AN EFFICIENT EVALUATION OF PERT NETWORK BY MONTE CARLO SIMULATION AND MULTI-OBJECTIVE GREY SITUATION DECISION-MAKING THEORY

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Abstract

Complex research and development projects can be managed effectively if the projectmanagers have the means to plan and control the schedules and cost of the work required to achieve their technical performance objectives. The direct methods used for solving project management problems are critical path method (CPM) and project evaluation and review technique (PERT). The effectiveness of direct methods in modeling the complex situation is limited when uncertainty arise in real world situation. Monte Carlo Simulation process for uncertain time duration of activities will be discussed briefly. Furthermore, the project's scheduling may change according to different parameters such as duration, cost, risk, visibility etc. The Multi-objective Grey Situation Decision-making theory(MGSD) has never adopted in project scheduling. So it's the first time to apply this method to analyze the project's schedule.

Keywords: Project Scheduling, Simulation of PERT Network, Multi-objective Grey Situation Decision-making theory (MGSD), Critical path.

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1. Introduction

A project is well defined set of activities, all of which must be completed to finish the project.Program Evaluation and Review Techniques (PERT) and Critical Path Method (CPM) are the techniques of operations research used for planning, scheduling and controlling of large and complex projects. These techniques are based on representation ofthe project as a network of activities. A network is a graphical plane consisting of acertain configuration of arrows and nodes for showing the logical sequence of variousactivities to be performed to achieve project objectives. PERT and CPM both of them share in common determination of critical path available in the network [1], [8].

As we know most of the activities in real world are uncertain so PERT and CPM failure in most of the cases. In this kind of situation one can make approach of simulation of activities of the project. Here we will apply concepts of Monte-Carlo Simulation for Simulation of PERT network [7].

In this research, Sometimes project include various parameters like cost, time duration, risk etc. projectevaluation and review of such a project, each parameter play some role. So it is necessaryto consider all in evaluation of project. The decision-making is known as to find out the optimal one from the possible option to achieve one or more established objectives.Such behaviors are undertaken by specific methods or means.It must be based on some particular information and experience and in accordance with the real condition and consideration on the existing environment's feasibility.The grey-decision-making as introduced in [5] is performed according to the grey system's thought and a grey element may be combined in the decision-making process. TheGrey Situation Decision-making (GSD) as in [4] is referred to the process of superiorselection and weakness elimination in whichdifferent countermeasures should be chosenaccording to the different events, then the extent of its effect and the objective will beconsidered. If there consider only a single objective for decision-making (SGSD). And ifthere consider several objectives for decision-making it is so-called Multi-objective GreySituation Decision-making theory (MGSD) is used for solving PERT network. The main research contribution of the present paper is to make the first attempt in the published literature to show how the MGSD can be applied for solution of PERT Network with ability and results are encouraging.

In the following section of this paper, Monte-Carlo Simulation for PERT Network is explained followed by MGSD for PERT network.

2. Monte-Carlo Simulation

Simulation is the imitation of the operation of a real-world process or system over time [7]. The act of simulating something firstrequires that a model be developed; this model represents the key characteristics orbehaviors/functions of the selected physical or abstract system or process. The modelrepresents the system itself, whereas the simulation represents the operation of the systemover time.

In simulation, probability distributions are used to define numerically outcomes in a samplespace by assigning a probability to each of possible outcomes.The principal behind the Monte Carlo Simulation technique is representative of the givensystem under analysis by system described by some known probability distribution and then drawing random samples from probability distribution by means of random numbers. In case it is not possible to describe a system in terms of standardprobability distribution such as normal, Poission, exponential, gamma, etc., an empiricalprobability distribution can be constructed [15] [16].

2.1 Steps for Monte-Carlo Simulation

Following are the steps used for simulation of PERT network by Monte Carlo Simulation:

Step 1: Setting up a probability distribution for variables to be analyzed.

Step 2: Building a cumulative probability distribution for each random variables.

Step 3: Generate random numbers. Assign an appropriate set of random numbers to represent value of interval of values for each random variable.

Step 4: Conduct the simulation experiment by means of random sampling.

Step 5: Repeat Step 4 until the required number of simulation run has been generated.

Step 6: Design and implement a course of action and maintain control.

2.2 Random Numbers

Monte Carlo Simulation requires the generation of a sequence of random numbers. This sequence of random number used for choosing random observations from the given probability distribution. Random Numbers can be generated by Arithmetic computation and by Computer generator [9].

2.2.1 Arithmetic computation

The n^{th} random number r_n consisting of k-digits generated by congruential method given by

$$r_n \equiv p.r_{n-1}(\text{modulo } m) \qquad (1)$$

where p and m are positive integers, p < m, $r_{n-1}$is a k-digit number and modulo mmeans thatr_n is the remainder when p.$r_{n-1}$is divided by m.To start the process of generating randomnumbers,the first random number (also called seed) $r_0$is specified by user. Then using above recurrence relation a sequence of k-digit random number with period h < m at which point the number $r_0$occurs again can be generated.

2.2.2 Computer generator

The random numbers that are generated by using computersoftware are uniformly distributed fractions between 0 and 1. the software works on theconcept of cumulative distribution function for the random variables for which we areseeking to generate random numbers.

For example, the negative exponential function with density function with density function

$$f(x) = \lambda e^{-\lambda x} dx = 1 - e^{-\lambda x} \qquad (2)$$

$$e^{-\lambda x} = 1 - f(x)$$

Taking log on both sides, we have

$$\lambda x = \log[1 - f(x)]$$

$$x = -(1/\lambda)\log[1 - f(x)]$$

If r = f(x) is uniformly distributed random decimal fraction between 0 and 1, then the exponential variable associated with r is given by

$$x_n = -(1/\lambda)\log(1-r) = (1/\lambda)\log r \tag{3}$$

3. The Multi objective Grey Decision-Making Theory

The Grey information becomes raw material for decision-making. The decision making in common includes four elements; event, strategy, effect and target. In our case of PERT network we will consider activity, parameters, effect and available paths. The combination of events p_i and strategy q_j is called situation (path) which can be defined as $S_{ij} = (p_i, q_j)$ [3].

The ratio of the effect measure r_{ij} and situation $r_{ij}/S_{ij} = r_{ij} / (p_i, q_j)$ is called as the decision unit which can be represented by δ_{ij}. Where r_{ij} is effect measure for s_{ij}. There is event cluster A = (p_1, p_2, p_3, … p_n) and strategy cluster B = (q_1, q_2, q_3,…q_n). For the same event p_i, there are n strategies. So (p_i, q_1), (p_i, q_2),… (p_i, q_n) turns to be m situation strategies S_{i1}, S_{i2},….S_{im}. Then these decision unit become the row vector:

$$\delta_i = \left[\frac{r_{i1}}{S_{i1}}, \frac{r_{i2}}{S_{i2}}, ..., \frac{r_{im}}{S_{im}}\right]$$

With the same reason we can get the column vector:

$$\delta_j = \left[\frac{r_{1j}}{S_{1j}}, \frac{r_{2j}}{S_{2j}}, ..., \frac{r_{nj}}{S_{nj}}\right]$$

Where δ_i is the row vector, i = 1, 2, 3,…n and δ_j is the column vector, j = 1, 2, 3,… m. The decision matrix S is formed by the decision column and the decision row.

$$\begin{bmatrix} \frac{r_{11}}{s_{11}} & \frac{r_{12}}{s_{12}} & ..... & \frac{r_{1m}}{s_{im}} \\ \frac{r_{21}}{s_{21}} & \frac{r_{22}}{s_{22}} & ..... & \frac{r_{2m}}{s_{2m}} \\ . & . & & . \\ . & . & & . \\ \frac{r_{n1}}{s_{n1}} & \frac{r_{n2}}{s_{n2}} & ..... & \frac{r_{nm}}{s_{nm}} \end{bmatrix}$$

3.1 The Effect Measure

There are three types of effect measures namely upper limit, lower limit and central limit measure.

The upper limit measure is used to find the maximum deviation data. Such induces as safety length(in case of critical path), speed etc. are the bigger the better [10] [17].

$$r_{ij} = \frac{u_{ij}}{u_{\max}} \tag{4}$$

Where u_{ij} is the actual effect measure for S_{ij} and $u_{\max}$ is the maximum data in S_{ij}.

$u_{ij} \le u_{\max} : r_{ij} \le 1$

The lower limit measure is used to find the minimum deviation data. Such induces as risk, cost, and time etc. are the smaller the better.

$$r_{ij} = \frac{u_{\min}}{u_{ij}} \tag{5}$$

Where u_{ij} is the actual effect measure for S_{ij} and $u_{\min}$ is the minimum data in S_{ij}.

$u_{ij} \ge u_{\min} : r_{ij} \ge 1$.

The central effect measure takes the value near by the specific goal as consideration scope.

$$r_{ij} = \min(u_{ij}, u_0) \Big/ \max(u_{ij}, u_0) \tag{6}$$

Where u_{ij} is the actual effect measure for S_{ij}, u_0 is the reference point and $r_{ij} \le 1$. And also we can get r_{ij} from:

$$r_{ij} = u_0 \Big/ (|u_{ij} - u_0| + u_0) \tag{7}$$

3.2 Situation Decision

Whenever there are several objectives in the given situation the comprehensive consideration on the objective is called multi-objective situation decision-making. The effect measure of k^{th} objective is remarked by $r_{ij}^{(k)}$. The corresponding decision unit given by $r_{ij}^{(k)} / S_{ij}$. The decision column is $\delta_i^{(k)}$, the decision row $\delta_j^{(k)}$ and the decision matrix $D^{(k)}$ [6] [17].

$$\begin{bmatrix} \frac{r_{11}^{(k)}}{s_{11}} & \frac{r_{12}^{(k)}}{s_{12}} & \ldots\ldots & \frac{r_{1m}^{(k)}}{s_{im}} \\ \frac{r_{21}^{(k)}}{s_{21}} & \frac{r_{22}^{(k)}}{s_{22}} & \ldots\ldots & \frac{r_{2m}^{(k)}}{s_{2m}} \\ . & . & & . \\ . & . & & . \\ \frac{r_{n1}^{(k)}}{s_{n1}} & \frac{r_{n2}^{(k)}}{s_{n2}} & \ldots\ldots & \frac{r_{nm}^{(k)}}{s_{nm}} \end{bmatrix}$$

The multi-objective situation decision comprehensive matrixis $D^{(\Sigma)}$.

$$\begin{bmatrix} \frac{r_{11}^{(\Sigma)}}{s_{11}} & \frac{r_{12}^{(\Sigma)}}{s_{12}} & \ldots\ldots & \frac{r_{1m}^{(\Sigma)}}{s_{im}} \\ \frac{r_{21}^{(\Sigma)}}{s_{21}} & \frac{r_{22}^{(\Sigma)}}{s_{22}} & \ldots\ldots & \frac{r_{2m}^{(\Sigma)}}{s_{2m}} \\ . & . & & . \\ . & . & & . \\ \frac{r_{n1}^{(\Sigma)}}{s_{n1}} & \frac{r_{n2}^{(\Sigma)}}{s_{n2}} & \ldots\ldots & \frac{r_{nm}^{(\Sigma)}}{s_{nm}} \end{bmatrix}$$

So the comprehensive effect measure is

$$r_{ij}^{(\Sigma)} = \frac{1}{N}\sum_{k=1}^{n} r_{ij}^{(k)} \quad (8)$$

4. Solution of PERT Network

Consider the following example Figure 1 of PERT network with given activities and their time duration.

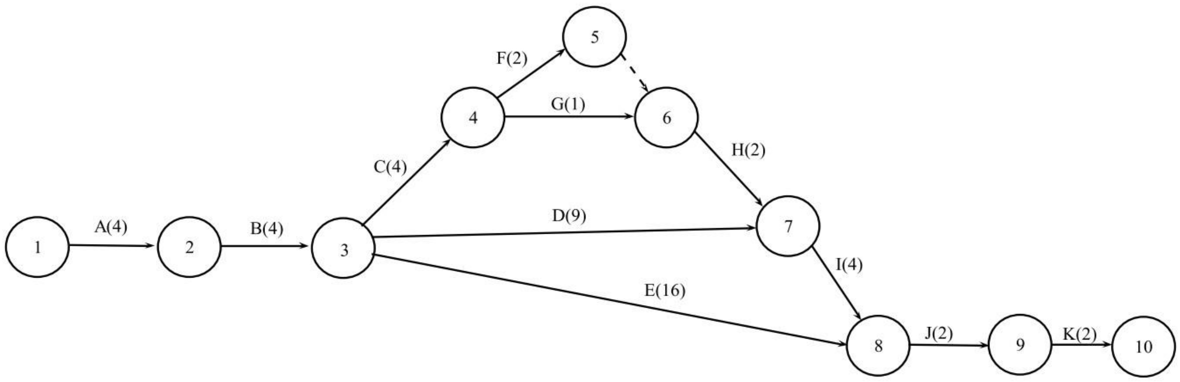


Figure 1

Activity denoted by dotted line is called dummy activity. Our target is to find critical path from the given PERT network.

4.1 Solution by Monte-Carlo Simulation

In real world whenever uncertainties arise in the activity time duration can be solved by Monte-Carlo Simulation method. Considering the given example with its probability distribution is shown in Table 1.

Activity	Probability / Day															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A		0.3		0.5			0.2									
B			0.1	0.7					0.2							
C		0.2			0.6			0.2								
D						0.1		0.1	0.7			0.1				
E									0.2	0.2			0.2			0.4
F		0.4	0.6													
G	0.7			0.3												
H		0.6												0.4		
I	0.1			0.3											0.6	
J		0.4	0.6													
K	0.2	0.6		0.2												

Table 1

The network diagram based on the precedence relationship and expected time duration is shown in Figure 2.

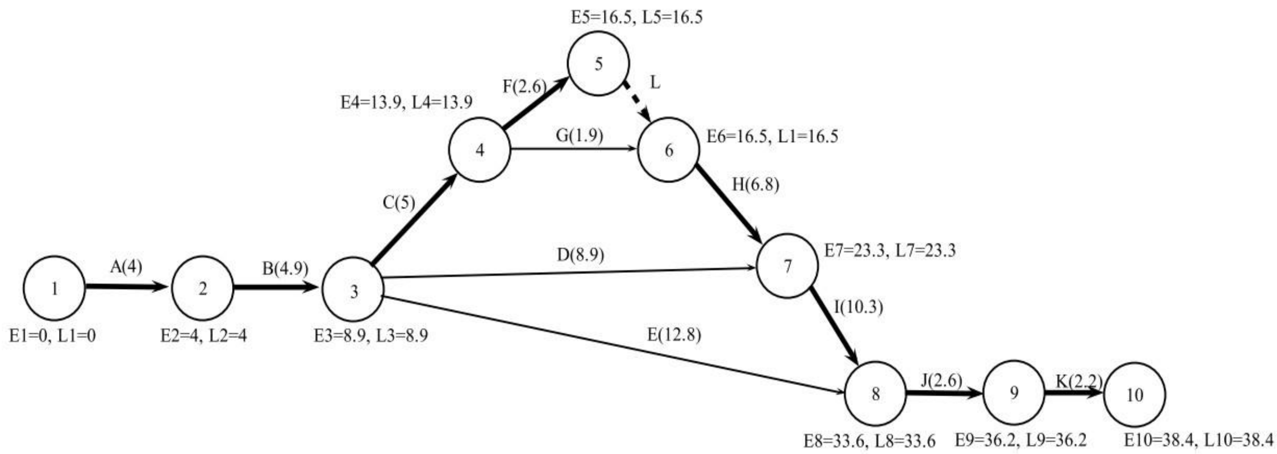


Figure 2

The expected completion time of each activity can be calculated by the equation:

$$\text{Expected time} = \sum(\text{Activity Time} \times \text{Probability}) \quad (9)$$

The simulation sheet for simulation run given in Table 2.

Activity	Time	Probability	Cumulative Probability	Random number range
A	2	0.3	0.3	00-29
	4	0.5	0.8	30-79
	7	0.2	1	80-99
B	3	0.1	0.1	00-09
	4	0.7	0.8	00-79
	9	0.2	1	80-99
C	2	0.2	0.2	00-19
	5	0.6	0.8	20-79
	8	0.2	1	80-99
D	6	0.1	0.1	00-09
	8	0.1	0.2	10-19
	9	0.7	0.9	20-89
	12	0.1	1	90-99
E	9	0.2	0.2	00-19
	10	0.2	0.4	20-39
	13	0.2	0.6	40-59
	16	0.4	1	60-99
F	2	0.4	0.4	00-39
	3	0.6	1	40-99
G	1	0.7	0.7	00-69
	4	0.3	1	70-99
H	2	0.6	0.6	00-59
	14	0.4	1	60-99
I	1	0.1	0.1	00-09
	4	0.3	0.4	10-39
	15	0.6	1	40-99
J	2	0.4	0.4	00-39
	3	0.6	1	40-99

K	1 2 4	0.2 0.6 0.2	0.2 0.8 1	00-19 20-79 80-99

Table 2

Simulation worksheet given by Table3 can be generated by using data given in Table2.

Run	R	A	R	B	R	C	R	D	R	E	R	F	R	G	R	H	R	I	R	J	R	K
1	22	2	7	3	68	5	99	12	15	9	68	3	95	4	23	2	29	4	9	2	85	4
2	92	7	95	9	81	8	9	6	28	10	95	3	6	1	87	14	88	15	61	3	52	2
3	2	2	22	4	57	5	51	9	89	16	24	2	82	4	3	2	7	1	32	2	5	1
4	47	4	99	9	16	2	17	8	92	16	46	3	13	1	9	14	38	4	92	3	30	2
5	56	4	37	4	13	2	85	12	52	13	5	2	90	4	62	14	91	15	27	2	62	2
Sum		19		29		22		47		67		13		14		46		39		12		11
Ave.		3.8		5.8		4.4		9.4		13		2.6		2.8		9.2		7.8		2.4		2.2

Table 3

Where in Table 3 the column R gives values of random numbers.

For each run the project time is obtained as follows and we have simulation result shown in Table4:

Simulation Run	Activity Time	Project Duration
1	2+3+5+2+9+2+4	37
2	7+9+8+14+15+3+2	58
3	2+4+5+12+9+2+1	35
4	4+9+2+14+16+3+2	51
5	4+4+4+15+2+2	31

Table 4

From Table4 total duration is 212. The Critical Path will be 212/5 = 42.4 days is nearby the expected critical path 38.4.

4.2 Solution by Multi Objective Grey Situation Decision Making Theory

Considering the same example and solving by Multi Objective Grey Situation Decision Making Theory and going for step by step procedure.

Step: 1 Define the strategy set (Available path in the network)

Step: 2 Establish the situation set

$S = (S_{11}, S_{12}, S_{13}, S_{14})$

Where: $S_{11} = (p_1, q_1)$, For path 1-2-3-4-5-6-7-8-9-10

$S_{12} = (p_1, q_2)$, For path 1-2-3-4-6-7-8-9-10

$S_{13} = (p_1, q_3)$, For path 1-2-3-7-8-9-10

$S_{14} = (p_1, q_4)$, For path 1-2-3-8-9-10

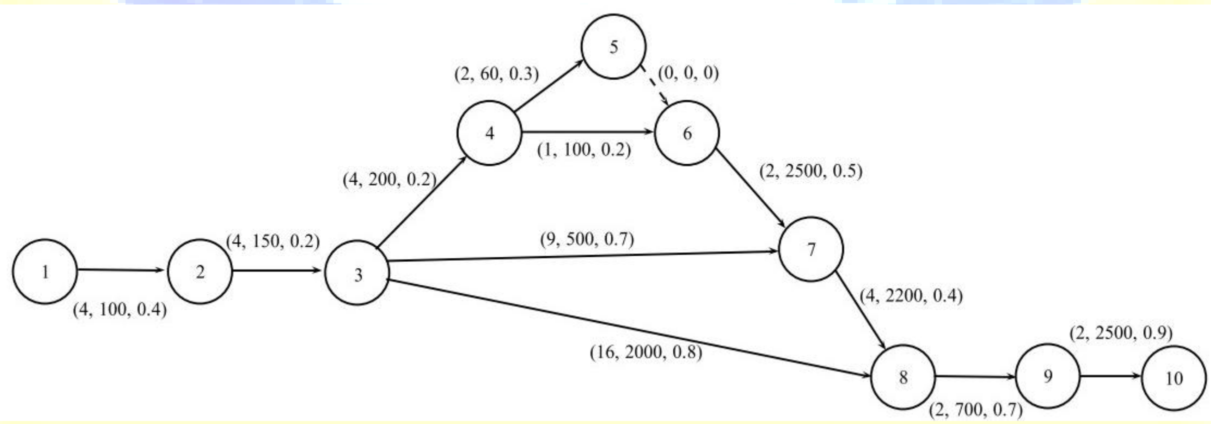
Step 3: Define the set of parameters.

Parameter 1: Path duration

Parameter 2: Cost

Parameter 3: Risk

Step 4: Draw PERT network with qualitative and the quantitative assessment to three parameter sets according to data given of the example Figure 1, we can draw Figure 3. And also calculate table for qualitative and the quantitative assessment shown in Table 5.



Where (Duration, Cost, Risk) for each activity.

Figure 3

Paths	Total Duration	µ1	Total Cost	µ2	Total Risk	µ3
1-2-3-4-5-6-7-8-9-10	24	μ_{11}=24	8410	μ_{21}=8410	3.6	μ_{31}=3.6
1-2-3-4-6-7-8-9-10	23	μ_{12}=23	8450	μ_{22}=8450	3.5	μ_{32}=3.5
1-2-3-7-8-9-10	25	μ_{13}=25	6150	μ_{23}=6150	3.3	μ_{33}=3.3

1-2-3-8-9-10	28	μ_{14}=28	5450	μ_{24}=5450	3	μ_{34}=3

Table 5

Step:5 Definethe effect measure under different parameters:

(a) Parameter 1: The longer duration of path considered for critical path, so upper limit effect measure isconsidered here.

$$r_{11}^{(1)} = \frac{24}{28}, \quad r_{12}^{(1)} = \frac{23}{28}, \quad r_{13}^{(1)} = \frac{25}{28}, \quad r_{14}^{(1)} = \frac{28}{28}$$

The effect matrix for parameter 1 is,

$$r_{ij}^{(1)} = \left[\frac{24}{28}, \frac{23}{28}, \frac{25}{28}, \frac{28}{28}\right]$$

(b) Parameter 2: The lower cost of path considered, so lower limit effect measure isconsidered here.

$$r_{11}^{(2)} = \frac{5450}{8410}, \quad r_{12}^{(2)} = \frac{5450}{8450}, \quad r_{13}^{(2)} = \frac{5450}{6150}, \quad r_{14}^{(2)} = \frac{5450}{5450}$$

The effect matrix for parameter 2 is,

$$r_{ij}^{(2)} = \left[\frac{5450}{8410}, \frac{5450}{8450}, \frac{5450}{6150}, \frac{5450}{5450}\right]$$

(c) Parameter 3: The lower the Risk for best path, so lower limit effect measure isconsidered here.

$$r_{11}^{(3)} = \frac{3}{3.6}, \quad r_{12}^{(3)} = \frac{3}{3.5}, \quad r_{13}^{(3)} = \frac{3}{3.3}, \quad r_{14}^{(3)} = \frac{3}{3}$$

The effect matrix for parameter 2 is,

$$r_{ij}^{(3)} = \left[\frac{3}{3.6}, \frac{3}{3.5}, \frac{3}{3.3}, \frac{3}{3}\right]$$

Step 6: Calculate the comprehensive effect measure.

$$r_{11}^{(\Sigma)} = \frac{1}{3}\left(\frac{24}{28} + \frac{5450}{8410} + \frac{3}{3.6}\right) = 0.78$$

$$r_{12}^{(\Sigma)} = \frac{1}{3}\left(\frac{23}{28} + \frac{5450}{8450} + \frac{3}{3.5}\right) = 0.77$$

$$r_{13}^{(\Sigma)} = \frac{1}{3}\left(\frac{25}{28} + \frac{5450}{6150} + \frac{3}{3.3}\right) = 0.90$$

$$r_{14}^{(\Sigma)} = \frac{1}{3}\left(\frac{28}{28} + \frac{5450}{5450} + \frac{3}{3}\right) = 1$$

Step 7: According to the sorting result of comprehensive effect measure the most superior situation can be selected. The comprehensive effect matrix for this example should be

$$r_{ij}^{(\Sigma)} = [r_{11}^{(\Sigma)}, r_{12}^{(\Sigma)}, r_{13}^{(\Sigma)}, r_{14}^{(\Sigma)}] = [0.78, 0.77, 0.90, 1.0]$$

Obviously there is

$$r_{14}^{(\Sigma)} > r_{13}^{(\Sigma)} > r_{11}^{(\Sigma)} > r_{12}^{(\Sigma)}$$

So we can find out that $r_{12}^{(\Sigma)}$ is worst one, its corresponding to situation S_{12} and $r_{14}^{(\Sigma)}$ is the best one which is critical path, its corresponding to situation S_{14}. In other words 1-2-3-8-9-10 is critical path.

Conclusion

If for the given PERT network activity time durations are uncertain then it is not possible to find critical path by traditional method. Here we have reviewed MonteCarlo Simulation process. It may be noted that simulated mean project completion time is nearby expected critical path and completed time indicated using expectedvalues alone.

In this research we explained the multiobjective Grey Decision-Making Theory (MGSD). This technique is useful when project involved with multiple parameters. This process will give efficient solution to find critical path of PERT network where activities are dependent upon multiple parameters.

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