# PERFORMANCE OF MIMO PRECODERSANDMIMO RECEIVERS IN TERM OF CAPACITY

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## ABSTRACT

The capacity of the channel is a very significant parameter on the performance of a numerical system. A numerical system can be telegraphic or without wire. To have a raised capacity, it is necessary to use MIMO antennas systems. This article compares the performances of the techniques used in MIMO in term of capacity. The number of antennas to the emission and the reception, the type of precoders and receivers used influence the capacity of MIMO channel.

**Keywords** :MIMO, capacity, precoder,receiver, SVD.

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## 1. Introduction

The multiple antennas MIMO technique presents several advantages in the mobile radiocommunication. The MIMO system is modelled by the matrixchannel $H$whose properties will determine the installation of aprecoder. It can be decomposed into several parallel channels (SISO) by using the singular values decomposition (SVD).

In this paper, we will be interested in the linear precoder. This precoder aims to optimize the minimal distance between the points representing the received symbols. In order to determine the performances of this precoder, we will be based on the singular values decomposition and with the calculation of the capacity.

The MIMO system uses various types of receivers which modify also the performance of the channel.

## 2. Diagonalisationof the channel

The SVD makes it possible to obtain the diagonalisation of the channel i.e. obtaining the virtual channel by modeling a MIMO channel in parallel SISO system.

It also makes it possible to determine the capacity of MIMO systems and to apply a linear pretreatment to the data to be transmitted and a post processing to the received signal.

The SVD is given by:

$$H = U \sum V^H \qquad (1)$$

$U$and$V$ are the unitary matrix, $\sum$ is a diagonal matrix such as the number of eigenvalues $r$ is the rank of the channel matrix $H$ and it is equal to $min(n_T, n_R)$.

That is to say the received signal defined by **:**

$$y = Hs + n \qquad (2)$$

By replacing the matrix $H$ in $y$by $U \sum V$, it obtains :

$$y = (U \sum V^H)\, s + n \qquad (3)$$

By applying a preprocessing to the symbols transmitted $(s)$ on the side of the transmitter and a postprocessing to the reception, i.e. by multiplying member to member by $U^H$,weobtain the relation:

$$U^H y = \tilde{y} = U^H((U \sum V^H)\, s + n)$$

$$= \sum V^H s + U^H n$$

Let us pose $U^H n = \tilde{n}$ (noise vector) and

$s = V\tilde{s}$ (symbol vector)

Weobtain :

$$\tilde{y} = \sum V^H V \tilde{s} + \tilde{n}$$

$$\tilde{\boldsymbol{y}} = \sum \tilde{\boldsymbol{s}} + \tilde{n} \qquad (4)$$

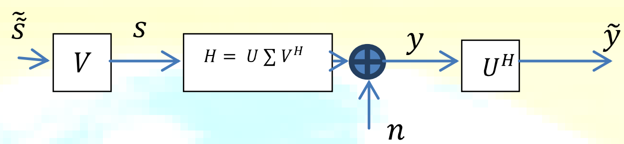


*Figure 1* :Representation of the received signal estimated$\tilde{y}$after decomposition SVD.

## 3. Capacity of a MIMO channel

The capacity of a channel is the maximum quantity of information being able to forward through this channel per unit of time.

If the transmitter does not have any information on the state of the channel, it is in the obligation to transmit on all the channels obtained previously using decomposition SVD with the same proportion of power.

The capacity resulting such a transmission is given by the following equation:

$$C = \sum_{i=1}^{b} log_2(1 + f_i^2 \sigma_i^2) \qquad (5)$$

## 4. MIMO Precoder

The precoding is a technique applied to the emission to optimize the performance of the channel.

### 4.1 PrecoderWater Filling (WF)

A precoder WF is a precoder which maximizes the capacity of a MIMO system. After the diagonalisation of the channel, we obtain *b* independent ways.

According to the constraint of following power to the emission:

$$trace\{F_d F_d^*\} = P$$

Then :

$$P = \sum_{i=1}^{b} f_i^2$$

And

$$\lambda_i = \sum_{i=1}^{b} \sigma_i^2$$

$P$is thetotal power available.

The solution optimizing the capacity is given by:

$$f_i^2 = \begin{Bmatrix} \Psi - \left(\frac{1}{\sigma_i}\right)^2 \; si \; \Psi > \left(\frac{1}{\sigma_i}\right)^2 \; pour \; i = 1, \dots, b \\ 0 \; ailleurs \end{Bmatrix} \qquad (6)$$

With :

$$\psi = \frac{P + \gamma_\psi}{b_\psi} \; and \; \gamma_\psi = \sum_{i=1}^{b_\psi} \frac{1}{\sigma_i^2}$$

$b_\psi$isthe number of ways used by the precoder.

**4.2 Precoder Mean Minimum Squared Error (MMSE)**

A precoder MMSE is a precoder which minimizes the squared error.

The equation of optimization is defined by:

$$\min_{F_d, G_d} E[\|y - s\|^2] \quad s \in \mathbb{C}^b \qquad (7)$$

$\mathbb{C}$isthe group of the symbols defined by the modulation.

After optimization, the coefficients of the precoderare described by the following system:

$$f_i^2 = \begin{Bmatrix} \frac{1}{\sigma_i}\left(\Psi - \frac{1}{\sigma_i}\right) \; si \; \Psi > \frac{1}{\sigma_i} \\ 0 \; ailleurs \end{Bmatrix} \qquad (8)$$

$$\psi = \frac{P + \gamma_\psi}{\sum_{i=1}^{b_\psi} \frac{1}{\sigma_i}}$$

$b_\psi$isthe number of ways used respecting$\sigma_i > \frac{1}{\Psi}$for $i = 1, \dots, b_\psi$ and $\sigma_i \le \frac{1}{\Psi}$ for $i = b_\psi + 1, \dots, b$.

## 5. MIMO Receiver

### 5.1 Receiver Zero Forcing (ZF)

The receiver ZF is a simple receiver which is based on the inversion of the matrix $H$. $H$ is square and invertible.

The objective of this receiver is to minimize the noise.

The noise is given by :

$$n = y - Hs$$

And the mean error is:

$$\|y - Hs\|^2 = (y - Hs)^T(y - Hs) = y^T y - s^T H^T y - y^T Hs + s^T H^T Hs$$

So that this value is minimal, it is necessary that its derivative compared to $s$ is zero.

$$\frac{d}{ds}\|y - Hs\|^2 = 0 - H^T y - H^T y + H^T Hs + H^T Hs = -2H^T y + 2H^T Hs$$

$$\frac{d}{ds}\|y - Hs\|^2 = 0 \leftrightarrow -2H^T y + 2H^T Hs = 0$$

$$H^T Hs = H^T y$$

$$\tilde{s} = (H^T H)^{-1} H^T y$$

If $H$ is complex, $H^T$ becomes $H^H$, so the estimated symbols are equal to :

$$\tilde{s} = (H^H H)^{-1} H^H y = H^\dagger y \qquad (09)$$

### 5.2 Receiver Mean Minimum Squared Error (MMSE)

With the difference of the ZF which reverses the matrix and which thus increases the level of noise, this receiver minimizes the total error caused by the contribution of the noise and the mutual interference of the signals as a result it resists the noise better by not separating perfectly the subchannels.

Like the ZF, the EQMM is a linear receiver, therefore the estimated symbol is formed by:

$$\tilde{s} = X^H y$$

For MMSE, chooses $X^H$ so that the average error $E\{\|\tilde{s} - s\|^2\}$ is minimal.

$$\|\tilde{s} - s\|^2 = (X^H y - s)^H (X^H y - s) = (X^H y - s)(X^H y - s)^H = X^H y y^H X - s y^H X - X^H y s + s s^H$$

$$E\{\|\tilde{s} - s\|^2\} = E(X^H y y^H X - s y^H X - X^H y s + s s^H)$$

However,

$$E(ss^H) = R_{ss} = E\left\{\begin{bmatrix} s_1 \\ s_2 \\ . \\ . \\ . \\ s_{n_T} \end{bmatrix}\begin{bmatrix} s_1^H s_2^H & \dots & s_{n_T}^H \end{bmatrix}\right\} = \begin{bmatrix} |s_1|^2 & 0 & \dots & 0 \\ 0 & |s_2|^2 & \dots & 0 \\ & \dots & & \\ 0 & 0 & & |s_{n_T}|^2 \end{bmatrix}$$

$$|s_1|^2 = |s_2|^2 = |s_{n_T}|^2 = \frac{P}{n_T}$$

$$E(ss^H) = R_{ss} = \frac{P}{n_T} I_{n_T}$$

$$E(yy^H) = R_{yy} = \frac{P}{n_T} HH^H + \sigma^2 I_{n_R}$$

$$E(ys^H) = R_{sy}^H = R_{ys} = \frac{P}{n_T} H$$

$$E(sy^H) = R_{sy} = E\{s(Hs+n)^H\}$$

So,

$$E\{\|\tilde{s} - s\|^2\} = X^H R_{yy} X - R_{sy} X - X^H R_{ys} + R_{ss} = X^H R_{yy} X - 2X^H R_{ys} + R_{ss}$$

We will be interested in the minimal value of $E\{\|\tilde{s} - s\|^2\}$.

So,

$$2R_{yy}X - 2R_{ys} = 0$$

$$X = R_{yy}^{-1} R_{ys} = \left(\frac{P}{n_T} HH^H + \sigma^2 I_{n_R}\right)^{-1} \frac{P}{n_T} H = \left(HH^H + \frac{n_T}{\rho} I_{n_R}\right)^{-1} H$$

Consequently, the expression of the estimated symbols is :

$$\tilde{s} = \left(H^H H + \frac{n_T}{\rho} I_{n_R}\right)^{-1} H^H y \qquad (10)$$

$\rho (= \frac{P}{\sigma^2})$ is the mean SNR per antenna receiver.

### 5.3 Receiver Successive Interference Cancellation (SIC)

The receiver SIC is a receiver employed if the receiver is not linear.

The equation $y = Hs + n$ is always valid

$$y = [\overline{h_1}|\overline{h_2}| \dots \overline{h_{n_T}}] \begin{bmatrix} s_1 \\ s_2 \\ \cdot \\ \cdot \\ \cdot \\ s_{n_T} \end{bmatrix} + n$$

$\overline{h_\iota}$is a vector column of $n_R$rows, however :

$$y = \overline{h_1}s_1 + \overline{h_2}s_2 + \cdots + \overline{h_{n_T}}s_{n_T} + n \qquad (11)$$

By considering the pseudo inverse of $H$,

$$Q = H^\dagger = \begin{bmatrix} q_1^H \\ q_2^H \\ \cdot \\ \cdot \\ \cdot \\ q_{n_T}^H \end{bmatrix} y \qquad (12)$$

$$QH = I_{n_T} = \begin{bmatrix} q_1^H \\ q_2^H \\ \cdot \\ \cdot \\ \cdot \\ q_{n_T}^H \end{bmatrix} [\overline{h_1}|\overline{h_2}| \dots \overline{h_{n_T}}]$$

Multiply $y$per $q_1^H$ :

$$\widetilde{y_1} = q_1^H y = q_1^H(\overline{h_1}s_1 + \overline{h_2}s_2 + \cdots + \overline{h_{n_T}}s_{n_T}) + q_1^H n$$

$$\widetilde{y_1} = s_1 + \tilde{n} \qquad (13)$$

With $\tilde{n} = q_1^H n$

The equation 15 makes it possible to decode the symbol $s_1$.

After,

$$\widetilde{y_2} = y - \overline{h_1}s_1 = (\overline{h_1}s_1 + \overline{h_2}s_2 + \cdots + \overline{h_{n_T}}s_{n_T}) + n - \overline{h_1}s_1 = \overline{h_2}s_2 + \cdots + \overline{h_{n_T}}s_{n_T} + n$$

$$\widetilde{y_2} = [\overline{h_2}|\overline{h_3}| \dots \overline{h_{n_T}}] \begin{bmatrix} s_2 \\ s_3 \\ \cdot \\ \cdot \\ \cdot \\ s_{n_T} \end{bmatrix} + n$$

Consider $H' = [\overline{h_2}|\overline{h_3}| \dots \overline{h_{n_T}}]$ is a matrix $n_R \times (n_T - 1)$,

$$\widetilde{y_2} = H' \begin{bmatrix} s_2 \\ s_3 \\ \cdot \\ \cdot \\ \cdot \\ s_{n_T} \end{bmatrix} + n \qquad (14)$$

Let us suppose, $Q' = (H')^{\dagger}$.

Now, let us take again the same procedures to decode $s_2, \ldots, s_{n_T}$.

## 6. Results and discussion

### 6.1 Influence of the precoders on the capacity of MIMO channel

The figure 2 represents the influence of the precoder WF on the capacity of MIMO channel.

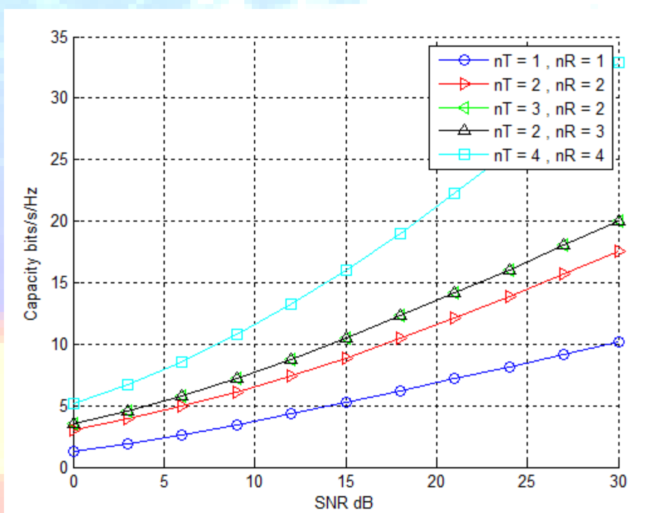


*Figure 2* : influence of the precoder WF on the capacity of MIMO channel.

The figure 2 shows us that:

- When the SNR increases, the capacity increases too
- The capacity increases with the number of antennas
- Capacity for $n_T = 3, n_R = 2$ and the same one as for $n_T = 2, n_R = 3$

The figure 3 represents the influence of the precoderMMSE on the capacity of MIMO channel.

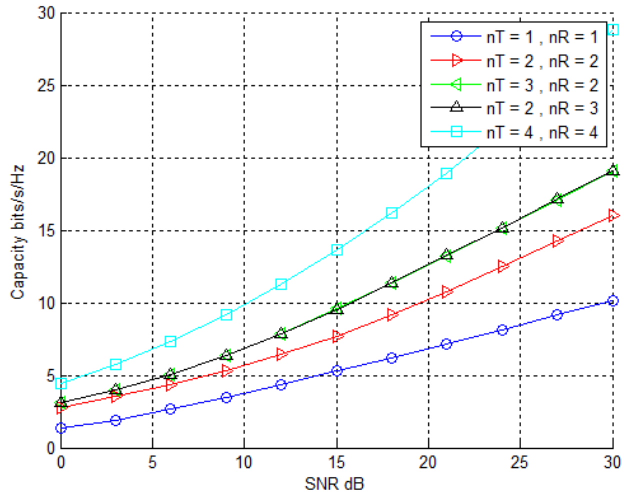


*Figure 3* : Influence of the precoder MMSE on the capacity of MIMO channel.

The results obtained on figure 1 are always valid for the MMSE.

By comparing figure 1 and figure 2, and by fixing the SNR at 15 dB; one can have table 1:

| | Number of transmit antennas and receive antennas | | | | |
|---|---|---|---|---|---|
| Precoder | 1-1 | 2-2 | 3-2 | 2-3 | 4-4 |
| WF | 5 | 9 | 11 | 11 | 17 |
| MMSE | 5 | 8 | 10 | 10 | 14 |

*Table 1* : Comparison of precoder WF and precoder MMSE.

The table 1 shows us that the precoder WF allows to have a capacity more raised compared to precodor MMSE.

**6.2 Influence of the receivers on the capacity of MIMO channel**

The figure 4 represents the influence of the receivers on the capacity of MIMOchannelwhere $n_T = n_R = 2$.

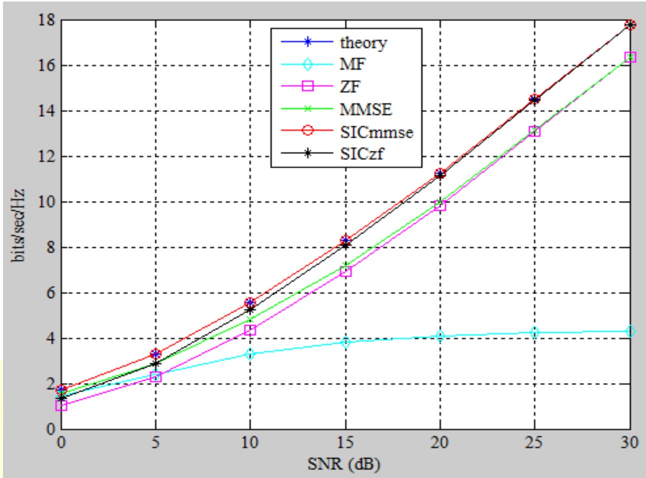


*Figure 4* : Influence of the receivers on the capacity of MIMO channel.

The figure 4 shows us that:

- For weak SNR, the capacities for the various receivers are almost identical
- The capacity for receiver SIC/MMSE represents the best performance
- The capacity for receiver SIC/MMSE has the same capacity as the theory
- For weak SNR, MMSE and MF are identical
- For raised SNR, MMSE and ZF are identical
- Receivers SIC are more powerful than the linear receivers

The figure 5 represents the ratio between the capacity of the various receivers and the theoretical capacity.

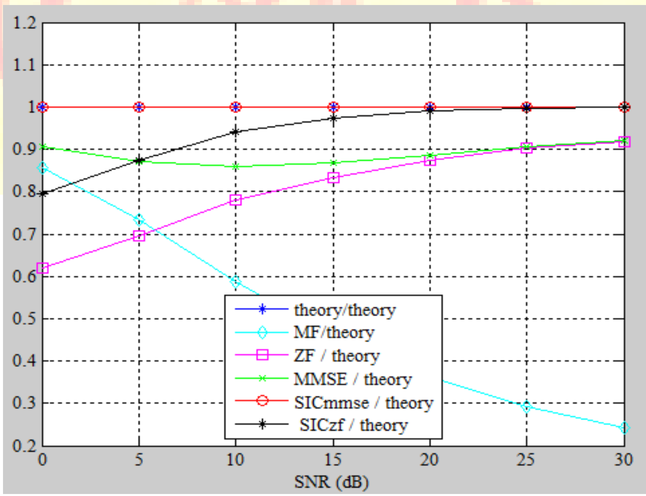


*Figure 5* : Ratio between the capacity of the various receivers and the theoretical capacity.

## 7. Conclusion

To send the signals with MIMO system, a coder should be used. In the case of the MIMO, the most adequate coder is the coder time spaces. In order to still increase the performances of MIMO systems in terms of robustness, flows, quality of service, there are methods to pre-equalize the data before the emission. To carry out a precodage, it is necessary to know the state of the channel on the level of the receiver.

The precoders each have their own asset:

The WF maximizes the capacity, the MMSE minimizes themean squared error in reception.

To estimate the symbols, the simplest receiver, the (ZF), is sensitive to the noise and the minimization of the MMSEdoes not remove all the IES. The most powerful receiver is the receiver SIC.

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