

## “CONSTRUCTING AN OPTIMAL PORTFOLIO USING SHARPE’S SINGLE INDEX MODEL”

**MR. ALPESH GAJERA** (MBA, NET)\*

**MR. KAUSHAL THAKAR** (MBA, NET)\*\*

### ABSTRACT

The foundation of Modern Portfolio Theory was laid by Markowitz in 1951. He began with the simple premise that since almost all investors invest in multiple securities rather than one, there must be some benefit in investing in a portfolio of securities. He measured riskiness of a portfolio through variability of returns and showed that investment in several securities reduced this risk. His work won him the Nobel prize for Economics in 1990. Markowitz’s work was extended by Sharpe in 1964, Lintner in 1965 and Mossin in 1966. Sharpe shared the Nobel prize for Economics in 1990 with Markowitz and Miller for his contribution to the Capital Asset Pricing Model (CAPM). This model breaks up the riskiness of each security into two components - the market related risk which cannot be diversified called systematic risk measured by the beta coefficient and another component which can be eliminated through diversification called unsystematic risk. In this research work, by using the concept of Sharpe Single Index Model I had try to create one portfolio which is optimum using real market date of SENSEX 30 Securities

**Key Words:** Portfolio, Sharpe, BSE Sensex, Optimal Portfolio.

\* ASST. PROFESSOR, SHRI SUNSHINE GROUP OF INSTITUTIONS, FACULTY OF MBA, RAJKOT

\*\* ASST. PROFESSOR, MARWADI EDUCATION FOUNDATION GROUP OF INSTITUTIONS, FACULTY OF MBA, RAJKOT

## Literature Review

1. “**Financial Portfolio Optimization through a Robust Beta Analysis**”: Published at Department of Mechanical and Industrial Engineering, University of Toronto This section of the topic based literature review discusses the various techniques for selecting security portfolios, specifically Harry M. Markowitz’s full covariance model and William F. Sharpe’s Single Index Model. The purpose of this section is to get a general understanding of the portfolio problem that investors face on a regular basis and the methods that can assist in determining its solution. Furthermore, the two models described below establish the context and motivation for the research done in this thesis study.

2. **Doron Avramov** : Titled “Bayesian Portfolio Analysis” published at Finance Department, The Hebrew University of Jerusalem, Mt. Scopus Jerusalem 91905, Israel; R.H. Smith School of Business, University of Maryland This paper reviews the literature on Bayesian portfolio analysis. Information about events, macro conditions, asset pricing theories, and security-driving forces can serve as useful priors in selecting optimal portfolios. Moreover, parameter uncertainty and model uncertainty are practical problems encountered by all investors. The Bayesian framework carefully accounts for these uncertainties, whereas standard statistical models often ignore them. We review Bayesian portfolio studies when asset returns are assumed both independently and identically distributed as well as predictable through time. We cover a range of applications, from investing in single assets and equity portfolios to mutual and hedge funds.

3. **Edwin J. Elton a, Martin J. Gruber b**: Titled “Modern portfolio theory, 1950 to date” published at a Management Education Center, 44 West 4th Street, Suite 9-190, Stern School of Business, New York, NY 10012-1126, USA b Department of Finance, Stern School of Business, New York University, Suite 9-190, 44 West 4th Street, New York, NY 10012-1126, USA In this article we have reviewed “Modern Portfolio Analysis” and outlined some important topics for further research. Issues discussed include the history and future of portfolio theory, the key inputs necessary to perform portfolio optimization, infected problems in applying portfolio theory to facial institutions, and the methods for evaluating how well portfolios are managed.

Emphasis is placed on both the history of major concepts and where further research is needed in each of these areas.

4. Research Paper Published on Titled “**Optimal portfolio selection with or without the procedure of short sales**” at Department of Commerce, Kurukshetra University, Kurukshetra, Haryana, The main purpose of this paper is to construct an optimal portfolio with the procedure of short sales and without the procedure of short sales by applying Sharpe’s single-index model. The present study is based on the secondary data. For the purpose of constructing an optimal portfolio, a sample of thirty stocks listed on Bombay Stock Exchange (BSE) was selected in this study. BSE Sensitive Index (Sensex) has been used as market index. Monthly closing prices of selected stocks as well as market index for the period of Jan 2011 to Dec 2012 were used in this study. A unique cut off point was computed. The results of the study constructed an optimal portfolio and also represented the optimal portfolio with the percentage invested in each stock. The present study found that the eleven out of thirty stocks have expected return greater than risk free rate of return and these eleven stocks have been used for optimal portfolio construction.

5. **Nadeem Aftab, Ingo Jungwirth, Tomás Sedliacik, Nadir Virk** : Titled “Estimating the Distribution of Sharpe Ratios” The Sharpe ratio is the most common measure for risk adjusted return of a financial asset. The question of its statistical distribution is of theoretical and practical importance and interest. We will use GMM and ML estimation methods to estimate the distribution of Sharpe ratios of 962 mutual funds and test on it. Sub-sampling gives further insight on the statistical patterns of the data and yields some rest results with respect to estimates on the distribution. However, we could not and statistical evidence in favor of the normal distribution, the gamma distribution and the chi-square distribution on the basis of different estimation methods and different tests on distribution.

## Objectives

- To create optimal portfolio using BSE SENSEX Scripts with the help of Sharpe Portfolio Optimization Theory.

- To apply Theoretical concept of Portfolio optimization theory on real market Movement and scripts.
- To help investors in taking investment decisions cautiously after studying risk and return involved in the security with the help of Sharpe Portfolio Optimization Theory.
- To aware the technical analyst regarding the practical applicability of Sharpe Portfolio Optimization Theory.

### **Hypothesis of Study**

According to above objectives, the researcher have been framed the following hypotheses.

**H<sub>0</sub>**, There is no significance difference in the monthly mean return of Sensex and portfolio created with the help of Sharpe single index model.

**H<sub>0</sub>**, There is no significance difference in the variance of the return of Sensex and portfolio created with the help of Sharpe single index model.

### **Research Design**

#### **Sample**

The universe of the study consist price of share of all the listed company at any of the stock exchange. For this research paper share price of 30 company included in BSE Sensex.

#### **The data collection and period of the study**

The study has been carried out by using the monthly closing price of 30 script and SENSEX for a time period January 2011 to December 2012.

#### **Tools and techniques**

As per the nature of study following tools and techniques are used for analyzing data and testing the hypotheses:

- **Descriptive Statistics** such as mean, standard deviation and variance is used.
- **Inferential Statistics** such as T test for two mean differences and F test for two variance difference is used.
- **Security Analysis theory** of Sharpe portfolio optimization is used finding the optimal portfolio.

**Data Analysis & Interpretation**

Table – 1 Security Analysis with the help of monthly closing price for selected time period.

Sr. No	Security	E(R)	$\beta$	
1	Bajaj Auto	2.26	0.71	5.76
2	Bharti Airtel	0.62	0.66	7.96
3	BHEL	-5.56	0.29	18.49
4	Cipla	1.07	0.42	6.35
5	CoalIndia	0.79	0.52	17.83
6	Dr. Reddy	-1.15	0.47	4.72
7	GAIL	0.81	0.65	4.28
8	HDFC	1.05	0.63	3.73
9	HDFC Bank	-1.14	1.45	16.28
10	Hero Moto Corp	0.83	0.55	8.15
11	Hindal Corp	-2.34	1.26	7.44
12	HUL	2.49	-0.06	5.52
13	ICICIBANK	1.22	1.88	4.55
14	Infosys	-0.09	0.89	7.5
15	ITC	2.79	0.51	4
16	Jindal Steel	1.39	1.52	6.21
17	L&T	0.34	1.63	7.29
18	M&M	1.25	0.91	6.35
19	MarutiSuzuki	1.43	1.42	6.17
20	NTPC	-0.56	0.97	4.41
21	ONCG	-2	1.12	16.63
22	Reliance Ind	0.1	1.08	4.32
23	SBI	0.14	1.44	6.62
24	Stelite Ind	-0.91	1.74	6.05
25	Sun Pharma	2.17	2.4	10.6
26	Tata Pharma	-1.45	2.4	17.7
27	Tata Power	-4.03	1.66	18.47
28	Tata Steel	-1.22	1.85	6.61
29	TCS	0.73	0.3	4.39
30	Wipro	-0.52	0.6	6.63

Risk free rate of return is assumed to be 6% p.a.

Table – 2 Calculation of cut off point with the help of Sharpe Portfolio Optimization

Stock	$R_i$	$\frac{(R_i - R_f)}{\beta}$	$\frac{\beta^2}{\sigma_e^2}$	$\frac{(R_i - R_f) * \beta}{\sigma_e^2}$	$\sum \frac{(R_i - R_f) * \beta}{\sigma_e^2}$	$\sum \frac{\beta^2}{\sigma_e^2}$	$C_i$
ITC	2.79	4.490	0.016	0.073	0.073	0.016	1.439
<b>Bajaj Auto</b>	<b>2.26</b>	<b>2.480</b>	<b>0.015</b>	<b>0.038</b>	<b>0.111</b>	<b>0.031</b>	<b>1.678*</b>
CIPLA	1.07	1.360	0.004	0.006	0.117	0.036	1.658
HDFC	1.05	0.870	0.029	0.025	0.142	0.064	1.432
M&M	1.25	0.820	0.021	0.017	0.158	0.085	1.327
TCS	0.73	0.770	0.005	0.004	0.162	0.090	1.306
Sun Pharma	2.17	0.700	0.051	0.036	0.198	0.141	1.128
Maruti Suzuki	1.43	0.650	0.053	0.035	0.232	0.194	1.018
Hero Moto Corp	0.83	0.600	0.005	0.003	0.235	0.198	1.010
Jindal Steel	1.39	0.590	0.060	0.035	0.270	0.258	0.923
Coal India	0.79	0.560	0.001	0.000	0.271	0.259	0.922
GAIL	0.81	0.480	0.023	0.011	0.282	0.282	0.889
ICICI BANK	1.22	0.380	0.171	0.065	0.347	0.453	0.712
Bharti Airtel	0.62	0.180	0.007	-0.001	0.348	0.460	0.705
L&T	0.34	-0.100	0.050	-0.005	0.343	0.510	0.631
SBI	0.14	-0.250	0.047	-0.012	0.332	0.557	0.560
Reliance Ind	0.10	-0.370	0.063	-0.023	0.308	0.620	0.472
Infosys	-0.09	-0.660	0.014	-0.009	0.299	0.634	0.448
Stelite Ind	-0.91	-0.810	0.083	-0.067	0.232	0.716	0.309
Tata Pharma	-1.45	-0.810	0.018	-0.015	0.217	0.735	0.282
Tata Steel	-1.22	-0.930	0.078	-0.073	0.144	0.813	0.170
NTPC	-0.56	-1.090	0.048	-0.053	0.091	0.861	0.102
HDFC Bank	-1.14	-1.130	0.008	-0.009	0.082	0.869	0.091
Wipro	-0.52	-1.700	0.008	-0.014	0.069	0.878	0.075
ONCG	-2.00	-2.230	0.005	-0.010	0.058	0.882	0.064
Hindal Corp	-2.34	-2.250	0.029	-0.065	-0.006	0.911	-0.007
Tata Power	-4.03	-2.730	0.008	-0.022	-0.028	0.919	-0.030
Dr. Reddy	-1.15	-3.511	0.010	-0.035	-0.063	0.929	-0.066
BHEL	-5.56	-20.897	0.000	-0.005	-0.068	0.929	-0.071
HUL	2.49	-33.167	0.000	-0.004	-0.072	0.929	-0.075

C\* in above  $C_i$  is **1.678** as upto this stage  $\frac{(R_i - R_f)}{\beta} > C_i$

**Formation of the optimal portfolio by calculating the percentage invested in each security**

As per this model first 2 securities should be there in optimal portfolio. Proportion for these securities can be found out with the help of below mention formula

$$X_i = \frac{Z_i}{\sum_{j=1}^n Z_j}$$

Where

$$Z_i = \frac{\beta_i}{\sigma_{e_i}^2} \cdot \left( \frac{R_i - R_f}{\beta_i} - C^@ \right)$$

**Table – 3 Composition of Optimal portfolio**

Stock	Zi	Xi	Proportion
ITC	0.089628	0.839532	0.84
Bajaj Auto	0.017131	0.160468	0.16
<b>Total</b>	0.10676	--	1

**Table – 4 Risk and Return of Constructed Portfolio**

Security	Average Monthly Return	σ	β
Sensex	0.48	5.383	1
ITC	2.79	4.840	0.51
<b>BAJAJ AUTO</b>	2.26	6.928	0.71
<b>PORTFOLIO</b>	2.67	4.371	0.54

**Analysis of Return of Portfolio (T - Test)**

H<sub>0</sub> There is no significance difference in the monthly mean return of Sensex and portfolio created with the help of Sharpe single index model.

H<sub>1</sub> There is significance difference in the monthly mean return of Sensex and portfolio created with the help of Sharpe single index model.

Table – 5 T – Test for difference in two population mean

Security	Return	Variance	No. of Observation	DF	T Stat	T Critical
Portfolio	2.673536	19.11255	24	44	-1.54969	1.30109
Sensex	0.479755	28.98322	24			

**Interpretation**

Calculated value of T test is falling in rejection region as it is higher than critical value, which shows that there are enough evidence to reject null hypotheses. It indicates that there is significance difference in the mean monthly return of sensex and portfolio created with the help of Sharpe single index model.

**Analysis of variance of Portfolio (F - Test)**

$H_0$  There is no significance difference in the variance of the return of Sensex and portfolio created with the help of Sharpe single index model.

$H_1$  There is significance difference in the variance of the return of Sensex and portfolio created with the help of Sharpe single index model.

Table – 6 F – Test for difference in two population variance

Security	Return	Variance	No. of Observation	DF	T Stat	T Critical
Portfolio	2.673536	19.11255	24	23	0.659435	0.49642
Sensex	0.479755	28.98322	24			

**Interpretation**

Calculated value of F test is falling in rejection region as it is higher than critical value, which shows that there is enough evidence to reject null hypotheses. It indicates that there is significance difference in the variance of return of Sensex and portfolio created with the help of Sharpe single index model.

### Findings

1. The optimum portfolio can be broken down into two parts i.e. the stocks that it comprises and the percentage of funds that go to acquire such stocks. The composition of the optimum portfolio would be:
  - 0.84 % of funds invested in ITC security
  - 0.16 % of funds invested in Bajaj Auto security
2. Monthly average return of sensex is 0.48 while if we see the monthly average return of selected security in optimal portfolio is 2.79 & 2.26 respectively for ITC & Bajaj Auto.
3. When hypothetic portfolio has been created by using selected security in decided composition, the return on that portfolio is **2.67** which is comparatively better than sensex and all other 30 security.

### Conclusion

Sharpe single Index Model is basically develop for creating a portfolio from list of securities in such a way which provide best return at lowest risk. In this research work we had try to do the same thing by selecting sensex 30 security as sample as on date 2<sup>nd</sup> March and then apply Shape Single Index Model on this securities for creating portfolio. That inputs required like  $\beta$ (beta) of security,  $\sigma$ (standard deviation) of security and index average monthly closing return of security and index error than standard deviation of error than etc...has been calculated by using closing price of sensex 30 security of last 2 years monthly basis. After applying Sharpe Single Index Model we come to know that as per this model only 2 securities are satisfying criteria for Sharpe Single Index Model. These securities are ITC and BAJAJ AUTO. The proportion of these securities is **0.84** and **0.16** respectively. Portfolio generated with the help of Sharpe Single Index model generates **2.67%** return which is best in all 30 security of Sensex.

## **References**

1. Donald E. Fischer, Ronaki J.Jordan “Security Analysis and Portfolio Management”
2. Edwin J. Elton and Martin J. Gruber “Modern Portfolio Theory and Investment Analysis”
3. M. Obaidullah “Indian Stock Market - Theories and Evidence”
4. Prasanna Chandra “The Investment Game - How to win”
5. Samir K. Barua, J .R. Varma and V.Raghunathan “Portfolio Management”
6. Zvi Bodie, Alex Kane and Alan J. Marcus “Investments”

## **Bibliography**

- [www.rbi.com](http://www.rbi.com)
- [www.bse.com](http://www.bse.com)
- [www.bseindia.com](http://www.bseindia.com)
- [www.moneycontrol.com](http://www.moneycontrol.com)