PREFERENTIAL SCALING FACTOR OF THE CONTINUOUS WAVELET TRANSFORM METHOD FOR THE IDENTIFICATION OF POTENTIAL FIELD SOURCES PROBLEM

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Abstract. Identification of source of potential field is one of most importance task in gravity and magnetic exploration. CWT is a relative new method for the identification of the sources. The scaling factor is the key parameter which influence the identification result of CWT method seriously. A novel method of estimating the preferential scaling factor is proposed based on the analysis the signal/noise ratio in the wavelet domain. The numerical result indicates that the accuracy of identification result can be improved obviously using the preferential scaling factor.

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Introduction

Gravity and magnetic exploration method are mature exploration methods, which widely used in oil and gas exploration, metal ore prospecting areas[1,2]. The identification of the source of potential field(gravity field or magnetic field) is one of the most important task of interpretation of gravity or magnetic data quantitatively. The classic identification of potential field sources include analytic signal[3,4] and Euler de-convolution[5] method which have some disadvantages such as dependence of prior structure index and interference of noise. The continuous wavelet transform(CWT) is relative new method for the identification the sources of potential field. It has advantages of independence of prior information and anti-noise ability. Moreau points out that the attenuation of noise is faster than that of signal when the scaling factor increasing, for the reason of the analyzing wavelet has high order vanishing moment[6]. But when the scaling factor increases too large, the signal is over attenuated while the noise is suppressed effectively. So, the scaling factor is a key parameter of CWT, which relates to the noise suppressing effect and influences the accuracy of the identification result seriously. The particular choice of scaling factor is still a open subject of investigation.

This paper describes the theory of identification of sources of potential field with CWT briefly firstly, then analyzes the influence of noise and the relationship between scaling factor and signal/noise ratio, proposes a novel optimizing method of scaling factor based on the signal/noise analysis and inspects the effectiveness at last.

Basic theory and method

In the two-dimensional potential field problem, the continuous wavelet transform is defined as a convolution product:

$$W_{g|\phi_0}(b,a) \equiv \int_R \frac{1}{a} g\left(\frac{b-x}{a}\right)\phi_0(x)dx = (D_a g * \phi_0)(b) \tag{1}$$

Where φ_0 is the observed potential field data on the ground, $g(x \in R)$ is the analyzing wavelet, $a \in R^+$ is the scaling factor, and dilation operator D_a is defined by the following action:

$$D_a g(x) \equiv \frac{1}{a} g\left(\frac{x}{a}\right) \tag{2}$$

It is defined by power law scaling property of the homogeneous source of order α located at at $x=0, z=0$:

$$\varphi_0(\lambda x) = \lambda^{\alpha}\varphi_0(x) \tag{3}$$

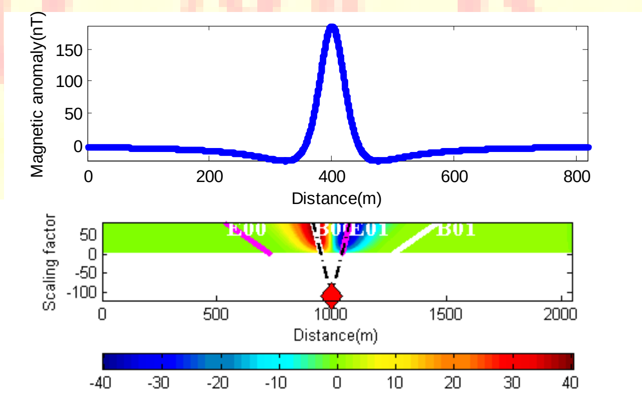
Considering the homogeneous source located at $x=0, z=z_0$, when a particular family of analyzing wavelet named Poisson wavelet is employed, the following formular is satisfied at the upper half space[7]:

$$W_{g|\phi(,z=0)}(x,a) = \left(\frac{a}{a'}\right)^{k}\left(\frac{a'+z_0}{a+z_0}\right)^{-\beta} W_{g|\phi(,z=0)}\left(x\frac{a'+z_0}{a+z_0}, a'\right) \tag{4}$$

where $\beta = \alpha - k + 1$, α is the homogeneous degree that is related to the shape of source, k is the order of Poisson wavelet. This scaling relation involves a cone-like structure of the wavelet domain with the apex located at the source place(Fig. 1).

I use following synthetic model illustrate the identification of sources of potential field with CWT.

Model 1: The model 1 is a infinite extend horizontal cylinder located at 110m depth and 1000m distance with a 60m section radius. The magnetization is 1A/m. The incline of earth magnetic field is 90 degree. The magnetic anomaly of this model is shown in Fig. 1(a), the CWT of the magnetic anomaly is shown in Fig.1(b). From Fig.1(b) we can see four straight line, named wavelet transform maximum line(WTMML), and the intersection point of them is the location of causing source. I can estimate the homogeneous degree of source along the WTMML after we estimate the location of source follow the formular (4).



(a) magnetic anomaly of model (b) identification result of potential field with CWT

Fig. 1. The identification of source of potential field with CWT

Optimization method of scaling factor

In order to increase the accuracy of identification result, preferential scaling factor must be chosen correctly. The lower altitude of scaling factor is considered to filter out noise, as a classical cut-off wavelength of low-pass filters[6]. The upper altitude of scaling factor is considered to only interpret wavelet transform coefficients at medium altitudes where effector of underground sources is still larger then the effect of measurement errors[6].

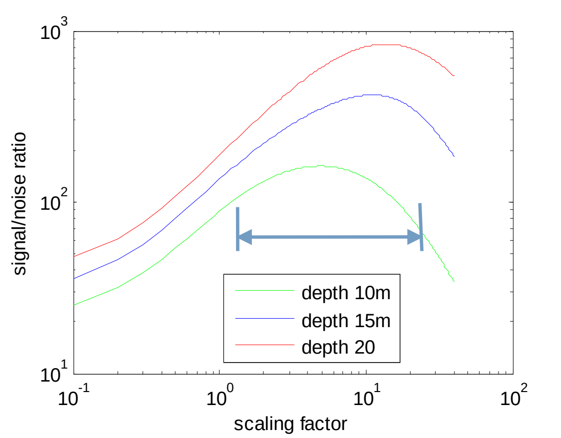
Considering the Gaussian white noise N (with zero mean and variance σ_N^2), the noise of the wavelet transform WN is a function of scaling factor a; the variance of the wavelet transformed noise and that of the noise are related by:

$$\sigma_{WN}^2 = \sigma_N^2 \frac{E_\phi}{a} \tag{5}$$

where $E_\phi = (2n)!2^{2n+1}$ is the energy of analyzing wavelet depending on the derivation order n.

Thus the wavelet transform of the signal decreases as $a^n(z_0 + a)^{\alpha-n}$, while the noise decreases as $a^{-1/2}$, which faster than signal.

The signal/noise ratio is a function of scaling factor a, which increases firstly and decreases secondly. I can estimates the optimum lower and upper border of scaling factor, illustrated in the Fig.2, analyzing the signal/noise ratio in the wavelet transform domain.



(c)

Fig.2 Estimating the optimum lower and upper border of scaling factor

Numerical result

The Fig.1 illustrates the identification of causing source of potential field when none of noise added to the magnetic data. However, the observed data contain the noise. Now we analyze the influence of noise to the identification result.

I employ the CWT method to identify the source of potential field from the synthetic data of model 1(Fig.1(a)) adding 0%, 5%, 10% , 15% and 20% noise respectively. The results are listed in the table 1.

Tab.1 identification result of synthetic data adding different level noise with CWT

	Distance(m)				Depth(m)			
Noise level (%)	True value	Identification result	std	Relative error (%)	True value	Identification result	std	Relative error (%)
0	1000	1000	0.0024	0	110	106.833	0.0071	2.88
5	1000	1002.2	0.0027	0.22	110	112.440	0.0066	2.22
10	1000	1003.7	0.0032	0.37	110	94.188	0.0072	14.37
15	1000	1001.3	0.0040	0.13	110	93.337	0.0089	15.15
20	1000	998.7	0.0027	0.13	110	128.352	0.0068	16.68

From Tab.1 we can see that the accuracy of identification result decreases rapidly as the level of noise increase from 0% to 20%. The influence of noise is seriously if we choose the bad scaling factor.

I analyzing the S/N of serval models of different depth and adding different level noise, using the proposed method, and obtain the optimum lower and upper border of scaling factor and list them in the table 2.

From the table 2, we can see that the preferential scaling factor is related to the depth and noise level. When the upper border is equal to the depth and the lower border is equal to the half of depth, the best S/N can be obtained.

Table 2 The relationship between lower and upper border to depth and noise level

Eepth(m)	Noise level(%)	Lower border of scaling factor	Upper border of scaling factor	S/N of lower scaling factor	S/N of upper scaling factor
10	10	9.1	10.3	39.6	48.9
10	20	9.4	10.8	11.5	12.5

30	10	10.3	31.1	121.5	131.6
30	20	14.4	44.4	93.6	101.9
50	10	16.0	37.0	1147.6	1259.6
50	20	20.4	52.8	292.5	321.2
70	10	24.2	66.4	2238.5	2465.6
70	20	35.1	92.5	929.0	1026.6
90	10	31.8	95.2	2670.0	2937.4
90	20	28.7	138.8	737.6	813.0
110	10	25.6	83.2	4705.6	5194.1
110	20	47.9	147.6	1477.6	1632.8

In order to check the effectiveness of the preferential scaling factor obtained in the table 2, we use the preferential scaling factor listed in the table 2 and identification the source of model 1 with adding 10% and 20% noise respectively. The identification results are listed in the table 3.

Table 3 identification result of synthetic data of model 1 adding 10% and 20% noise respectively

	Distance(m)				Depth(m)			
Noise level (%)	True value	Identification result	std	Relative error(%)	True value	Identification result	std	Relative error(%)
10	1000	1000.1	0.0001	0.01	110	108.564	0.0024	1.31
20	1000	998.5	0.0011	0.15	110	107.495	0.0026	2.27

Comparing the result data listed in Tab.1 and Tab.3, we can see the accuracy of identification results increase obviously--relative error decreases from 14.3% to 1.31% at the noise level 10% and from 16.68% to 2.27% at the level of 20%. The results indicate the proposed method for estimating the preferential scaling factor can improve the accuracy of identification of potential field effectively.

Summary

I proposed a method to estimate the preferential scaling factor based on analyzing the S/N ratio in the wavelet transform domain. The numerical results indicate that the accuracy of identification result can be improved obviously using the preferential scaling factor. I can draw a conclusion as follow:

(1) The S/N ratio increases firstly and decreases secondly as scaling factor increasing;

(2) Optimal lower and upper border of scaling factor can be obtain by analyzing the S/N in WT domain;

(3) Using preferential scaling factor can improve the accuracy of identification result.

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