ADAPTABILITY OF MATHEMATICAL/STATISTICAL METHOD OF ALLOCATION OF SCARCE RESOURCES TO ACHIEVE A STATED OBJECTIVE

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ABSTRACT:

Linear Programming is a mathematical method of allocation of scarce resources to achieve a stated objective. The method is useful wherever both the objective and environmental constraints, which limit the degree of achievement of the objective, can be expressed in the form of linear equations and/or in-equations. Linear programming can help the management to determine how it can best use the resources of a business to achieve a stated objective such as maximum profit or minimum cost, when the resources have alternative uses. Linear Programming is extremely useful technique for dealing with situation of scare resources in a variety of situations, but it suffers From the following drawbacks. It is necessary for its use to specify exact contributions and constraints, which presents practical problems. All relationships be must true linear relationships. This is difficult as demand and price are unlikely to conform to

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a linear relationship. Fractional solution must be accepted. In production plan, fractional units represent work in-progress.

Keywords: linear Programming, Mathematical application, Product Mix, formulation, simplex, non-negativity, environmental constraint, graphic analysis, linear relationship. Fractional solution.

INTRODUCTION:

Linear Programming enables the management to determine how it can best use the resources of the organization to achieve a stated objective such as maximizing profit or minimizing cost, when the resources have alternative uses. When there are more than two products and more than one limiting / key factor, simplex method can be used. It is desirable to use simplex method, where graphical method will not suffice because of the additional information generated specially dual values. The dual values represent the amount the organization is willing to pay in order to acquire one extra unit of the scarce resource, in other words, the internal opportunity cost of the resource. Simplex can be used to solve minimizing or maximizing problems. The graphical analysis involves steps such as:

- Formulation of appropriate linear programming problem i.e. express the problem in standardized manner
- Construct the graph for the problem formulated i.e. the objective line.
- Draw a constraint line for each of the limiting factors.
- Identify the feasible region. The feasible region is that space, which satisfies all of the constraints simultaneously.
- Locate the solution points. This is done by identifying the corner points in the feasible region.
- Evaluate the objective function at each of the solution points that were identified.

- Identify the optional solution. This is accomplished by selecting the corner points, which correspond to the optimal value of the objective function.

Mathematical Application: Data procured from a manufacturing company shows that it produces two products coded A and B, and wishes to determine the optimum product mix. Two processes are required for the production of the products, process 1 and process 2.

Each unit of A requires 12 hrs processing by process 1 and 6 hours by process 2. Each unit of B required 8 hours processing by process 1 and 5 hours by process 2. Process 1 has a maximum capacity of 12000 hours and process 2, a maximum capacity of 7200 hours. Maximum demand is 1,100 units of A and 1,400 units of B. Each unit of A yields a contribution of N20 per unit and B, a contribution of Nl4 per unit.

ADAPTABILITY OF GRAPHICAL SOLUTION FOR OPTIMUM PRODUCT MIX:

STEP 1: Formulation of equations for each constraint

Let a = quantity of A to be produced

Let b = quantify of B to be produced.

Process 1 $12000 \geq 12a + 8b$

Process 2 $72000 \geq 6a + 5b$

Negativity $a \geq 0$

$b \geq 0$

Demand $1100 \geq a$

$1400 \geq b$

STEP 2: Formulation of equation for the objective function

Maximize $20a + 14b$

In order to plot this on the graph it is necessary to make the objective function equal to a number. Any Convenient figure can do in this case i.e. 14000

20a+14b = 14000

STEP 3: Plot the lines on the graph.

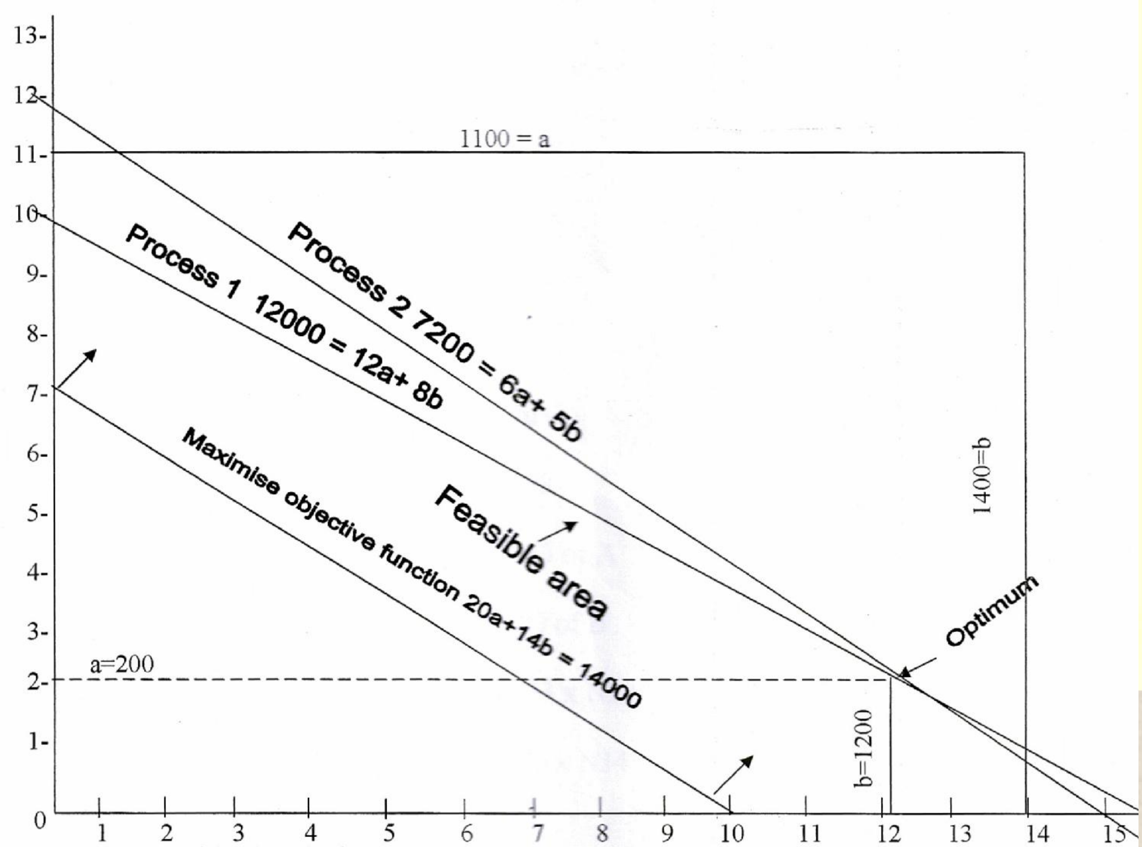
Take Process 1 as an example;

When b is zero a = 1000

When a is zero b = 1500

STEP 4: Read off solution at maximized intersect i.e.

a = 200 Units of A b =l200 units of B



Graphical Solution to problem of more than one scare resource

APPLYING SIMULTANEOUS EQUATIONS FOR OPTIMUM PRODUCT MIX:

Determining the optimum mix through simultaneous equations will give the intersect of any two lines. Care must be taken to ensure that the correct intersect is found. Possible solutions are to be found at the following intersects:

1 i Process 1 constraint with a – axis

ii Process 2 constraint with demand b constraint

iii Demand b constraint with b — axis

iv Process 1 constraint with process 2 constraint.

(i) Working for this is:

1000 of A, 0 of B

Contribution 1000 x N20 = N20000

(ii) (1) 7,200 =6a+ 5b

(2) 1,400 = b or b = 1400 from (2)

substitute in (1): a = 33.33

33.33 of A

1400 of B

Contribution 33.33 x N2O = N666.60

l400 x N14 = N19,600.00

N20,266.60

(iii) 1400 of B, 0 of A

Contribution 1400 xNl4 = N19,600

(iv) (1) 12000 = 12a + 8b

(2) 7200= 6a + 5b

Multiplying equation 2 by two and subtracting equation (1) l2000 = l2a + 8b

2 x (2) 14, 400= 12a + l0b

or b = l,200, a = 200

Solution 200 of A, 1200 of B

Contribution		N
200 x N20	=	4,000
1200 x N14		16,800
		N20,800

This is the optimum solution as it 3ields the highest contribution.

ADOPTING SIMPLEX SOLUTION FOR OPTIMUM PRODUCT MIX

The objective function is to minimize

$20a + 14b$ subject to;

$12a + 8b \leq 12000$

$6a+5b \leq 7200$

$a \leq 1{,}100$

$b \leq 1400$

Non - negativity, $a \geq 0$

$b \geq 0$

It must be remembered that it is usual to exclude the non-negativity constraint when constructing the simplex table. One slack variable is inserted for each constraint, four in all (S_1 to S_4) and the matrix derived. It is conventional to use negative values in the contribution line for the initial table - this means that an optimum will be reached when all these values are positive. The initial table assumes that there will be no production whatsoever, so there will be total spare capacity, and no contribution.

Initial table

Row number	Solution variable	Production variables		Slack variables				Solution quantity
		a	b	S_1	S_2	S_3	S_4	
1	S_1	12	8	1	0	0	0	12000
2	S_2	6	5	0	1	0	0	7200
3	S_3	1	0	0	0	1	0	1100
4	S_4	0	1	0	0	0	1	1400
5	C	-20	-14	0	0	0	0	0

The pivot element (ringed) is found in the following manner:

1. Select the product with the highest contribution ('a')
2. Find factor which limits quantity of 'a' product (S_1-only,

   1000 can be produced as opposed to 1200 under S_2 and 1100 under S_3).

The next step is to make the pivot element = 1 by dividing the row, in this case by 12. This means that the new row 1 on the next table becomes.

a*	1	$^{2}/_{3}$	$_{1}/_{12}$	0	0	0	1000

'a' replaces S_1 as it is the new solution variable

The other rows are now evaluated, making the pivot element column = 0 in each row by subtracting, as though solving simultaneous equations.

New row 2

= old row 2 — (6 x new row 1

= 0 1 $-^{1}/_{2}$ 1 0 0 1200

New row 3

= Old row 3 - (1 x New row 1)

= 0 -$^2/_3$ -½ 0 1 0 100

New row 4

= Old row 4 (no change necessary)

= 0 1 0 0 0 1 1400

New Row 5

= Oldrow5 + (2O x New row l)

= 0 -$^2/_3$ $^{20}/_{12}$ 0 0 0 20,000

New table from these rows

Solution Variables	Production variables		Slack variables				Solution quantity
	A	b	S_1	S_2	S_3	S_1	
a	1	$^2/_3$	$^1/_{12}$	0	0	0	1000
b	0	1	-½	1	0	0	1200
S_3	0	-$^2/_5$	-½	0	1	0	100
S_4	0	1	0	0	0	1	1,400
C	0	-$^2/_3$	^{20}/12	0	0	0	20,800

The table still has a negative on the contribution line, hence not an optimum. However, it is a feasible solution and represents the solution point in the top left - hand corner of the graph. (Simplex tests all the intersect points in rotation).

The new pivot element is marked with a circle again. Going through the same steps as before, the new row 2 will be:

b 0 1 -½ 1 0 0 1200

(Note that division is not necessary as the pivot element is already equal to 1) carrying out row operations once again;

New row 1 = Old row l- ($^2/_3$ x new row 2)

= 1 0 $^5/_{12}$ -$^2/_3$ 0 0 200

New row 3 = Old row 3 + ($^2/_3$ x new row 2)

= 1 0 -$^5/_{12}$ $^2/_3$ 1 0 900

Old row 4 - (1 x new row 2)

= 0 0 -½ -1 0 1 200

Newrow5 = Old row 5 - (2/3 x new row 2)

= 0 0 $^8/_6$ $^2/_3$ 0 0 20,800

The final optimal table can now be reconstructed.

Solution Variables	Production variables		Slack variables				Solution quantity
	a	b	S_1	S_2	S_3	S_1	
a	1	0	$^5/_{12}$	-$^2/_3$	0	0	200
b	0	1	-½	1	0	0	1200
S_3	1	0	-$^5/_{12}$	$^2/_3$	1	0	900
S_4	0	0	½	$^{-1}$	0	1	200
C	0	0	$^8/_6$	$^2/_3$	0	0	20,800

INTERPRETATION

The solution quantity column shows that 200 of A should be made together with 1200 of B. At this level of production, there will be 900 spare Units of demand A constraint and 200 spare units of demand B Constraint (i.e. these two factors are not material constraints in this case).

DUAL VALUES (SHADOW PRICES)

Most important are the dual values (referred to as shadow prices); these can only have any value for a limiting factor. In this case the values are:

S_1 $^8/_6$ i.e. N1.33

Or

S_2 $^2/_3$ i.e. N0.67

This is the contribution lost by not having more of these resources. Putting it in another way, the dual value is the maximum premium the business organization is prepared to pay for one extra unit of these resources. For example, suppose each hour of process 1 time (S_1) cost N3 per hour, the business is prepared to pay N3 + N 1.33 = N4.33 for one extra hour of process 1 time. The figures in the column above these values show what business will do with the extra one hour.

Taking S_1 (i.e process 1 time) as an example, with one extra hour, the business will produce $^5/_{12}$ of a unit more of A, ½ a unit less of B, which will leave $^5/_{12}$ of a unit less spare on demand A Constraint, and ½ a unit more spare of the demand B constraint. This can be proved: each unit of A takes 12 hours of process 1 time. So $^5/_{12}$ of a unit will take 5 hours, less ½ a unit not made of B (8 hrs per unit) equals one extra hour. The contribution increase will be $^5/_{12}$ x N20 = N8.33 less than lost from B ½ x N14 = N7, net increase N 1.33, the value of one extra hour of process 1 time.

Conclusion:

Caution must be exercised by the management in applying this technique in resolving managerial problems such as allocation of scarce resources to achieve its stated objective. The environmental constraints which limit the degree of management achievement must be capable of being expressed in linear equation. Hence the formulation of linear equation might impose a considerable difficulty.

Selected References

- **Akinsuel Ire Oye** (2006):Financial Management 4th Education Ceemol Nig. Ltd.
- **Christopher J.N John Neter, Michael H. Kutner and William**
- **Wasserman** (1996) Applied linear Statistics McGraw Hill New York U.S.A.
- **M.Y Kan and P.K. Jain** (1998): Cost and Management Account
- Tata. MaGraw-Hill Publishing Co. Ltd, New Delhi
- Journal of Chartered Institute of Cost and Management Accountants of India 1992 Vol. XL.