CHEMICAL REACTION AND THERMO DIFFUSION EFFECTS ON MHD FREE CONVECTIVE FLOW PAST A MOVING VERTICAL PLATE WITH TIME DEPENDENT SUCTION AND HEAT SOURCE IN A SLIP FLOW REGIME

K.S. Balamurugan[*]
S.V.K. Varma[**]
V.C.C. Raju[***]

Abstract

In this article, the effects of chemical reaction and thermo diffusion on unsteady hydro magnetic free convective flow of a viscous, incompressible, electrically conducting fluid with heat and mass transfer past a moving porous vertical plate of infinite length with time dependent suction in the presence of heat source in a slip flow regime have been investigated. A uniform transverse magnetic field is applied. The effects of various physical parameters over the velocity, temperature and concentration distribution as well as on skin friction coefficient, dimensionless rate of heat transfer and dimensionless rate of mass transfer at the plate are discussed in detail. It is inferred from the graphs that the velocity increases due to the effects of velocity ratio parameter and thermodiffusion parameter where as the velocity decreases by magnetic parameter and chemical reaction parameter.

Keywords: Free Convection, Slip Flow, Vertical Plate, Chemical Reaction, Thermodiffusion.

---

[*] *Department of Mathematics, RVR & JC College of Engineering, Guntur, Andhra Pradesh, India*

[**] *Department of Mathematics, Sri Venkateswara University, Tirupati, Andhra Pradesh, India*

[***] *Department of Mathematics, University of Botswana, Gaborone, Botswana*

1. Introduction

Free convective flow of an electrically conducting fluid past a vertical plate under the influence of magnetic field has attracted the attention of many scientists in recent years in view of its vast applications e.g. in Aerodynamics, Astrophysics, Geophysics and Engineering. The phenomenon of free convection arises in the fluid when the temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. There are many transport processes occurring in nature due to temperature and chemical differences. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the fluid is a working medium.

Pop[1] studied an unsteady flow past an infinite porous plate with variable suction for hydromagnetic case. Soundalgekar[2] analyzed the effects of variable suction and the horizontal magnetic field on the free convection flow past an infinite vertical porous plate and made a comparative discussion of different parameters and the free convection flow of mercury and ionized air. A study on the steady laminar free convection flow in an electrically conducting fluid along a porous vertical plate in the presence of heat source was carried out by Sharma and Pankaj Kumar[3]. Heat transfer in MHD free convection flow over an infinite vertical plate with time-dependent suction was investigated in detail by Basant Kumar Mishra[4]. A study of vorticity of fluctuating flow of a visco-elastic fluid past an infinite plate with variable suction in slip flow regime was made by Mittal and Mukesh Bijalwan[5].

The combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore received a considerable amount of attention in recent years. Chambre and Young[6] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Apelblat[7] studied analytical solution for mass with a chemical reaction of first order. Das *et al.*[8] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Dulal Pal and Babulal Talukdar[9] reported perturbation analysis of unsteady magneto hydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction neglecting the Soret, Chemical reaction and Slip due to jump in temperature. Anjali Devi and Wilfred Samuel Raj[10] investigated the thermo diffusion effects on unsteady hydromagnetic free convection flow with heat and mass transfer past a moving vertical plate with time dependent suction and heat source in a slip flow regime. Masthanrao *et al.*[11] analyzed chemical reaction and combined buoyancy effects of thermal and mass diffusion on MHD convective flow along an infinite vertical porous plate in the presence of hall current with variable suction and heat generation.

The main objective of the present investigation is to study the effects of chemical reaction and thermo diffusion on unsteady hydro magnetic free convection flow of a viscous, incompressible, electrically conducting fluid with heat and mass transfer past a moving porous vertical plate of infinite length with time dependent suction in the presence of heat source in a slip flow regime. Slip flow conditions for the velocity and jump in temperature are taken into account in the boundary conditions.

2. Mathematical Formulation

The x^* axis is taken along the plate in the upward direction and y^* axis is normal to it. Due to semi-infinite plane surface assumptions, all the flow variables except pressure are functions of y^* and t^* only. A constant transverse magnetic field is applied in the direction of y^* axis.

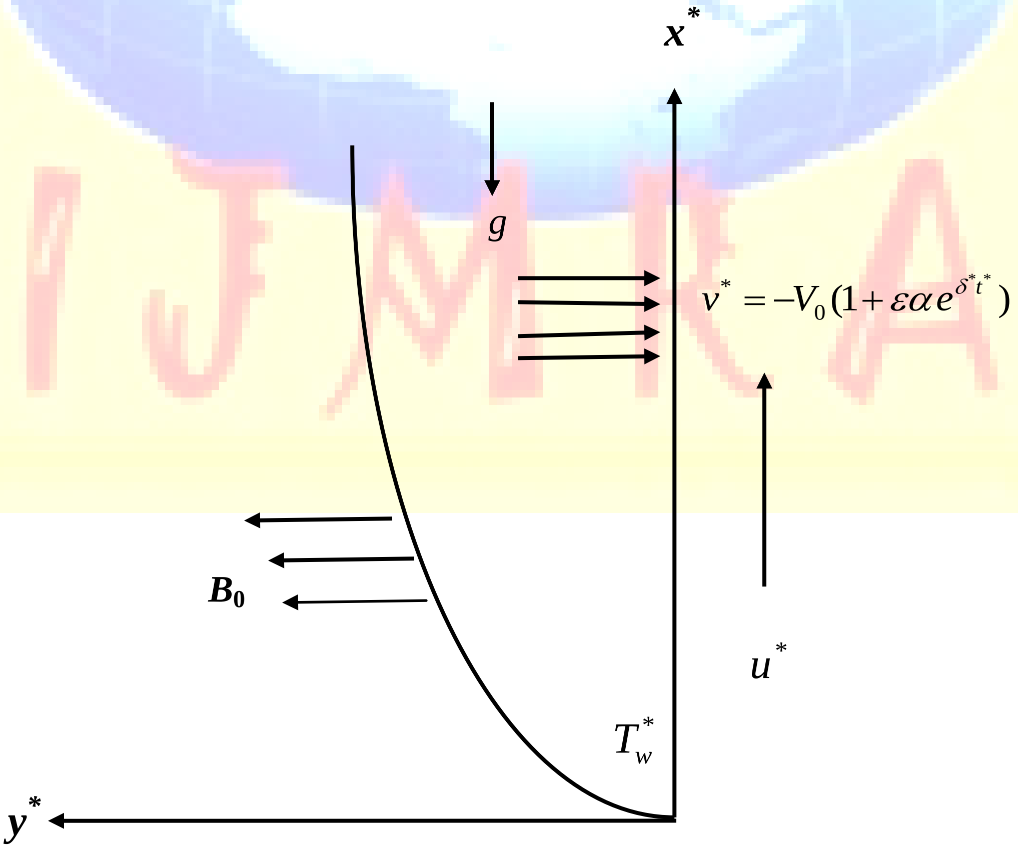

Figure 1 Physical configuration

In the present work, the following assumptions are made:

- The flow is unsteady and laminar, and the magnetic field is applied perpendicularly to the plate.
- The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field can be neglected.
- The effects of viscous and Joule's dissipation are assumed to be negligible in the energy equation.

Under, these assumptions, the governing boundary layer equations of the problem are given by

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{dU_\infty^*}{dt^*} - \frac{\sigma B_o^2}{\rho}(u^* - U_\infty^*) + \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f (T^* - T_\infty^*) + g\beta_c (C^* - C_\infty^*) \tag{2}$$

$$\rho C_p \left(\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} \right) = K \frac{\partial^2 T^*}{\partial y^{*2}} + S^* (T^* - T_\infty^*) \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} + D_1 \frac{\partial^2 T^*}{\partial y^{*2}} + D_2 (C^* - C_\infty^*) \tag{4}$$

The slip flow boundary conditions are given by

$$\begin{aligned} &u^* = u_w^* + h_1^* \frac{\partial u^*}{\partial y^*}; \quad T^* = T_w^* + h_2^* \frac{\partial T^*}{\partial y^*}; \quad C^* = C_w^* \quad at\ y^* = 0 \\ &u^* \to U_\infty^* = U_0 (1 + \varepsilon e^{\delta^* t^*}); \quad T^* \to T_\infty^*; \quad C^* \to C_\infty^* \quad as\ y^* \to \infty \end{aligned} \tag{5}$$

where u^* and v^* are components of velocities along and perpendicular to the plate x^* and y^* are distances along and perpendicular to the plate respectively, ρ is the density of the fluid, g is the acceleration due to gravity, σ is the electrical conductivity, B_0 is Magnetic flux density, β_f is the coefficient of volume expansion of the working fluid, β_c is the coefficient of volumetric expansion with concentration, U_∞^* is the velocity of the fluid in the free stream, υ is the kinematic viscosity, T^* is the temperature of the fluid, T_∞^* is the temperature of the fluid in the free stream, K is thermal conductivity, S^* is coefficient of heat source, C^* is the concentration of the fluid, C_∞^* is concentration at infinity, D is chemical molecular diffusivity, D_1 is thermal diffusivity, D_2 the chemical reaction rate constant, C_p is the specific heat at constant pressure, u_w^* the velocity at the wall, T_w^* is temperature at the wall, C_w^* is the concentration at the wall, ε and δ^* are scalar constants which are less than unity and $\varepsilon << 1$ and U_0 is the scale of stream velocity.

The plate is subjected to variable suction and from the equation of continuity, it can be written as

$$v^* = -V_0(1+\varepsilon\alpha e^{\delta^* t^*}) \qquad (6)$$

where α a real positive constant less than unity and V_0 is the scale of the suction velocity which has a non-zero positive constant.

and $$h_1^* = \left(\frac{2-f_1}{f_1}\right)\xi_1, \quad \xi_1 = \left(\frac{\pi}{2p\rho}\right)^{\frac{1}{2}}, h_2^* = \left(\frac{2-a}{a}\right)\xi_2, \quad \xi_2 = \left(\frac{2\gamma}{\gamma+1}\right)\frac{\xi_1}{\Pr} \qquad (7)$$

where ξ_1 is mean free path constant, f_1 Maxwell's reflection coefficient, γ Ratio of specific heats, a thermal accommodation coefficient.

Introducing the following non dimensional scheme

$$u = \frac{u^*}{U_0}, \quad v = \frac{v^*}{V_0}, \quad t = \frac{t^* V_0^2}{\upsilon}, \quad y = \frac{V_0 y^*}{\upsilon}, \delta = \frac{\delta^* \upsilon}{V_0^2}, \quad u_w = \frac{u_w^*}{U_0}, \quad U = \frac{U_\infty^*}{U_0}, \quad Sc = \frac{\upsilon}{D},$$

$$\theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \quad \phi = \frac{C^* - C_\infty^*}{C_w^\infty - C_\infty^*}, \quad K = \frac{K^* V_0^2}{\upsilon^2}, \quad Gr = \frac{\upsilon g \beta_f}{U_0 V_0^2}(T_w^* - T_\infty^*), \quad \Pr = \frac{\mu C_p}{k},$$

$$Gm = \frac{\upsilon g \beta_c}{U_0 V_0^2}(C_w^* - C_\infty^*), \quad M = \frac{\sigma B_0^2 \upsilon}{\rho V_0^2}, \quad S = \frac{S^* \upsilon^2}{K V_0^2}, \quad So = \frac{D_1(T_w^\infty - T_\infty^*)}{\upsilon(C_w^* - C_\infty^*)}, Kr = \frac{D_2 \upsilon}{V_0^2}$$

where Gr is the Grashof number, Gm is the Modified Grashof number, M is the magnetic field parameter, Pr is the Prandtl number, S is the Heat source parameter, Sc is the Schmidt number, Kr is the Chemical reaction parameter, So is the Soret number, θ is the dimensionless temperature and ϕ is the dimensionless concentration; the governing equations of the problem in non dimensional form are given by

$$\frac{\partial u}{\partial t}-(1+\varepsilon\alpha e^{\delta t})\frac{\partial u}{\partial y}=\frac{dU}{dt}+\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gm\phi-M[u-U(t)] \tag{8}$$

$$\frac{\partial \theta}{\partial t}-(1+\varepsilon\alpha e^{\delta t})\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+\frac{S}{\Pr}\theta \tag{9}$$

$$\frac{\partial \phi}{\partial t}-(1+\varepsilon\alpha e^{\delta t})\frac{\partial \phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}+So\frac{\partial^2 \theta}{\partial y^2}+Kr\phi \tag{10}$$

The corresponding boundary conditions in non dimensional form are

$$\begin{aligned}&u=u_w+h_1\frac{\partial u}{\partial y};\quad \theta=1+h_2\frac{\partial \theta}{\partial y};\quad \phi=1\quad at\quad y=0\\&u\to U(t);\quad \theta\to 0;\quad \phi\to 0\quad as\ y\to\infty\end{aligned} \tag{11}$$

where U (t) = 1+ε e$^{\delta t}$, $u_w=\frac{u_w^*}{U_0}$ (velocity ratio parameter), $h_1=\frac{h_1^* V_0}{\upsilon}$ (slip parameter due to velocity), $h_2=\frac{h_2^* V_0}{\upsilon}$ (slip parameter due to jump in temperature).

3. Solution of the Problem

In order to solve the nonlinear partial differential equations, the above systems of partial differential equations are reduced to a system of ordinary differential equations in a dimensionless form. The velocity, temperature and concentration are assumed in the following form:

$$F(y)=F_0(y)+\varepsilon e^{\delta t}F_1(y)+O(\varepsilon^2) \tag{12}$$

where F stands for any value of u, θ and ϕ. By substituting (12) in equations (8) to (10), the following equations are obtained.

$$u_0^{''} + u_0^{'} - Mu_0 = -Gr\theta_0 - Gm\phi_0 - M \tag{13}$$

$$u_1^{''} + u_1^{'} - (M+\delta)u_1 = -Gr\theta_1 - Gm\phi_1 - \alpha u_0^{'} - (M+\delta) \tag{14}$$

$$\theta_0^{''} + \Pr\theta_0^{'} + S\theta_0 = 0 \tag{15}$$

$$\theta_1^{''} + \Pr\theta_1^{'} + (S - \Pr\delta)\theta_1 = -\alpha \Pr\theta_0^{1} \tag{16}$$

$$\phi_0^{''} + Sc\phi_0^{'} + KrSc\phi_0 = -ScSo\theta_0^{''} \tag{17}$$

$$\phi_1^{''} + Sc\phi_1^{'} + (Kr - \delta)Sc\phi_1 = -ScSo\theta_1^{''} - \alpha Sc\phi_o^{'} \tag{18}$$

with the corresponding boundary conditions

$$\begin{aligned} &u_0 = u_w + h_1\frac{\partial u_0}{\partial y};\ u_1 = h_1\frac{\partial u_1}{\partial y};\ \theta_0 = 1 + h_2\frac{\partial \theta_0}{\partial y};\ \theta_1 = h_2\frac{\partial \theta_1}{\partial y};\ \phi_0 = 1; \\ &\phi_1 = 0 \ at \ y = 0 \\ &u_0 \to 1;\ u_1 \to 1;\ \theta_0 \to 0;\ \theta_1 \to 0;\ \phi_0 \to 0; \quad \phi_1 \to 0 \quad as \ y \to \infty \end{aligned} \tag{19}$$

On solving (13) to (18) subject to boundary conditions in (19), we get

$$u_0 = 1 + C_4e^{-m_3y} - C_5e^{-m_1y} - C_6e^{-m_2y} \tag{20}$$

$$u_1 = C_{13}e^{-m_6y} + C_{14}e^{-m_1y} - C_{15}e^{-m_2y} + C_{16}e^{-m_3y} + C_{17}e^{-m_4y} - C_{18}e^{-m_5y} + 1 \tag{21}$$

$$\theta_0 = C_1e^{-m_1y} \tag{22}$$

$$\theta_1 = C_8e^{-m_1y} - C_7e^{-m_4y} \tag{23}$$

$$\phi_0 = C_2e^{-m_2y} + C_3e^{-m_1y} \tag{24}$$

$$\phi_1 = C_9e^{-m_5y} + C_{10}e^{-m_1y} + C_{11}e^{-m_2y} + C_{12}e^{-m_4y} \tag{25}$$

3.1 Skin Friction

The skin friction coefficient at the plate is given by

$$C_f = \left(\frac{\partial u}{\partial y}\right)_{y=0} = m_1 C_5 + m_2 C_6 - m_3 C_4 + \varepsilon\, e^{\delta t}[-m_6 C_{13} - m_1 C_{14} + m_2 C_{15} - m_3 C_{16} + m_4 C_{17} - m_5 C_{18}] \tag{26}$$

3.2 Nusselt Number

The rate of heat transfer in terms of the Nusselt number Nu is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -m_1 C_1 + \varepsilon\, e^{\delta t}[-m_1 C_8 + m_4 C_7] \tag{27}$$

3.3 Sherwood Number

The rate of mass transfer in non dimensional form is given by

$$Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = -m_2 C_2 - m_1 C_3 + \varepsilon\, e^{\delta t}[-m_5 C_9 - m_1 C_{10} - m_2 C_{11} - m_4 C_{12}] \tag{28}$$

4. Results and Discussion

In the present work, we have chosen t = 1, δ = 0.1, ε = 0.01, α = 1 while the other non dimensional parameters take various values. The results obtained show that the dimensionless velocity is affected by physical parameters such as velocity ratio parameter u_w, magnetic field parameter M, thermo diffusion parameter So and chemical reaction parameter Kr respectively. Figures 2-5 show the effects of these parameters over the dimensionless velocity profiles. Figure 2 depicts the effect of velocity ratio parameter over the dimensionless velocity profiles. The effect of velocity ratio parameter is to accelerate the velocity and its influence is highly dominant near the plate whereas it remains uniform as we move far away from the plate. Figure 3 elucidates the influence of magnetic field parameter on the fluid velocity distribution, and we observed that for an increase in magnetic field parameter, the velocity of the flow field decreases. It is due to the presence of magnetic field normal to the flow in an electrically conducting fluid which introduces a Lorentz force which acts against the flow.

Figure 4 portrays the dimensionless velocity profiles for different values of chemical reaction parameter. Due to the effect of chemical reaction parameter the velocity of the flow field gets decelerated. The hydrodynamic boundary layer becomes thin as the chemical reaction parameter increases. Figure 5 displays the influence of thermo diffusion parameter over the dimensionless velocity. It is evident that the thermo diffusion parameter accelerates the velocity of the flow field.

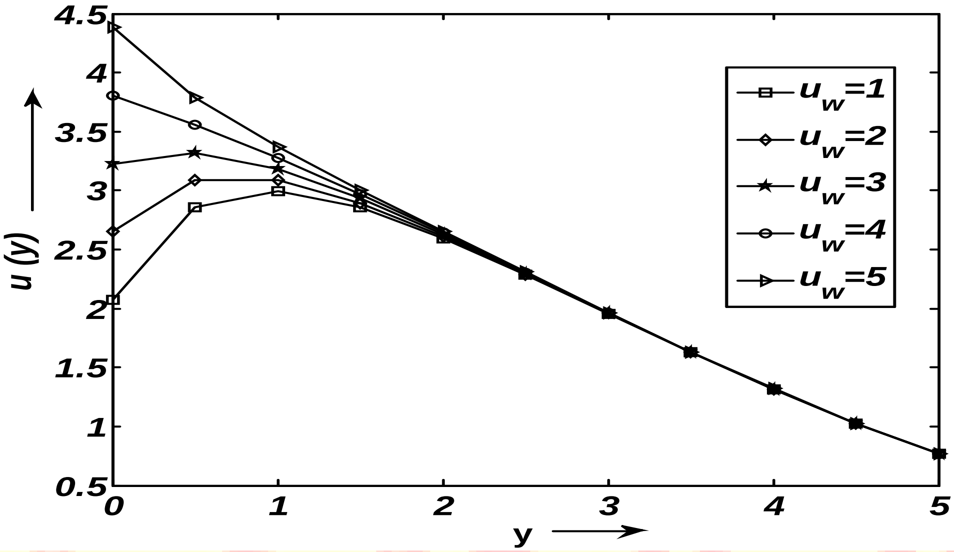

Figure 2 Velocity profiles when *Gr* = 0.9, *Gm* = 4, *Pr* = 0.71, *Sc* = 0.22, *S* = 0.1, *So* = 1.5, *M* = 5, h_1 = 0.4, h_2 = 0.5, Kr = 0.5.

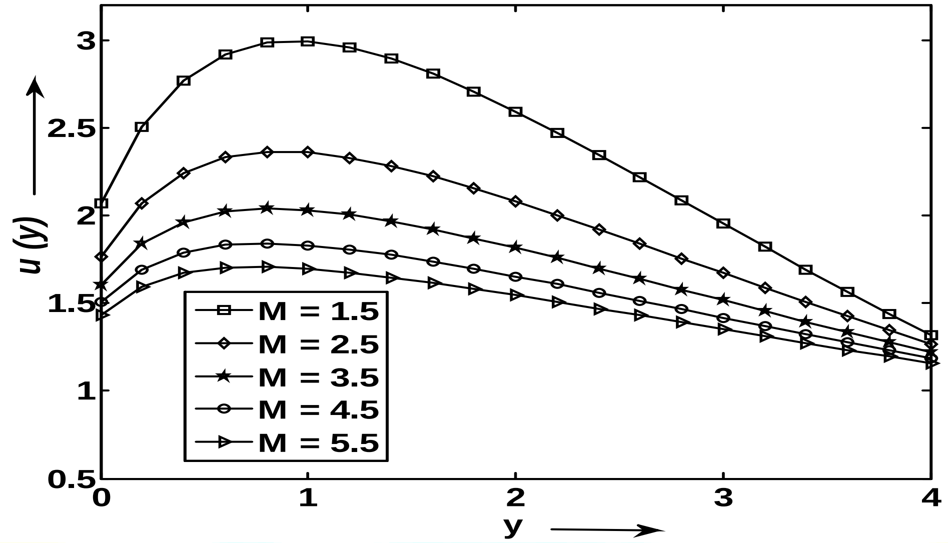

Figure 3 Velocity profiles when $Gr = 0.9$, $Gm = 4$, $Pr = 0.71$, $Sc = 0.22$, $S = 0.1$, $So = 1.5$, $u_w = 1$, $h_1 = 0.4$, $h_2 = 0.5$, $Kr = 0.5$.

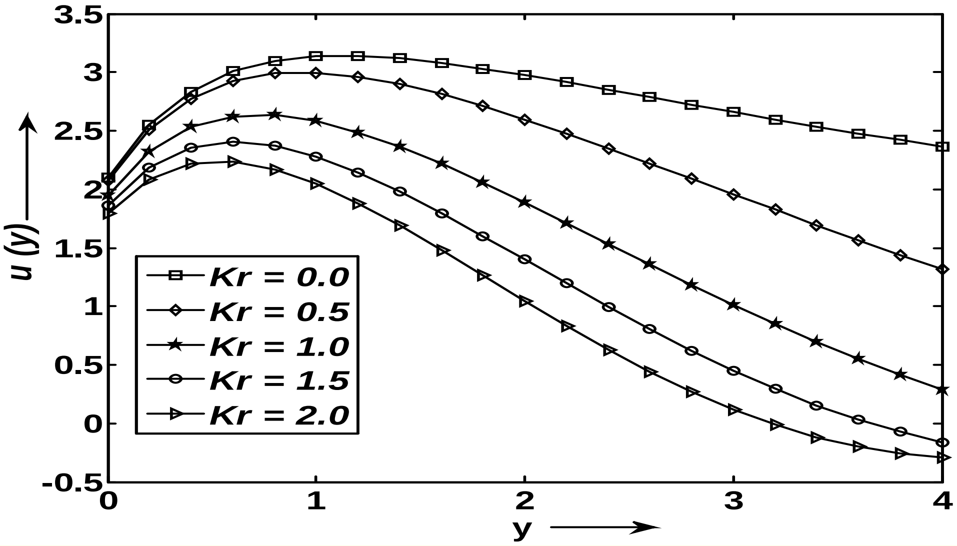

Figure 4 Velocity profiles when $Gr = 0.9$, $Gm = 4$, $M = 5$, $Pr = 0.71$, $So = 1.5$, $Sc = 0.22$, $u_w = 1$, $S = 0.1$, $h_1 = 0.4$, $h_2 = 0.5$.

Figure 6 illustrates the temperature profiles for different values of Prandtl numbers *Pr*. The results show that the increasing Prandtl number results in a decrease in the thermal boundary layer and in general lower average temperature within the boundary layer. The reason is that smaller *Pr* is equivalent to the increase in the thermal conductivity of the fluid, and heat is able to diffuse away from the heated surface more rapidly for higher values of *Pr*. Therefore, in the case of smaller Prandtl numbers, the thermal boundary layer is thicker, and the rate of heat transfer is reduced.

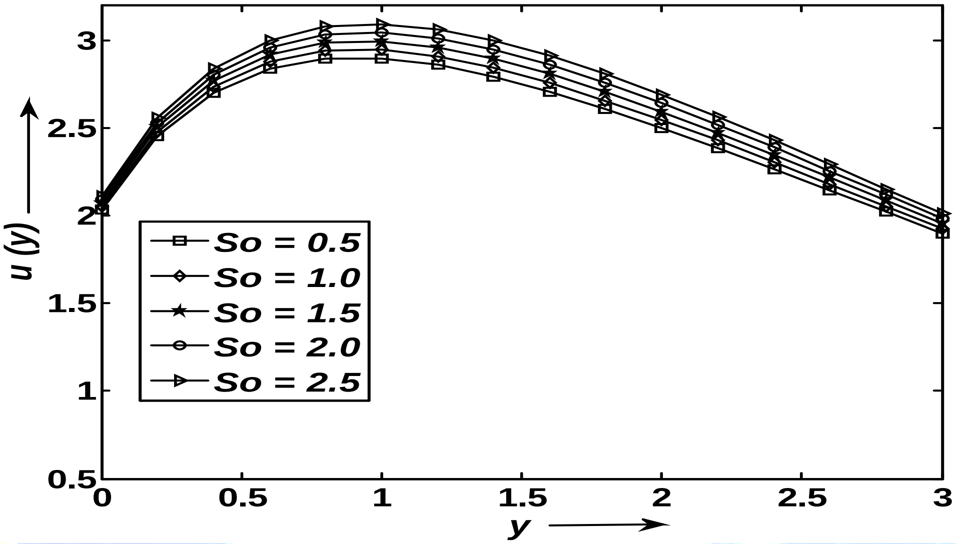

Figure 5 Velocity profiles when $Gr = 0.9$, $Gm = 4$, $M = 5$, $Pr = 0.71$, $S = 0.1$, $Sc = 0.22$, $u_w = 1$, $h_1 = 0.4$, $h_2 = 0.5$, $Kr = 0.5$.

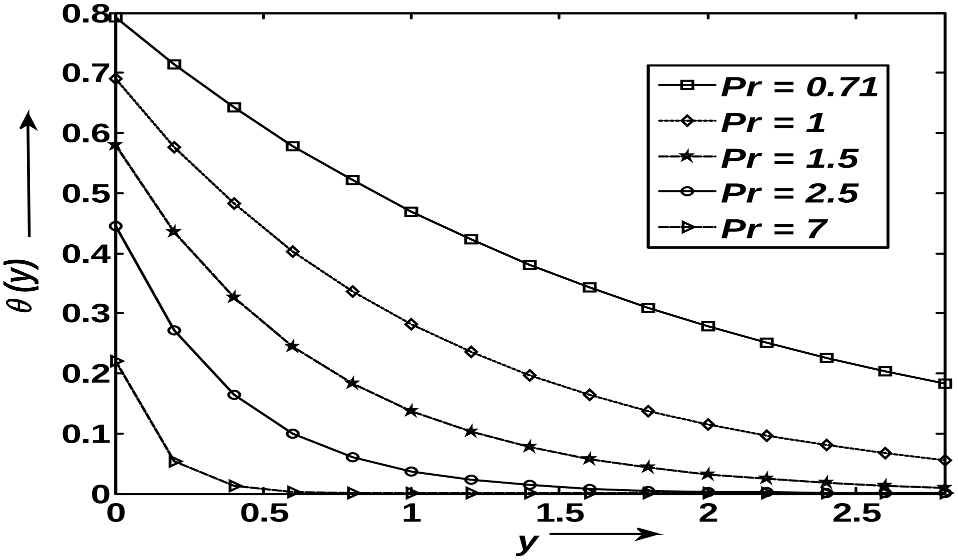

Figure 6 Temperature profiles when $Sc = 0.22$, $h_2 = 0.5$, $S = 0.1$.

Figure 7 deals with the effect of chemical reaction parameter over the concentration profiles. Due to increase in *Kr* the concentration of the flow decreases. It is evident that the increase in the chemical reaction *Kr* significantly alters the concentration boundary layer thickness but does not alter the momentum boundary layer. In the absence of chemical reaction, these results are in good agreement with the results of Anjali Devi and Wilfred Samuel Raj[10].

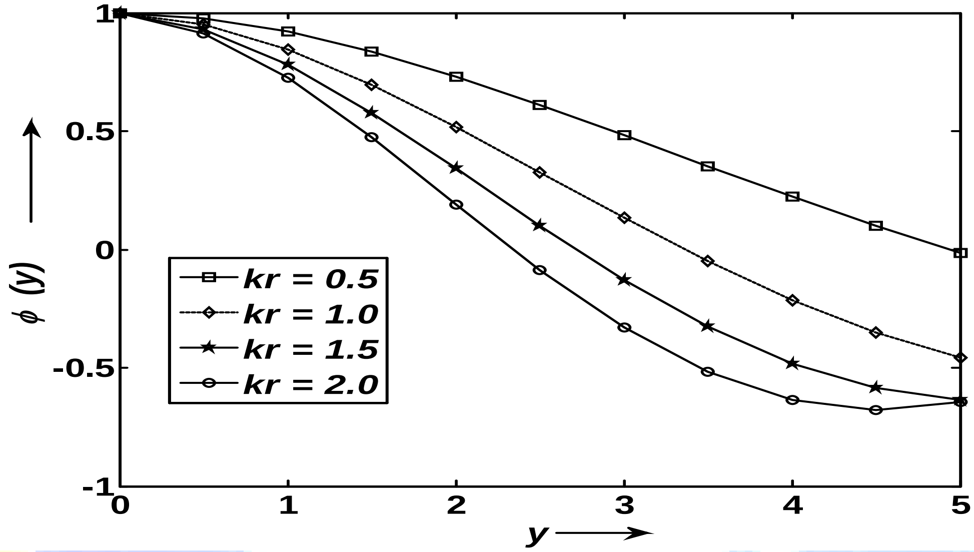

Figure 7 Concentration profiles when $S = 0.1$, $Pr = 0.71$, $Sc = 0.22$, $So = 1.5$, $h_2 = 0.5$.

5. Conclusions

Based on the discussion above, it was found that the velocity profiles increased due to increase in velocity ratio parameter u_w and thermo diffusion parameter *So* while it decreased due to increase in magnetic parameter *M* and chemical reaction parameter *Kr*. The temperature profiles decrease with the increasing values of Prandtl number *Pr*. The concentration profiles decrease with chemical reaction parameter *Kr*.

Acknowledgments

The authors thank their respective Departments of Mathematics at the University of Botswana, RVR & JC College of Engineering and Sri Venkateswara University for the excellent research-stimulating environment.

References

[1] I. Pop, Unsteady hydro magnetic free convection flow form a vertical infinite flat plate, *Z. Angew. Math. Mech.* 49(12) (1969) 756-757.

[2] V. M. Soundalgekar, Unsteady MHD free convection flow past an infinite plate with variable suction, *Indian J. of Pure Appl. Math*. 3(3) (1972).426-436.

[3] P. R. Sharma, Pankaj Kumar. Steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source/sink, *Indian J. of Pure Appl. Math*. 26(11) (1995) 1125-1134.

[4] Basant Kumar Mishra, Heat transfer in MHD free convection flow over an infinite vertical plate with time-dependent suction, *Acta Ciencia Indica.* XXXI M No. 2, (2005) 371.

[5] P. K. Mittal, Mukesh Bijalwan, A study of vorticity of fluctuating flow of a visco-elastic fluid past an infinite plate with variable suction in slip flow regime, *Acta Ciencia Indica. XXXI M* No. 3 (2005) 903-910.

[6] P. L. Chambre and J. D. Young, On the diffusion of a chemically reactive species in a laminar boundary layer flow, *Phys. of Fluids.* 1(1) (1958) 48-54.

[7] Alexander Apelblat, Mass Transfer with a chemical reaction of the first order, *Chem. Eng. J.* 19 (1980) 19-37

[8] U. N. Das, R. K. Deka, V. M. Soundalgekar, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, *Forschung in Ingenieurwsen*. 60 (1994) 284-287.

[9] Dulal Pal, Babulal Talukdar, Perturbation analysis of unsteady magneto hydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction, *Communications in Nonlinear Science and Numerical Simulation.* 15(7) (2010) 1813-1810.

[10] S. P. Anjali Devi, J. Wilfred Samuel Raj, Thermo diffusion effects on unsteady hydro magnetic free convection flow with heat and mass transfer past a moving vertical plate with time dependent suction and heat source in a slip flow regime, *Int. J. of Appl. Math and Mech*. 7(21) (2011). 20-51

[11] S. Masthanrao, K.S. Balamurugan, S.V.K. Varma, V.C.C. Raju, Chemical reaction and hall effects on MHD convective flow along an infinite vertical porous plate with variable suction and heat absorption, *Applications and Applied Mathematics: An International Journal (AAM)*, 8(1) (2013) 268-288 .