INTRODUCE A MATHEMATICAL MODEL FOR THE METRO PASSENGERS WAITING TIME

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Abstract:

Can be say all or most people tend to do the work faster and also move faster from place to other place, and fed up with the factors which could cause things done and moving on slowly and keep aloof too waiting. In this paper, given the importance of passenger waiting time in systems of Public transport such as Metro and being an effective amount of it on the amount of the people welcomed, therefore we attempted to identify factors that can be expected over Passengers waiting time and with regard to relationship between these factors, we introduce a Mathematical model for minimizing the average waiting time. For comparison Mathematical model with the actual model, simulation was required that using simulation software Arena we achieved this goal. We also solve the Mathematical model using MATLAB software and the output was created by simulation model. To obtain the necessary parameters, sampling was done from the station of Tehran TARASHT , 8 time, an hour a day in the middle of the week. Solution of these models was done in two forms.When there is an operator at the box office and also when there are two operators. Finally After solving the Both models in two-forms and 8 time, The Obtained Output were compared .

Keywords: Queue length , average waiting time , mathematical modeling , pubic transportation, Metro

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1- Introduction:

The increase number of vehicles and the willingness of citizens to the use of private vehicles have created problems for citizens. Blockade Long, losing time, the uncontrolled consumption of fuel and pollution are samples of these problems. Heavy traffic in the cities also is increased travel time [1]. Encourage citizens to use public transportation especially subway can be a suitable way to reduce the challenges which these Systems is involved it and face to this system requires that the proper services that can be Encourage passengers to use it by identification factors which are important to citizens and improving these factors Encourage passengers to use this system. Determination of the waiting time at the stations to benefit from the service provided is so important. Because if this time was more than significant amount is makes the part of applicants can be deterred to use this system. One important factor in attracting people to this System is reducing passengers waiting time. But with restrictions such as capacity, or capacity of each train and the number of operators are also facing.operational research and Mathematical Modeling can be used to minimize these cases. In studies by Janis P.LI in the context of simulating a station, important activities that are done at the station and relationship between them have been identified. Time required for this activity is a function of rental process and how collect it. With the simulation tools can evaluate and make decision about the following [3]:

- To optimum of order equipment related to obtaining rental
- Changing Method of Obtaining rental
- To modify equipment related to obtaining rental

Some studies also for coordination of transportation networks by BUKKAPATNAM, DESSOUKY & ZHAO were as conducted. With using simulation techniques were done modeling the scheduling and dispatching for different conditions [4]. The objectives of these studies were:

- Achieving to process for mixing different policy for inputs.
- Evaluation of Various methods by simulation.

One of the effective factors in the passengers waiting time,is ratio of those factors to the headway of dispatching. it determined in studies of KNOPPERS & MULLER by simulation model for urban Leeds and Manchester, this ratio is about 0.5. (That means $W_i = 0.5h_i$ where W_i is

Passenger waiting time for the route i and h_i is the average time between dispatching trains on the route i). Expected waiting time $E(W)$ at the urban public transport Station is a function of headway trains dispatch (H) and its changes (Variance) $Var(H)$ that is as equation (1) :

$$E(W) = E(H)/2\ 1 + Var(H)/E^2(H) \qquad (1)$$

They also were obtained the average waiting time for passengers moving between the two systems $E(W)$ through the equation (2):

$$E(W) = \int_{-\infty}^{+\infty} W_{(p)} P_r(P) d_p \qquad (2)$$

Studies were conducted by BOOKBINDDER & DESILET at the crossing stations and two points was considered:

- creating an objective function that indicate a general dissatisfaction.
- Creating an Algorithm for finding schedual table.

Some studies were done by VANSTEENWEGEN & VAN OUDHEUSDEN that their aim was reducing the waiting time of passengers at suburban railway transportation, they tried to minimize the function include travel time parameters in the network and by linear programming method optimized schedual table of Trains movement.

In continuing studies, they with using the techniques of Linear programming and Simulation method, were obtained model of trains and suburban movement planning aimed at increasing service level to customers and to minimize the travel times and the cost of Passengers waiting time in the system.

For conditions which stations and terminals are faced with overcrowding, with regard to safety issues, studies by CHAW & CANDY was performed that for comparison normal state with the state that occurs congestion, simulation was used.

2- Mathematical model :

The Passengers who enter system at time t for got on train, it is possible they have ticket $P_{bn}(t)$ or have no ticket $P_b(t)$ if they have any ticket, they must go to box office. At box office may some passengers at time t be in line $L_g(t)$ who is not yet their turn, so at time t , $P_{bn}(t)$ passengers are added to this queue. For passengers who in t have to wait, we consider the average waiting time in queue and show as $T_g(t)$. The number of existing operators in the box office show as integer variable x and we assume that it can be 1 or 2. Passengers after buying the ticket go to ticket control . The $\mu(t)$ is also Service rates of any operator at the box office at t. that is assumed same for all operators. The $L_g(t=0)$ will be zero.

If passengers who are in the box office, could not buy ticket before t, then new passengers will be added to this queue. Total passengers who can not buy ticket at t, waiting time will be defined for them. This number of passenger could be as sentences (3) and (4):

$$P_{bn}(t)+L_g(t)-(x\times\mu(t))\begin{cases}\leq 0 & (3)\\ >0 & (4)\end{cases}$$

If the state (3) occurs, that means at the end of t we have no queue, but when the state (4) occurs that means at t the queue be formed and queue length can also show as (5) :

$$L_g(t+1)=P_{bn}(t)+L_g(t)-(x\times\mu(t)) \qquad (5)$$

In general, all people who at Study period are waiting in line at the box office show as (6):

$$\sum_{t}^{T} P_{bn}(t)+L_g(t)-(x\times\mu(t)) \qquad (6)$$

$T_g(t)$, is the average waiting time for per passenger at the box office who at the end of t could not buy Ticket. So, total waiting time of passengers in the queue can be show as (7) that is one of the objective functions In order to minimize:

$$Min \sum_{t}^{T}(P_{bn}(t)+L_g(t)-(x\times\mu(t)))\times T_g(t) \qquad (7)$$

On the other hand, if the queue formed at the box office, passengers waiting time can be show as (8):

$$T_g(t) = \frac{L_g(t+1)}{\lambda(t)} \qquad (8)$$

$\lambda(t)$, is passenger entrance rate during the T. it can be vary and can be show as (9):

$$\lambda(t) = \begin{cases} \lambda_1 & t_0 < t \le t_1 \\ \lambda 2 & t_1 < t \le t_2 \\ . & \\ . & \\ . & \\ \lambda n & t_{n-1} < t \le t_n \end{cases} \qquad (9)$$

$P_{gc}(t)$ is the number of passengers who at t from box office go to control, that is expressed by two states. Now two variables 0 and 1 are defined as follows:

$V_1(t)$: If at t,at box office queue be formed, it will be equal 1 and otherwise it'll be 0.

$V_2(t)$:If at t, at box office queue be formed, it will be equal 0 and otherwise it'll be 0.

In addition restriction (10) must be added. Because at time t, may be there is a queue or not:

$$V_1(t) + V_2(t) = 1 \qquad \forall t \in T \qquad (10)$$

However, total output in the box office at T can be expressed depending on whether at the box office a queue is formed or not as (11) :

$$\sum_{t}^{T} P_{gc}(t) = \sum_{t}^{T} V_1(t) \times (x \times \mu(t)) + V_2(t) \times (P_{bn}(t) + L_g(t)) \qquad (11)$$

This phrase is composed of two parts. When $V_1(t)$ is equal 1 that means the queue is formed at the box office and $T_g(t)$ is greater than zero and the output of box office to control is equal to first phrase and when $V_2(t)$ is equal 1 that means the queue is not formed at the box office and $T_g(t)$ is equal to zero and the box office output to control is equal to second phrase. Relations between these variables can be show as (12):

$$\begin{cases} V_1(t)=0 \ \rightarrow T_g(t)=0 \\ V_1(t)=1 \ \rightarrow T_g(t)>0 \end{cases} \rightarrow \varepsilon . V_1(t) \le T_g(t) \le M . V_1(t) \quad \forall t \in T \qquad (12)$$

Where M , is a very large number and ε is a very small number. If the queue is formed at the box office, $V_1(t)$ must be 1 and if the queue is not formed, $V_1(t)$ must be 0 that these states show as (13):

$$V_1(t) \ge \frac{1}{M} \times (P_{bn}(t) + L_g(t) - (x \times \mu(t))) \quad \forall t \in T \qquad (13)$$

At the ticket control may be number of passengers be in queue that we show as $L_c(t)$, that $P_b(t)$ and $P_{gc}(t)$ will be added to it. Therefore, all passengers who are at ticket control at time t, can be show as (14):

$$P_b(t) + P_{gc}(t) + L_c(t) \qquad (14)$$

Assuming the ticket control has the service rate equal to μ_c ,conditions that may occur, can be expressed as (15) and (16):

$$P_b(t) + P_{gc}(t) + L_c(t) - \mu_c \begin{cases} \le 0 & (15) \\ > 0 & (16) \end{cases}$$

If the state (15) occurs that means at t at the Control gate, queue is not formed and if (16) occurs, that means at t at the Control gate, queue will formed. In general, all passengers who at the study time T stand in queu at the control gates can be show as (17):

$$\sum_{t}^{T} (P_b(t) + P_{gc}(t) + L_c(t) - \mu_c) \qquad (17)$$

$T_c(t)$, is the average waiting time for per passenger who at t could not go to the platform. So the total waiting time of passengers in the queue can be as (18) that is another objective function to minimize:

$$Min \sum_{t}^{T} (P_b(t)+P_{gc}(t)+L_c(t)-\mu_c)\times T_c(t) \qquad (18)$$

The average waiting time of passengers in the queue can be show as (19):

$$T_c(t)=\frac{L_c(t+1)}{\lambda'(t)} \qquad (19)$$

$\lambda'(t)$, is entrance rate of passengers to control that at T it can be varied and show as (20):

$$\lambda'(t)=\begin{cases}\lambda'_1 & t_0<t\le t_1\\ \lambda'_2 & t_1<t\le t_2\\ . & \\ . & \\ . & \\ \lambda'_n & t_{n-1}<t\le t_n\end{cases} \qquad (20)$$

$P_{cs}(t)$ is the number of passengers who at t go to the platform from control that has two types. Two 0-1 variables are defined as follows :

$V_3(t)$:If at t, at the control, queue be formed, it'll be equal 1 and otherwise it will be 0.

$V_4(t)$: If at t, at control,queue not be formed, it'll be equal 1 and otherwise it will be 0.

In addition restrictions (21) must be added. Because at time t, may be there is a queue or not:

$$V_3(t)+V_4(t)=1 \qquad \forall t\in T \qquad (21)$$

However, the total control output to the platform at T depending on the control gate the queue is formed or not, show as phrase (22):

$$\sum_{t}^{T} P_{cs}(t)=\sum_{t}^{T} (V_3(t)\times\mu_c)+(V_4(t)\times(P_b(t)+P_{gc}(t)+L_c(t)) \qquad (22)$$

When $V_3(t)$ is equial 1, that means the queue is formed at the control gate and $T_c(t)$ is greater than 0 and the output of box office to platform is equal to first phrase and when $V_4(t)$ is equial 1, that means the queue is not formed at the control gate and $T_c(t)$ is equal 0 and the box office output to control is equal to second phrase. Relations between these variables can be show as (23):

$$\begin{cases} V_3(t)=0 & \to T_c(t)=0 \\ V_3(t)=1 & \to T_c(t)>0 \end{cases} \to \varepsilon . V_3(t) \le T_c(t) \le M . V_3(t) \quad \forall t \in T \qquad (23)$$

Where M , is a very large number and ε is a very small number. If the queue is formed at the control, $V_3(t)$ must be 1 and if the queue is not formed, $V_3(t)$ must be 0 that these states show as (24):

$$V_3(t) \ge \frac{1}{M} \times (P_b(t) + P_{gc}(t) + L_c(t) - \mu_c) \quad \forall t \in T \qquad (24)$$

To people who are on Platforms at t ($L_s(t)$) the Number of Passengers $P_{cs}(t)$ is added and they formed all people who are in queue. Also depending on at t the Train stops or not, however, $L_s(t+1)$ can be defined as (25):

$$L_s(t+1) = P_{cs}(t) + L_s(t) - y_i(t) \times Q_i(t) \qquad (25)$$

That $L_s(t+1)$ is the number of passengers who are on Platform and waiting time is defined for them. $Q_i(t)$, is capacity of train i at t and $y_i(t)$ also can be defined as Phrase (26):

$$y_i(t):\begin{cases}1\\0\end{cases} \quad \forall i = 1,2,...,n \qquad (26)$$

If the train i at t stop at the station, it will be equal 1,and otherwise is 0. In general, all passengers who at T stand on queue at the platforms show as (27):

$$\sum_{t}^{T}(P_{cs}(t)+L_s(t)-y_i(t)\times Q_i(t)) \qquad (27)$$

The total waiting time on the Platform is as (28):

$$Min\sum_{t}^{T}\sum_{i=1}^{n}(P_{cs}(t)+L_s(t)-(y_i(t)\times Q_i(t)))\times T_s(t) \qquad (28)$$

We assume that for each t more than one train does not have permission to stop and that restriction show as Phrase (29):

$$\sum_{i=1}^{n} y_i(t) \le 1 \quad \forall t \in T \qquad (29)$$

And we assume at T each train can be stop maximum once at the station and we show this as phrase (30):

$$\sum_{t}^{T} y_i(t) \le 1 \quad \forall i \qquad (30)$$

The average waiting time for each passenger on the platform can be show as (31):

$$T_s(t) = \frac{L_s(t+1)}{\lambda''(t)} \quad (31)$$

Passengers entrance rate to queue on the platform $\lambda''(t)$ is different at different time and can be show as (32):

$$\lambda''(t) = \begin{cases} \lambda''_1 & t_0 < t \le t_1 \\ \lambda''_2 & t_1 < t \le t_2 \\ . \\ . \\ . \\ \lambda''_n & t_{n-1} < t \le t_n \end{cases} \quad (32)$$

the total volume of passengers who get service, must be compatible with thecapacity of the trains that can show as equation (33):

$$\sum_{t}^{T} \sum_{i=1}^{n} y_i(t) \times Q_i(t) \ge \sum_{t}^{T} P_{bn}(t) + P_b(t) \quad (33)$$

3 - Simulation model: In the designed simulation model using the software Arena,the process of subway station is similar to a mathematical model that is described completely. Simulation model has shown in two forms: (1) and (2).

Figure 1: Simulation model-part 1

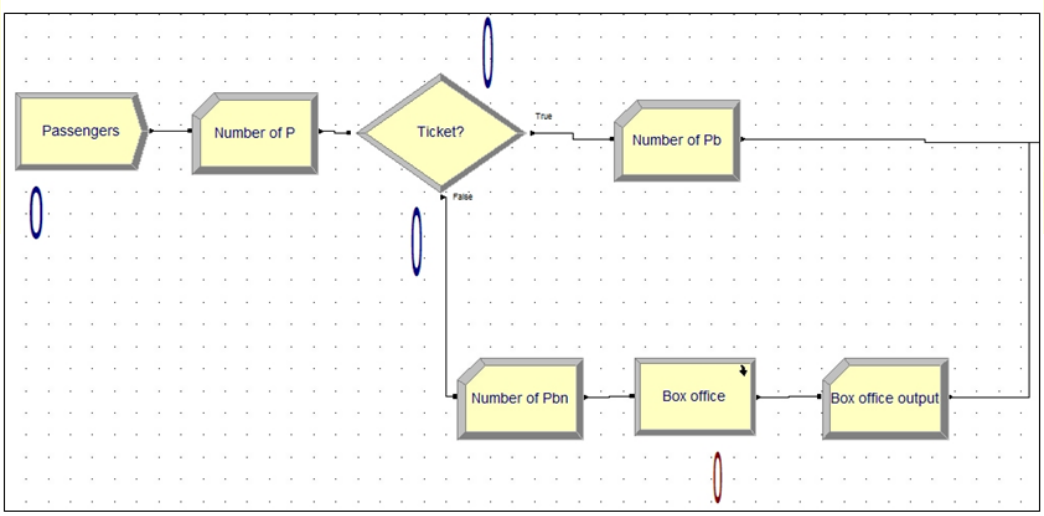
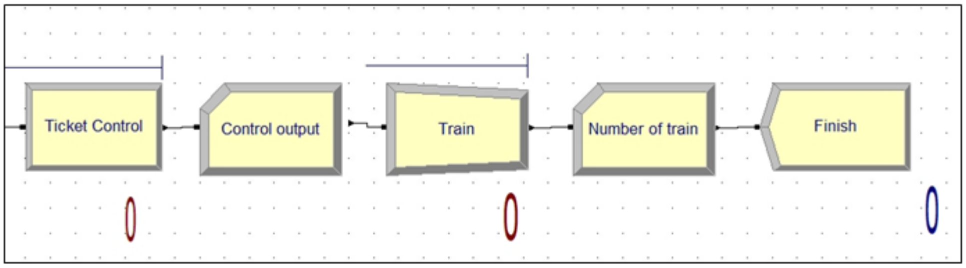


Figure 2: Simulation model-part 2



4 – solve: To determine various factors, we did the sampling in the 8 period (length of one hour) in one day in the middle of the week from TARASHT station of Tehran. Using MATLAB software, we solved the mathematical model. Finally, we compared the current conditions (simulation output) with optimum conditions resulting from the mathematical model. In addition to examin different policies about the number of operators at the box office, we solved the mathematical model in two types 1 and 2 operator existence.

4-1 - A mode with one operator: In the first stage, total passengers entering the subway system was 1440 people which 876 people had no ticket and 564 people had ticket. Output of stages is shown in Table 1. The output of the simulation model is presented in the Parenthesis. Average queue length at the box office, ticket control and platform in optimum situation is 6, 4 and 84 and these numbers for simulation mode, respectively is 8, 6 102. In the first phase, with comparison two outputs obtained is observed that there is possibility of improvement and decrease in the length of queues and waiting time at different levels. The same comparison can be made for seven other stages.

Table 1: the mathematical model and the simulation model outputs-one operator

	One operator							
	Periods							
	1	2	3	4	5	6	7	8
All passengers entering the system	1440	1284	1034	954	928	842	747	816
all passengers without a ticket	876	783	624	576	472	507	372	484
all passengers who have a ticket	564	501	410	373	456	335	375	332
the average queue length at the Box office	6(8)	6(7)	6(6)	5(6)	4(5)	5(5)	2(4)	4(4)
the average queue length at the Control	4(6)	3(4)	2(4)	1(2)	2(2)	1(2)	1(2)	1(2)
the average queue length at the Platform	84(102)	71(93)	59(83)	55(78)	51(78)	46(70)	41(61)	44(66)
the average waiting time at the Box office	0.88(1.23)	0.74(1.08)	0.68(0.99)	0.61(0.9)	0.48(0.73)	0.53(0.8)	0.42(0.68)	0.49(0.73)
the average waiting time at the Control	0.21(0.3)	0.18(0.19)	0.14(0.17)	0.14(0.16)	0.14(0.14)	0.13(0.14)	0.13(0.12)	0.14(0.14)
the average waiting time at the Platform	2.16(2.77)	2.09(2.73)	2.01(2.52)	2(2.46)	2(2.43)	1.89(2.4)	1.72(2.26)	1.86(2.37)

4-2 - A mode with two operators: Outputs of all these steps is given in Table (2). Average waiting time at the Box office, control and platform is 0.61, 0.31 and 2.3 min. These numbers in the simulation mode, respectively is 0.8, 0.37 and 3.4 min.

Table 2: the mathematical model and the simulation model outputs-Two operators

	Two operators							
	Periods							
	1	2	3	4	5	6	7	8
All passengers entering the system	1440	1284	1034	954	928	842	747	816
all passengers without a ticket	876	783	624	576	472	507	372	484
all passengers who have a ticket	564	501	410	373	456	335	375	332
the average queue length at the Box office	4(6)	5(6)	4(5)	4(5)	3(4)	4(5)	1(2)	2(3)
the average queue length at the Control	5(8)	4(6)	3(5)	2(4)	2(4)	2(3)	1(2)	1(3)
the average queue length at the Platform	93(112)	80(99)	72(91)	66(87)	63(80)	60(77)	53(70)	59(75)
the average waiting time at the Box office	0.61(0.8)	0.56(0.72)	0.49(0.64)	0.38(0.5)	0.27(0.44)	0.29(0.46)	0.21(0.36)	0.27(0.45)
the average waiting time at the Control	0.31(0.37)	0.2(0.24)	0.16(0.19)	0.15(0.18)	0.15(0.170	0.14(0.15)	0.13(0.13)	0.14(0.15)
the average waiting time at the Platform	2.3(3.4)	2.18(2.18)	2.14(2.59)	2.13(2.57)	2.13(2.5)	1.94(2.5)	1.86(2.44)	1.86(2.45)

In the first phase, with comparison two obtained outputs are observed that there is possibility of improvement and decrease in the queues length and waiting time at different parts. The same comparison can be made for seven other stages.

Now,we can analyzed different scenarios in terms of employment or putting 1 or 2 operators at different part. For example, refer to table (3):

Table 3: Comparison of output of two models with one and two-operators

	One operator	Two operators
	Period 1	
the average queue length at the Box office	6(8)	4(6)
the average queue length at the Control	4(6)	5(8)
the average queue length at the Platform	84(102)	93(112)
the average waiting time at the Box office	0.88(1.23)	0.61(0.8)
the average waiting time at the Control	0.21(0.3)	0.31(0.37)
the average waiting time at the Platform	2.16(2.77)	2.3(3.4)

with increasing number of operators to 2, the average queue length at the box office has declined. But the average queue length at the control and platform has increased. also the average waiting time of passengers at the box office is declined. But the waiting time at the platform and the average waiting time at the platform has increased respectively 0.1 and 0.14 minutes.

5 – Results: In this paper a mathematical model for minimizing the waiting time of metro passengers was described in three parts included, box office, ticket control and the platform. due to influential factors on the average waiting time, Relations between them were identified . In the given model assumes that passengers after entering the platform are placed in a queue and after the arrival of the train are mounted. Then presented a simulation model and we analyzed outputs.

6 - References

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