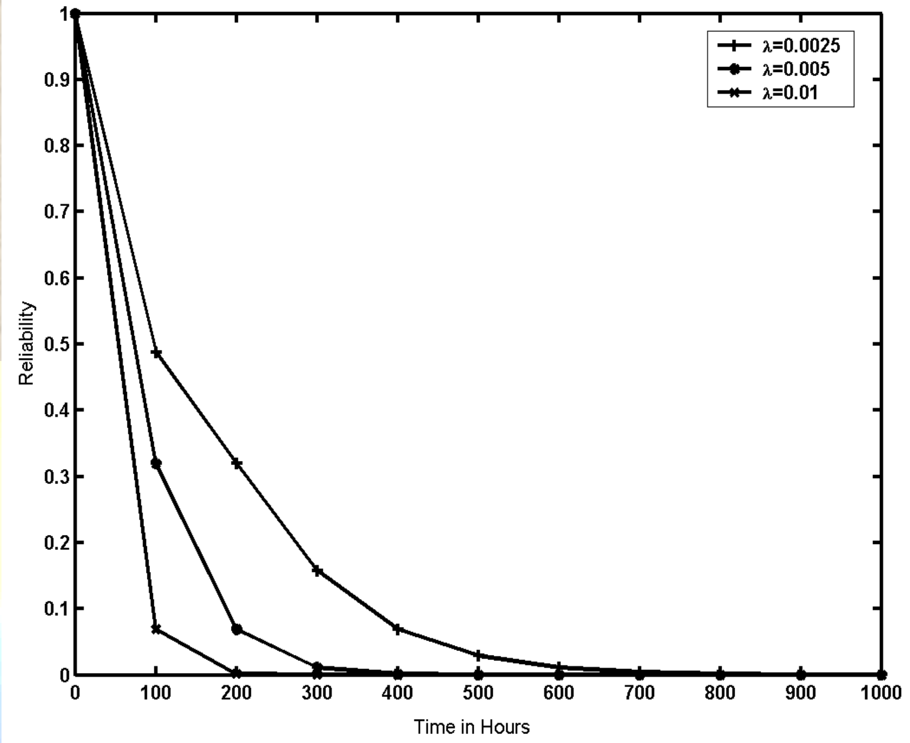


Figure 5: Reliability of Hypercube (n=3) with imperfect node

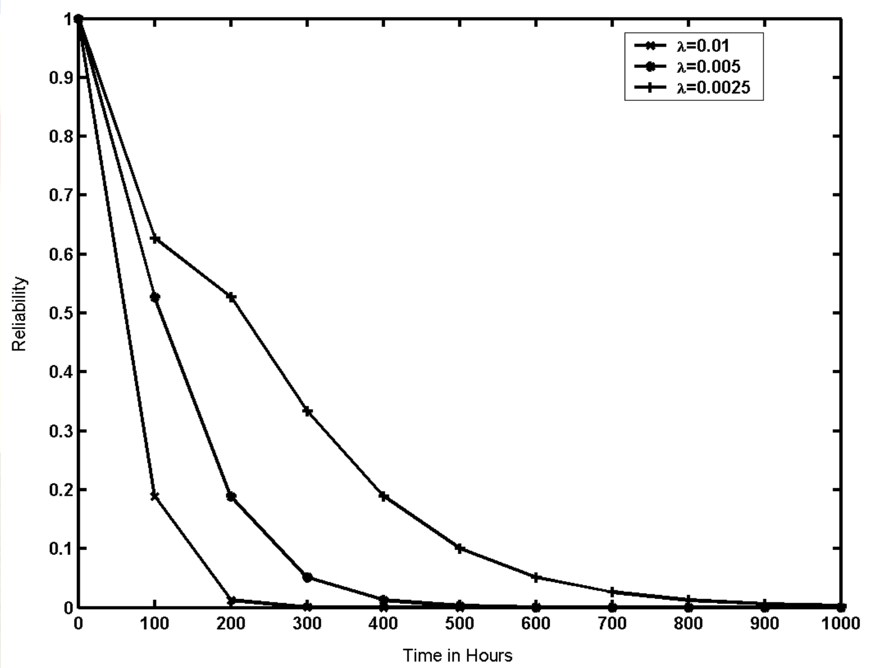


Figure 6: Reliability of Crossed cube (n=3) with imperfect node

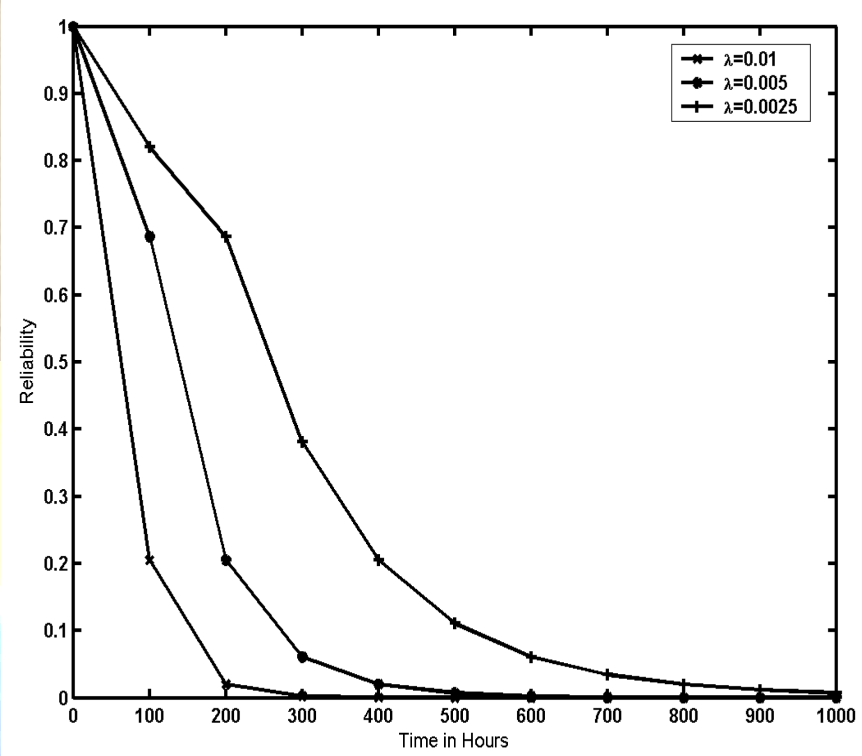


Figure 7: Reliability of Folded Hypercube (n=3) with imperfect node

The reliability of all the three multicomputer networks has been compared under the same node failure rate $\lambda$=0.005 and plotted as shown in Fig.8. From the figure, it can be observed that the reliability of Folded hypercube is found to be the highest followed by Crossed cube and Hypercube at mission time 100 hours. At this mission time Folded hypercube has a reliability as high as 83%, where as the other two networks are having only 55% and 33% respectively. After that reliability values decreases and becomes zero within mission time 600 hours.

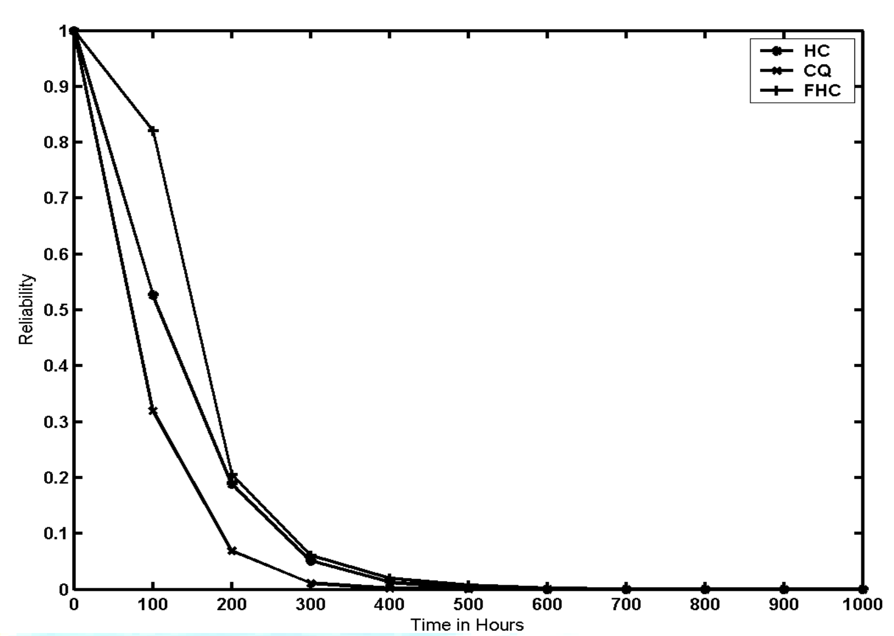


Figure 8: Comparison of multi computers under the same node failure rate (λ)=0.005

From the comparative curve of reliabilities of the said networks, the networks are arranged in the following order in terms of their terminal reliability

$$FH_n > CQ_n > HC_n$$

## Conclusion:

An efficient and effective method has been proposed for identifying all maximal fault free incomplete sub cubes from a faulty cube by taking maximum fault tolerance capacity is equal to the system dimension. In this process every fault free node is required to participate in the identification process. An efficient recursive algorithm has been proposed to generate reliability expression of the incomplete cube-base topology. The terminal reliability of three cube based topology namely Hypercube ($HC_n$), Crossed cube ($CQ_n$) and Folded hypercube ($FH_n$) are evaluated and compared. From this comparison, it can be concluded that $FH_n$ is better than its counterparts in term of their computed terminal reliability.

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