A NUMERICAL STUDY ON MHD LAMINAR MIXED CONVECTION IN DUCTS WITH ASYMMETRIC WALL HEAT FLUXES

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Abstract:

In this paper, the effect of magnetic field on a mixed convection in ducts with asymmetric wall heat fluxes, under the influence of transverse magnetic field is discussed. The flow problem is described by means of partial differential equations and the solutions are obtained by using an implicit finite difference technique. The velocity, the temperature and the pressure profiles are obtained and their behaviour is discussed computationally for different values of governing parameters like magnetic field parameter M, Prandtl number Pr, ratio of heat flux r_H and Gr/Re. It is observed that the velocity decreases with the increase in magnetic parameter M. It is noticed that the increase in Gr/Re will decrease the temperature for fixed r_H=0.5 and for fixed magnetic field parameter M.

Keywords: Mixed Convection, Uniform Heat Flux and MHD.

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1. INTRODUCTION

Considerable attention has been given to laminar mixed convection problems in vertical channels. The majority of the recent studies have dealt with the circular tube geometry, but increasing attention is being focused on the parallel-plate duct. This configuration is relevant to solar energy collection as in the convectional flat plate collector and Trombe wall, and in the cooling of the modern electronic system. In the latter application, electronic components are mounted on circuit cards, an array of which is positioned vertically in a cabinet forming vertical flat channels through coolants are passed [6, 8]. The coolant may be propelled by free convection, forced convection, or mixed convection, depending on the application.

The subject of laminar mixed convection in vertical or inclined ducts deserves a special interest for its applications in the design of cooling systems for electronic devices or for solar collectors. Several authors have presented theoretical or experimental investigations most of which have been reviewed, for instance, in Aung and Worku [1]. The literature of the last decades includes many papers presenting theoretical investigations of buoyancy-induced flows in vertical or inclined ducts. Some of these papers (Lavine [15], Barletta and Zanchini [7], Buhler [10], Weidman [20], Magyari [16]) describe analytical solutions with reference to fully developed parallel flows. Kim et al. [14] studied numerically the effect of wall conduction on laminar free convection between parallel vertical plates subjected to asymmetric heating plates are heated by subjecting the external surfaces of the plates to uniform heat flux (UHF) conditions. An implicit finite difference method was used to solve the governing equations.

Aung and Worku [3] presented the numerical results for the effects of buoyancy on the hydro-dynamic and thermal parameters in the laminar, vertically upward flow of a viscous fluid in a parallel plate channel. A numerical study dealing with combined free and forced laminar convection in a parallel plate vertical channel with asymmetric heating at uniform heat fluxes (UHF) was considered by Aung and Worku [2]. Mixed convection effects on fully developed flow (FDF) in a parallel plate vertical channel with asymmetric wall temperatures were studied by Aung and Worku [4]. Habchi and Acharya [12] studied numerically the laminar mixed convection in a symmetrically or asymmetrically heated vertical channel.

Barletta [5] studied fully developed mixed convection and through reversal in a vertical rectangular duct with uniform wall heat flux. Barletta and Zinchini [6] studied mixed convection with viscous dissipation in an inclined channel with prescribed wall temperatures. Barletta et al.

[8] studied combined forced and free flow in a vertical rectangular duct with prescribed wall heat flux. Lavine [15] Analyzed fully developed apposing mixed convection between inclined parallel plates.

Fully developed MHD free convection flow of a viscous electrically conducting fluid in a vertical parallel plate channel was studied by Hughes and Young [13]. Chamkha [11] studied on laminar hydro-magnetic mixed convection in a vertical channel with symmetric and asymmetric wall heating conditions. Umavathy and Malashetty [19] studied magneto-hydrodynamic mixed convection in a vertical channel.

2. NOMENCLATURE

b	Spacing between plates
C_p	Specific heat at constant pressure
g	Acceleration due to gravity
Gr	Grashof number, $g\,\beta q_2\, b^4 / \upsilon^2{}_k$
h	Heat transfer coefficient
H_0	Applied magnetic field
$\bar{j}$	Current density vector
k	Thermal conductivity
M	Magnetic parameter
Nu	Nusselt number
p	Pressure difference, p′ - p″
p′	Static pressure
p″	Hydrostatic pressure
P	Dimensionless pressure difference, $(p' - p'') / \rho_u{}^2$
Pr	Prandtl number
q	Heat transfer for unit surface area per unit time
$\bar{q}$	Velocity vector, (u, v, 0)
r_H	q_1 / q_2
Re	Reynolds number, $u_0\, b / \upsilon$

T	Temperature
u	Axial velocity
u_0	Average fluid velocity
v	Transverse velocity
U	Dimensionless stream wise velocity, u / u_0
V	Dimensionless transverse velocity, vb / υ
x	Stream wise distance from channel entrance
y	Transverse coordinate (measured from cool wall)
X	Dimensionless stream wise distance from channel entrance, x / (b Re)
Y	Dimensionless transverse coordinate, y / b
β	Thermal expansion coefficient
μ	Dynamic viscosity
μ_e	Magnetic permeability
υ	Kinematics viscosity
ρ	Density
ρ_0	Fluid density at ambient temperature
σ	Electrical conductivity
θ	Dimensionless temperature difference, $(T - T_0) / (q_2 b / k)$

SUBSCRIPTS:

0	Value at channel entrance (at x = 0)
1	Cool wall (i.e. value at y = 0)
2	Hot wall (i.e. value at y = b)
B	Bulk value
c	Value at center line
w	Condition at wall

3. FORMULATION OF THE PROBLEM

We consider the effect of magnetic field on mixed convection in ducts with asymmetric wall heat fluxes. The distance between the plates is taken to be 'b' and the walls are kept at y=0 and y=b. The flow is assumed to be two-dimensional and the form of the vertical channel is given in figure.1. A forced flow approaches the bottom of the duct with a flat upward (positive x-direction) velocity profile and a uniform temperature. The right wall is heated uniformly by external means at a rate q_2 and the left wall heat flux is q_1. The ratio of the heat flux is r_H, where $0 \le r_H \le 1$, which characterizes the degree of asymmetric heating. A uniform transverse magnetic field of strength H_0 is applied perpendicular to the walls (i.e. in the Y-direction).

The governing equations for the steady viscous flow of an electrically conducting fluid in the presence of an external magnetic field are

(i) he flow is steady, viscous, incompressible, and developed.

(ii) The flow is assumed to be two-dimensional steady, and the fluid properties are constant except for the variation of density in the buoyancy term of the momentum equation.

(iii) The electric field $\bar{E}$, and induced magnetic field are neglected [17, 18].

(iv) Energy dissipation is neglected.

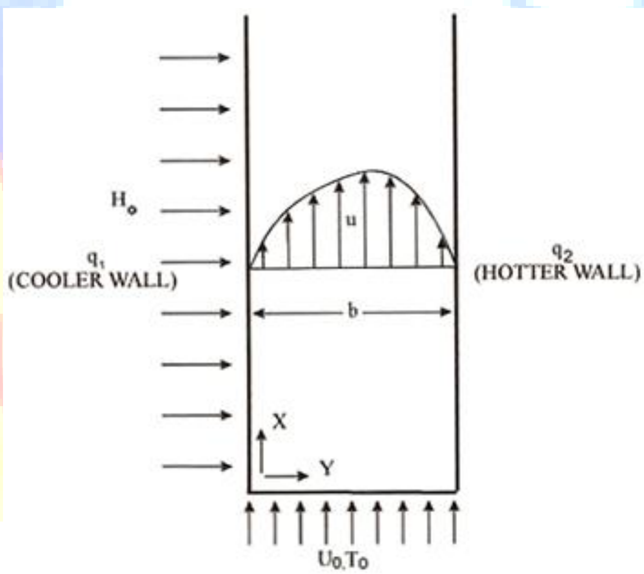

Figure 1: Two Dimensional Channel

Using the above assumptions the governing equations become

Continuity

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

X - momentum

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{Gr}{\mathrm{Re}}\theta+\frac{\partial^2 U}{\partial Y^2}-M^2U \qquad (2)$$

Y - momentum

$$\frac{\partial P}{\partial Y}=0 \qquad (3)$$

Energy

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial Y^2} \qquad (4)$$

Where $M^2 = \sigma \mu_e^2 H_0^2 b^2 / \mu$

In equation (2), use has been made of the Boussinisq equation of state, $\rho - \rho_0 = -\rho\, \beta(T-T_0)$, and of the definition $p = p' - p''$, where p'' is the pressure at any stream-wise position if the temperature were T_0 everywhere. The latter definition gives $dp''/dx = -\rho_0 g$.

Hence

$$-\frac{dp'}{dx}-\rho\, g=-\frac{dp}{dx}+\rho g\beta(T-T_0)$$

It is noted that the dimensionless pressure is $P = (p' - p'') / \rho\, u_0^{\,2}$, if the channel were horizontal, we would have $P = p' / \rho\, u_0^{\,2}$, the convectional definition in pure forced flow.

The boundary conditions are

At $X = 0,\ 0 \le Y \le 1$: $U = 1, V = 0, \theta = 0, P = 0$

At $X > 0, Y = 0$: $U = 0, V = 0, \frac{\partial\theta}{\partial Y} = -r_H$ (5)

At $X > 0, Y = 1$: $U = 0, V = 0, \frac{\partial\theta}{\partial Y} = 1$

In the above, dimensionless parameters have been depended as

$U = u / u_0$, $V = vb / \upsilon$, $X = x / (b\ \mathrm{Re})$, $Y = y / b$

$p = (p' - p'') / \rho\, u_0^{\,2}$, $\mathrm{Pr} = \mu C_p / k$, $\mathrm{Re} = b\, u_0 / \upsilon$ (6)

$Gr = g\, \beta\, q_2\, b^4 / \upsilon^2 \kappa$, $\theta = (T - T_0) / (q_2 b / k)$

To obtain a solution of the mixed convection problem formulated above, an additional equation expressing the global conservation of mass at any cross section in the channel is also required. This becomes

$$\int_0^1 U \ dY = 1 \tag{7}$$

The systems of non-linear equations (1) to (4) are solved by a numerical method based on finite difference approximations. An implicit difference technique is employed whereby the differential equations are transformed into a set of simultaneous linear algebraic equations.

4. NUMERICAL SOLUTION

The solution of the governing equations for developing flow is discussed in this section. Considering the finite difference grid net work of figure.2, equations (2) and (4) are replaced by the following difference equations which were also used in [9].

$$U(i,j)\frac{U(i+1,j)-U(i,j)}{\Delta X}+V(i,j)\frac{U(i+1,j+1)-U(i+1,j-1)}{2\Delta Y}$$
$$=\frac{U(i+1,j+1)-2U(i+1,j)+U(i+1,j-1)}{(\Delta Y)^2}-M^2U(i+1,j) \tag{8}$$
$$-\frac{P(i+1)-P(i)}{\Delta X}+\frac{Gr}{\text{Re}}\theta(i+1,j)$$

$$U(i,j)\frac{\theta(i+1,j)-\theta(i,j)}{\Delta X}+V(i,j)\frac{\theta(i+1,j+1)-\theta(i+1,j-1)}{2\Delta Y}$$
$$=\frac{1}{\text{Pr}}\frac{\theta(i+1,j+1)-2\theta(i+1,j)+\theta(i+1,j-1)}{(\Delta Y)^2} \tag{9}$$

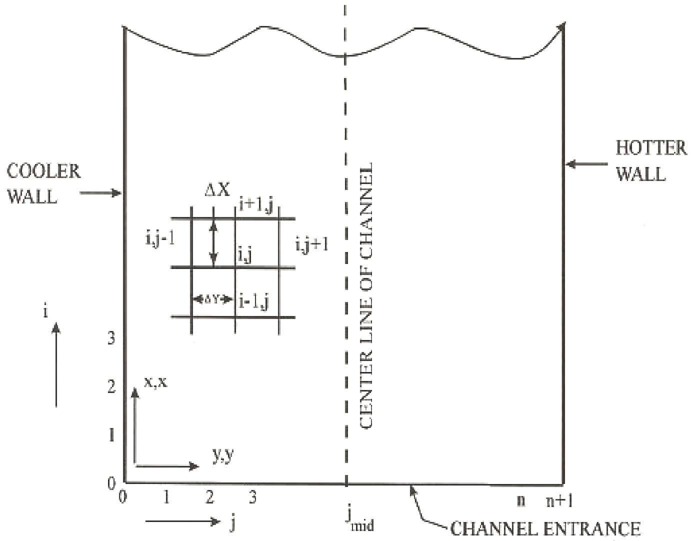

Figure 2: Finite Difference Network

To calculate V, the transverse component of velocity, the centerline value of the index is denoted (Figure2) j_{mid} = (n+3) / 2, where (n+2) is the total number of nodes along Y and is an odd integer. For j < j_{mid}, the continuity equation (2) can be written

$$V(i+1, j) = V(i+1, j-1) - \frac{\Delta Y}{2\Delta X}(U(i+1, j) + U(i+1, j-1) - U(i, j)$$

$$-U(i, j-1)), j = 1,2,....., j_{mid} - 1 \qquad (10a)$$

For j>j_{mid} , the above equation is modified and the following form is used:

$$V(i+1, j) = V(i+1, j+1) + \frac{\Delta Y}{2\Delta X}(U(i+1, j) + U(i+1, j+1) - U(i, j)$$

$$-U(i, j+1)), j = n, n-1,......, j_{mid} + 1 \qquad (10b)$$

In principle, either equation (10a) or equation (10b) could be used to evaluate the centerline velocity V(i+1, j_{mid}). However, since both equations employ one-sided differences, a different value of V(i+1, j_{mid}) could result depending on which equation is considered. Consequently, the transverse velocity at the centerline is calculated by fitting a third order polynomial through the values on the two immediate mesh points on both sides of the centerline.

By means of Simpson's rule, we write equation (7) as

4U(i+1,1)+2U(i+1,2) + 4U(i+1, 3) +. . .+ 4U(i+1, n) = 3(n+1) (11)

A set of finite-difference equations written about each mesh point in a column for the equation (8) as shown:

$$\beta_1 U(i+1,1)+\gamma_1 U(i+1,2)+\xi P(i+1)+\frac{Gr}{\mathrm{Re}}\theta(i+1,1)=\phi_1$$

$$\alpha_2 U(i+1,1)+\beta_2 U(i+1,2)+\gamma_2 U(i+1,3)+\xi P(i+1)+\frac{Gr}{\mathrm{Re}}\theta(i+1,2)=\phi_2$$

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$$\alpha_n U(i+1,n-1)+\beta_n U(i+1,n)+\xi P(i+1)+\frac{Gr}{\mathrm{Re}}\theta(i+1,n)=\phi_n$$

where

$$\alpha_k=\frac{1}{(\Delta Y)^2}+\frac{V(i,j)}{2\Delta Y},\qquad \beta_k=-\left[\frac{2}{(\Delta Y)^2}+M^2+\frac{U(i,j)}{\Delta X}\right]$$

$$\gamma_k=\frac{1}{(\Delta Y)^2}-\frac{V(i,j)}{2\Delta Y},\qquad \xi=\frac{-1}{\Delta X},\ \phi_k=-\left[\frac{P(i)+U^2(i,j)}{\Delta X}\right]$$

for k = 1,2… n

A set of finite-difference equations written about each mesh point in a column for the equation (9) as shown:

$$\bar{\beta}_0\theta(i+1,0)+\left(\bar{\alpha}_0+\bar{\gamma}_0\right)\theta(i+1,1)+-----------=\bar{\phi}_0-2r_H\Delta y\bar{\alpha}_0,$$

$$\bar{\alpha}_1\theta(i+1,0)+\bar{\beta}_1\theta(i+1,1)+\bar{\gamma}_1\theta(i+1,2)+------------=\bar{\phi}_1$$

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$$\bar{\alpha}_n\theta(i+1,n-1)+\bar{\beta}_n\theta(i+1,n)+\bar{\gamma}_n\theta(i+1,n+1)+--------=\bar{\phi}_n$$

$$\left(\bar{\alpha}_{n+1}+\bar{\gamma}_{n+1}\right)\theta(i+1,n)+\bar{\beta}_{n+1}\theta(i+1,n+1)=\bar{\phi}_{n+1}-2\Delta y\,\bar{\gamma}_{n+1}$$

where

$$\bar{\alpha}_k=\frac{1}{\mathrm{Pr}(\Delta Y)^2}+\frac{V(i,j)}{2\Delta Y},\qquad \bar{\beta}_k=-\left[\frac{2}{\mathrm{Pr}(\Delta Y)^2}+\frac{U(i,j)}{\Delta X}\right]$$

$$\bar{\gamma}_k = \frac{1}{\Pr(\Delta Y)^2} - \frac{V(i,j)}{2\Delta Y}, \qquad \bar{\phi}_k = -\frac{U(i,j)\theta(i,j)}{\Delta X}$$

for k = 0,1,2… n+1

The solutions of the difference equations are obtained by first selecting values for Pr, Gr/Re, M and r_H and then by means of a marching procedure the variables U, V, θ and P for each row beginning at row (i+1) = 2 are obtained using the values at the previous row 'i' . Thus, by applying equations (8), (9) and (11) to the points 1, 2, ..., n on row i, 2n+3 algebraic equations with the 2n+3 unknowns U(i+1,1), U(i+1,2), . ……... ., U(i+1,n), P(i+1), θ(I+1,0), θ(i+1,1), θ(i+1,2), ... ,θ(i+1,n), θ(i+1,n+1) are obtained. This system of equations is then solved by a matrix reduction technique. Equations (10) are then used to calculate V(i+1,1),V(i+1,2), ..., V(i+1, n).

5. RESULTS AND DISCUSSION

In the present study the effects of buoyancy and asymmetric heating have obtained for Pr=0.72 at Gr/Re=0, 250, 500 for different values of magnetic field parameter M.

In the UWT-case, the effect of buoyancy is most noticeable on the velocity profile. The profile in that case is highly distorted during flow development. The distortion diminishes in FDF when r_T< 1 and completely disappears when r_T=1 (symmetric wall temperature) for all Gr/Re values in a parabolic profile. In the present UHF case, a similar situation exists for r_H<1 but when symmetric heating occurs (r_H=1), the velocity becomes parabolic only for Gr/Re=0. Figures 3(a) to 3(c) illustrates this point axial variation of the center line velocity for fixed M. For symmetric heating (r_H=1) the centerline velocity into the FDF of U_C=1.5 when Gr/Re=0 and full development is accomplished at a very small value of dimensionless distance X when compared to the UWT case. When Gr/Re> 0 the development length, as indicated by the region where U_C is changing, is increased and this trend is consistent with the UWT case; however, unlike UWT, the centerline velocity fails to develop into the universal value of 1.5, but instead assumes values progressively smaller as Gr/Re increases. Thus, a major difference between UWT and UHF (when r_T=r_H=1) lines in FDF, where in UWT an identical profile is assumed by the fluid at all values of Gr/Re, whereas in UHF the profile suffers a center depression or concavity that becomes increasingly pronounced as Gr/Re increases.

For r_H=0.5, the present result so that the profile at a relatively large value of X is not affected by the asymmetric heating when Gr/Re=0, and a parabolic profile is attained by the fluid for fixed M. This is shown in figures 4(a) to 4(c). The same figures also show that for Gr/Re >0, the fluid flow is more and more channeled toward the hot wall (Y=1). There is only a negligible effect on the fluid flow near the cool wall, and at Gr/Re=500, the maximum velocity is still not much larger than 1.5 for fixed M. The implication of the above discussion is that in UHF the likelihood of flow separation is reduced if not completely eliminated. Within the ranges of parameters studied herein, i.e. $0.3 \leq r_H \leq 1$, $0 \leq$ Gr/Re≤ 500, no flow reversal was observed for fixed M.

In the present study, heat transfer results have been obtained for Pr=0.72. Figures 5(a) to 5(c) depicts typical development of the temperature profiles for r_H=1 for a fixed value of Gr/Re = 250 and M. At large X, the curves become parallel to one another, suggesting equal axial temperature gradient throughout the flow. A contrast between UHF and UWT in the thermal profiles is provided in figures 6(a) to 6(c) for fixed M. The profiles in UWT degenerate into a linear one at large X, while in UHF they maintain a self similar concave shape in FDF.

The axial variations of the wall temperature are given in to figures 7(a) to 7(c) and 8(a) to 8(c) with Gr/Re as a parameter for fixed M. The effect of buoyancy is to decrease the wall temperatures, but the impact on the cool wall, when r_H=0.5, is negligible. In this case, the temperature on the cool wall is found to be virtually unchanged as Gr/Re increases, but the hot wall temperature is reduced for fixed M.

The wall temperature, particularly the cool wall temperature, are sensitive the parameter r_H. In figures 10(a) to 10(c) cool wall temperature is plotted versus X for a fixed Gr/Re=250, but r_H is allowed to vary for fixed M. Note the drastic variation of the curves for different r_H. In figures 9(a) to 9(c), the corresponding plot for the hot wall temperature for fixed M is presented. The trend is the same but the variations on the hot wall due to r_H are much smaller.

The axial distribution of the dimensionless pressure for r_H=1 shows a trend that is similar to that in UWT; however, here too the effect of buoyancy is not significance. (see figures 11(a) to 11(c)). The available information suggest that even at a relatively small value of Gr/Re, the pressure P eventually becomes positive when X is sufficiently large for fixed M.

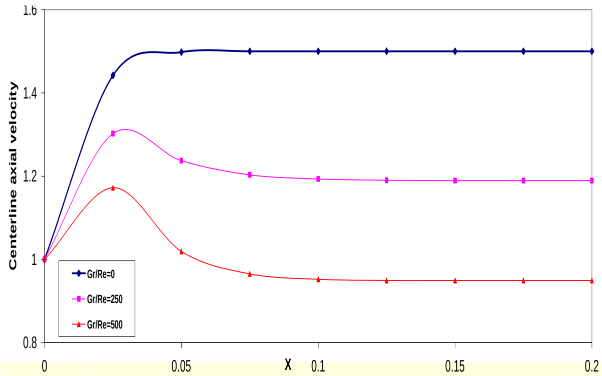

Fig. 3(a) : Axial variation of the centerline velocity for fixed r_H=1 and M=0

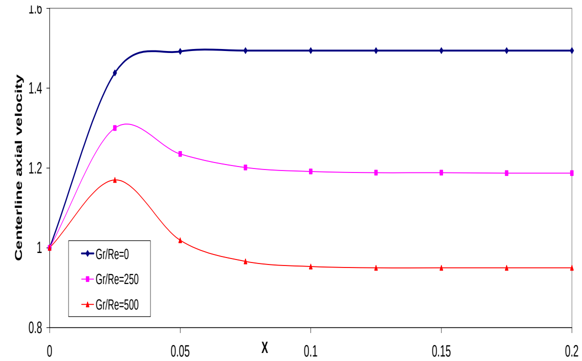

Fig. 3(b): Axial variation of the centerline velocity fixed r_H=1 and M =1

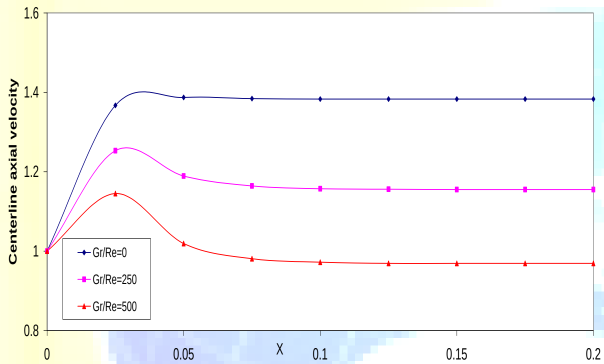

Fig.3(c): Axial variation of the centerline velocity for fixed r_H=1 and M =5

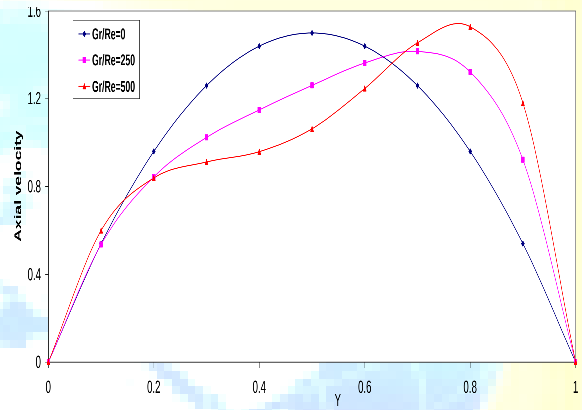

Fig. 4(a): Variation of the velocity profile with the buoyancy parameter for fixed r_H=0.5, X=0.1 and M=0

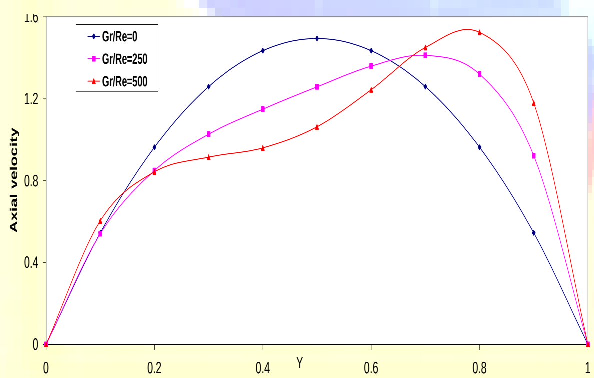

Fig. 4(b): Variation of the velocity profile with the buoyancy parameter for fixed r_H=0.5, X=0.1 and M=1

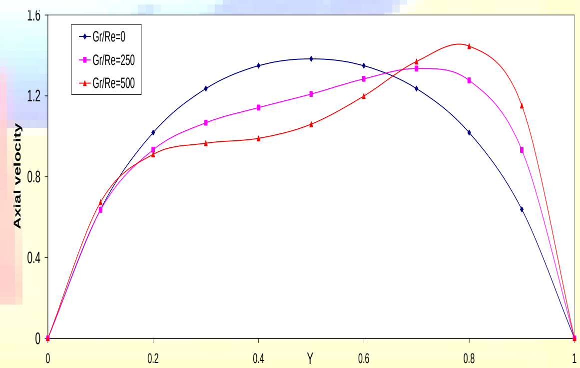

Fig. 4(c) : Variation of the velocity profile with the buoyancy parameter for fixed r_H=0.5, X=0.1 and M=5

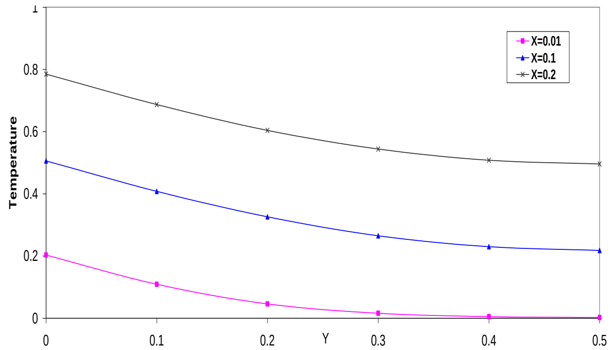

Fig. 5(a) Development of the temperature profile for fixed r_H=1, Gr/Re=250 and M=0

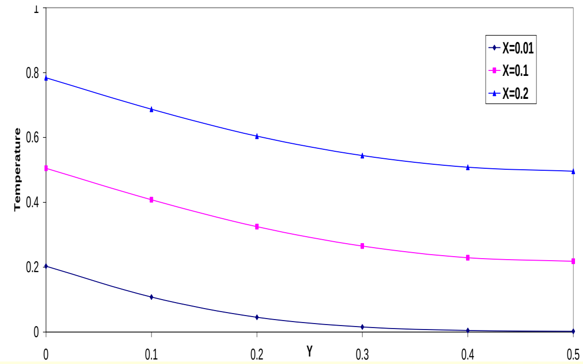

Fig. 5(b): Development of the temperature profile for fixed r_H=1, Gr/Re=250 and M=1

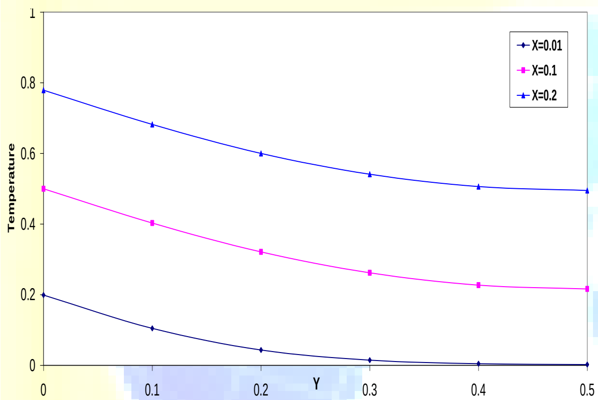

Fig. 5(c) : Development of the temperature profile for fixed r_H=1, Gr/Re=250 and M=5

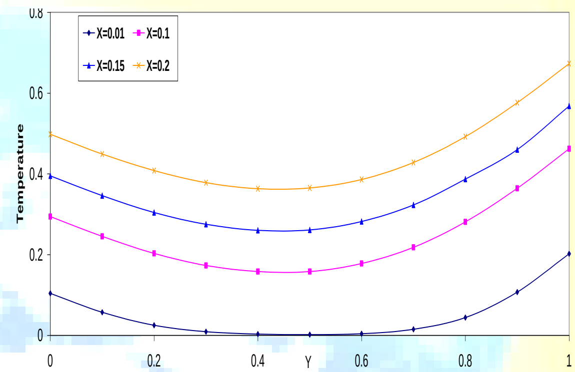

Fig. 6(a): Development of temperature profile for fixed r_H= 0.5, Gr/Re=250 and M=0

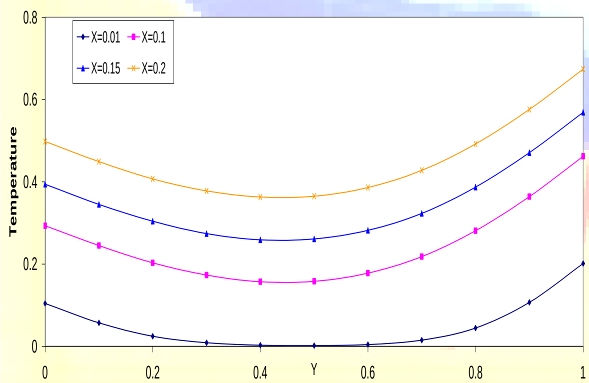

Fig. 6(b): Development of temperature profile for fixed r_H =0.5, Gr/Re=250 and M =1

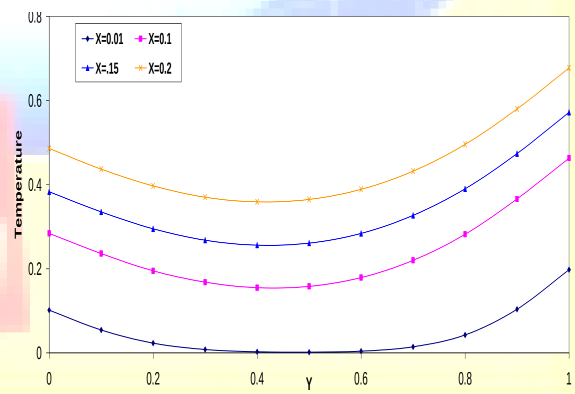

Fig. 6(c): Development of temperature profile for fixed r_H=0.5, Gr/Re=250 and M=5

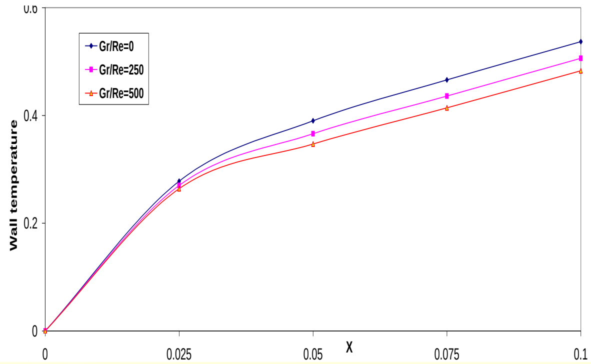

Fig. 7(a): Axial variation of the wall temperature for r_H=1.0 and M=0

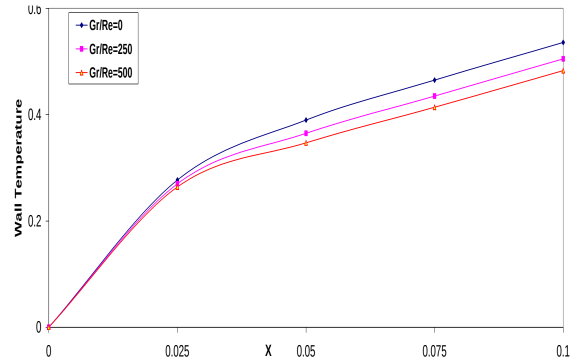

Fig. 7(b): Axial variation of the wall temperature for r_H=1.0 and M=1

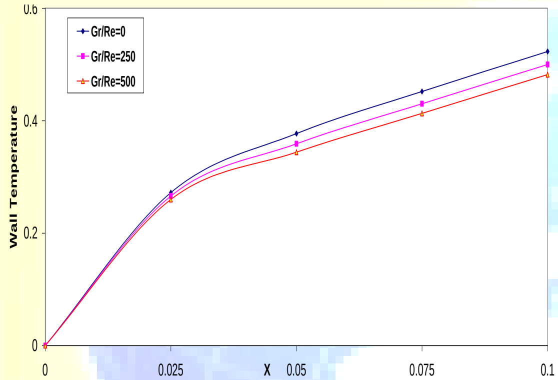

Fig. 7(c) : Axial variation of the wall temperature for r_H=1.0 and M=5

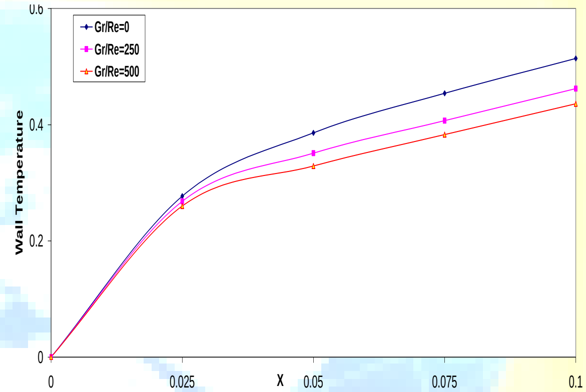

Fig. 8(a) : Axial variation of the wall temperature for r_H=0.5 and M=0

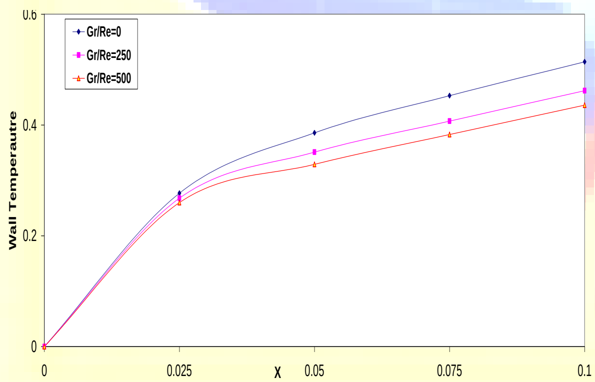

Fig. 8(b): Axial variation of the wall temperature for r_H=0.5 and M=1

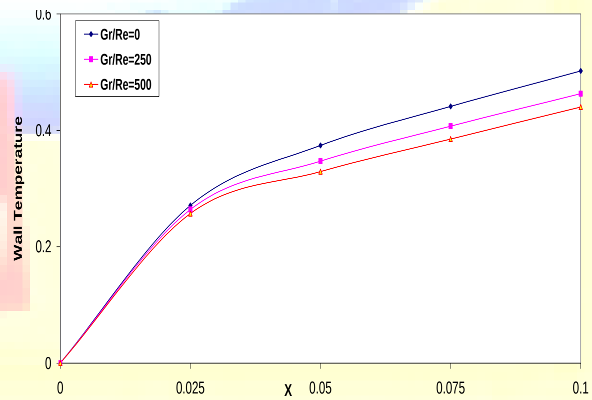

Fig. 8(c) : Axial variation of the wall temperature for r_H=0.5 and M=5

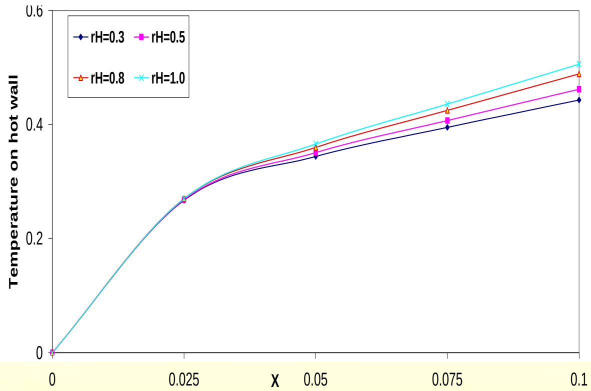

Fig. 9(a) : Axial variation of the temperture on the hot wall for fixed Gr/Re=250 and M=0

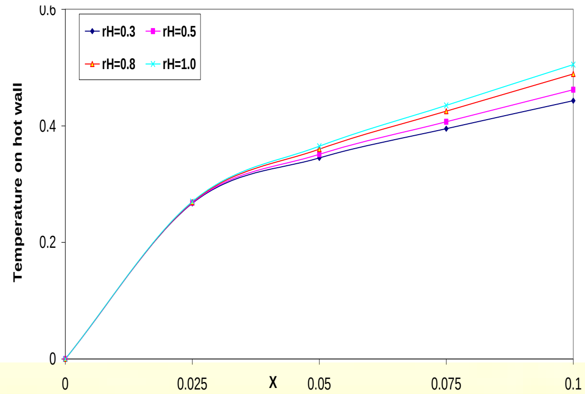

Fig. 9(b) : Axial variation of the temperture on the hot wall for Gr/Re=250 and M=1

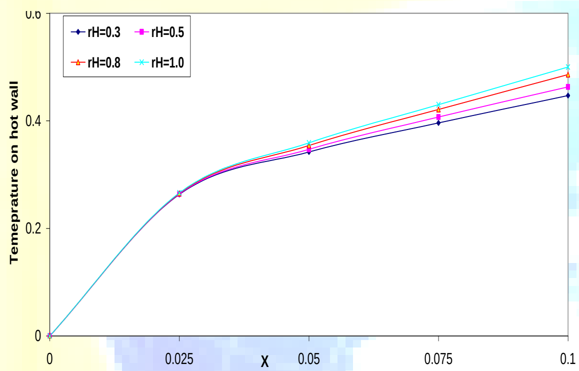

Fig. 9(c) : Axial variation of the temperture on the hot wall for Gr/Re=250 and M=5

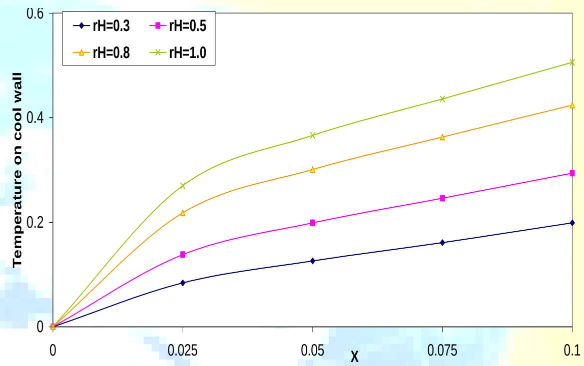

Fig. 10(a) Axial variation of temperatures on the cool wall for Gr/Re=250 and M=0

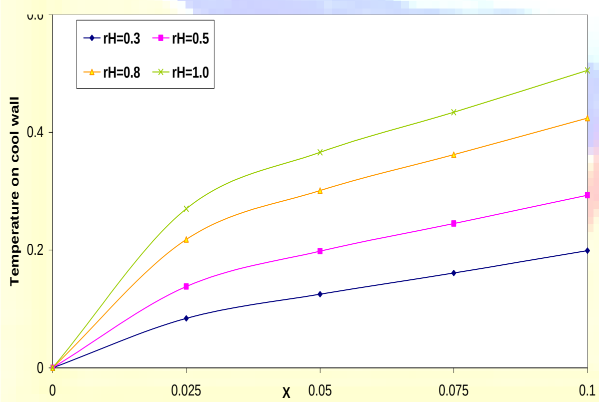

Fig. 10(b): Axial variation of the temperature on the cool wall for Gr/Re=250 and M=1

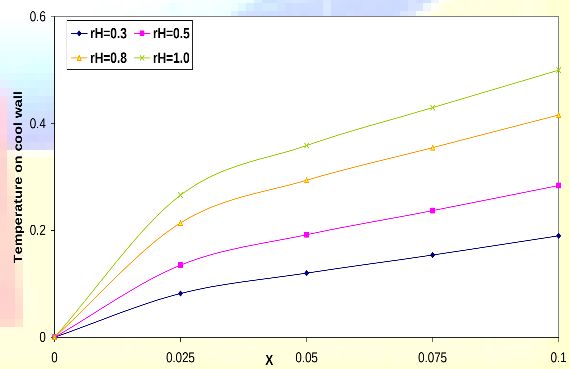

Fig. 10(c): Axial variation of the temperatures on the cool wall for Gr/Re=250 and M=5

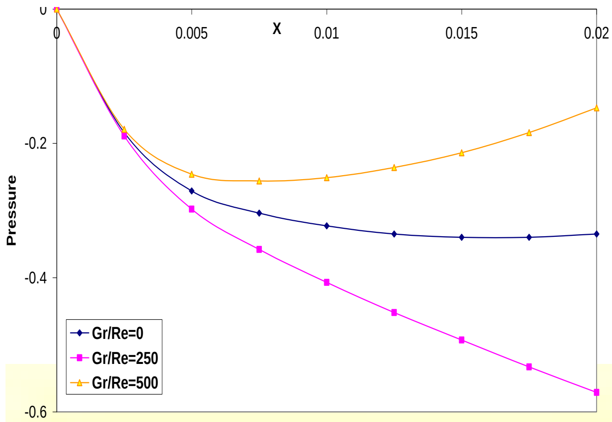

Fig. 11(a) Pressure profile for fixed r_H=1.0 and M=0

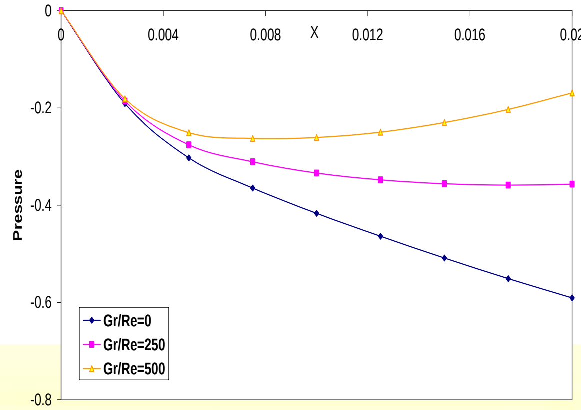

Fig. 11(b) Pressure profile for fixed r_H=1.0 and M=1

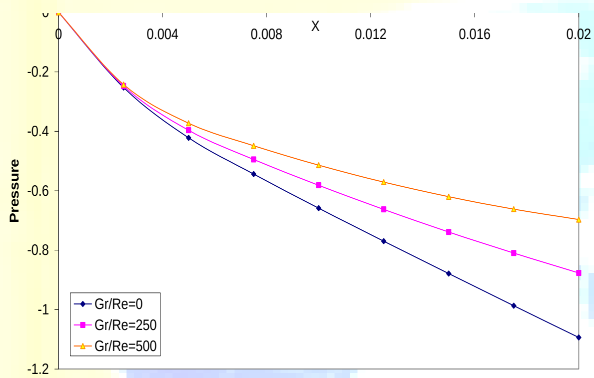

Fig. 11(c): Pressure profile for fixed r_H=1.0 and M=5

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