AN APPLICATION OF THE ASSIGNMENT PROBLEMS

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Abstract

In this paper deals with the Assignment problems arise in different situation where we have to find an optimal way to assign n objects to m other objects. These problems to find numerical application in production planning, Sales proportion, air –line operators etc. For example using maximizes (or) minimizes assignment methods and the existing Hungarian methods have been solved.

Keywords: Assignment Problems, Hungarian method, optimum solution

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1 Introduction

An assignment problem is a particular case of transportation problem where the objective is to assign a number of resources to an equal number of activities on a one to one basis so as to minimize total cost or maximize total profit of allocation. The problem of assignment arises because available resources such as men, machines, production etc. Thus, the problem is how the assignments should be made so as to optimize the given objective. The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operations research in Mathematics.

It is worth to recall that the assignment problem has been used in a variety of application contexts such as personnel scheduling, manpower planning and resource allocation. The standard assignment problem can be seen as a relaxation of more complex combinatorial optimization problems such as traveling salesman problem [5-6], quadratic assignment problem [7], etc. It can also be considered as a particular transportation problem with all supplies and demands equal to 1. The assignment problem has also several variations such as the semi-assignment problem and the k-cardinality assignment problem. The reader interested in more details about these two problems or other variations can see [3] for a comprehensive survey of the assignment problem variations.

2. Mathematical Statement of the Problem

The (standard) assignment problem consists of assigning a number of tasks to an equal number of agents (each agent is assigned to exactly one task and each task has exactly one agent assigned to perform it) in such a way to minimize the overall cost of assigning agents to tasks, given the cost of the assignment of each agent to each task. The mathematical formulation of the standard assignment problem (SAP) is as follows:

$$\text{Min } Z= \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to

$$\sum_{i=1}^{1} x_{ij} = 1 \qquad i=1,2\text{----------}n$$

$$\sum_{j=1}^{n} x_{ij} =1 \qquad j= 1,2 \text{--------}n$$

$$x_{ij} = \{0 \text{ or } 1\} \quad i ,j =1, 2\text{-------}n$$

where for all i, j = 1, …, n, cij is the cost of assigning agent I to task j, Xij = 1 means that agent i is assigned to task j and Xij = 0 means that agent i is not assigned to task j.The first set of constraints implies that each agent is assigned to one and only one task and the second set of constraints implies that to each task is assigned one and only one agent.

In addition to the minimization of assignment cost, an assignment problem may consider other objective functions such as the minimization of completion time. When the assignment problem is considered with the minimization of assignment cost as the objective function, it is called the cost minimizing assignment problem.

3. Methods of Assignment Problem:

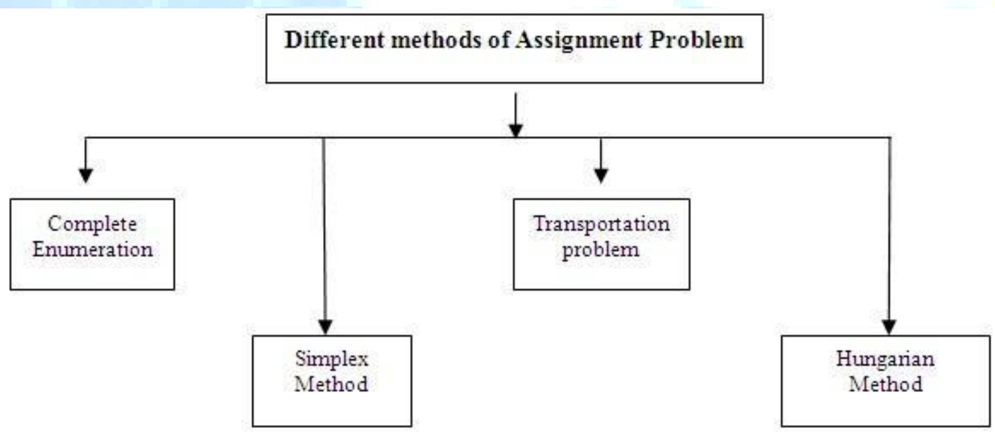

Fig.1 The different methods or solving an assignment problem.

Note:- It may be noted that assignment problem is a variation of transportation problem with two characteristics 1. The cost matrix is a square matrix 2. the optimum solution for the problem would always be such that there would be only one assignment in a given row or column of the cost matrix.

4. Maximization Assignment Problem:

Sometimes the assignment problem may deal with maximization of the objective function. The maximization problem has to be changed to minimization before the Hungarian method may be applied. This transformation may be done in either of the following two ways:

a. by subtracting all the elements from the largest element of the matrix.

b. by multiplying the matrix elements by-1

Example 4. 1:

A company has four territories open and four salesmen available for assignment. The territories are not equally rich in their sales potential. It is estimated that a typical salesman operation in each territory would bring in the following annual sales:

Territory:	I	II	III	IV
Annual sales (RS) :	60,000	50,000	40,000	30,000

The four salesmen are also considered to differ in ability: it is estimated that working under the same conditions, their yearly sales would be proportionately as follows:

Salesman:	A	B	C	D
Proportion:	7	5	5	4

If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest territory; the next best salesmen to the second richest territory and so on verify this answer by the assignment method

Solution:

Step 1: To construct the effectiveness of the matrix .By taking Rs. 10000/- as one unit and the sales proportion and the maximum sales matrix is obtained as follows:

Table (a)

Sales in 10 thousand of rupees

Sales Proportion

		6	5	4	3
		I	II	III	IV
7	A	42	35	28	21
5	B	30	25	20	15

5	C	30	25	20	15
4	D	24	20	16	12

Find the value of C_{11} = Sales Proportion × Sales Territory

7 ×6 = 42

In the same it is continued for the remaining cells

Step 2:

To convert the maximum sales matrix to minimum sales matrix.By simply multiplying each element of given matrix by -1. Thus resulting matrix becomes:

Table (b)

	I	II	III	IV
A	-42	-35	-28	-21
B	-30	-25	-20	-15
C	-30	-25	-20	-15
D	-24	-20	-16	-12

Step 3:

Select the most negative in the matrix (i.e) is -42. With this element subtract all the Elements in the matrix. MinZ = -(-MaxZ),the resulting is minimization table

Table (C)

	I	II	III	IV
A	0	7	14	21
B	12	17	22	27
C	12	17	22	27
D	18	22	26	30

Find the value of C_{11} = -42 – (-42) =0, in the same it is continued for the remaining cells

Now, using the above table we can apply the Hungarian method to find the assignment for the given problem and the value should be taken from the original table since, it is a maximization problem

Step 4. Iterate towards an Optimal Solution. We proceed according to the Hungarian algorithm and we get optimal solution

0	2	4	7
0	**0**	0	1
0	0	**0**	1
0	0	0	**0**

N=n, 4=4, the assignment of the given problem

A--------I,B-------II,C-------III,D------IV

Example4. 2:

Beta Corporation has four plants each of which can manufacture any one of four products Production costs differ from one plant to another as do sales revenue. Given the revenue and cost data below, obtain which product each plant should produce to maximize profit.

Sales revenue (Rs. 000s Product)

Plant

	1	2	3	4
A	50	68	49	62
B	60	70	51	74
C	55	67	53	70
D	58	65	54	69

Production costs (Rs. 000s Product)

Plant

	1	2	3	4
A	49	60	45	61
B	55	63	45	69
C	52	62	49	68
D	55	64	48	66

Solution:

Step 1:

Now, we have found the profit matrix by using sales revenue and production cost.

Profit = sales - cost

Profit matrix

	1	2	3	4
A	1	8	4	1
B	5	7	6	5
C	3	5	4	2
D	3	1	6	3

Step 2:

To convert the maximum sales matrix to minimum sales matrix.By simply multiplying each element of given matrix by -1. Thus resulting matrix becomes:

	1	2	3	4
A	-1	-8	-4	-1
B	-5	-7	-6	-5
C	-3	-5	-4	-2
D	-3	-1	-6	-3

Step 3: Select the most negative in the matrix (i.e) is -8. With this element subtract all the Elements in the matrix. MinZ = -(-MaxZ),the resulting is minimization table

	1	2	3	4
A	7	0	4	7
B	3	1	2	3
C	5	3	4	6
D	5	7	2	5

Find the value of $C_{11} = -1 - (-8) = 0$, in the same it is continued for the remaining cells

Now, using the above table we can apply the Hungarian method to find the assignment for the given problem and the value should be taken from the original table since, it is a maximization problem,Using Hungarian method

Step 4: Iterate towards an Optimal Solution. We proceed according to the Hungarian algorithm and we get optimal solution

	1	2	3	4
A	5	[0]	4	5
B	0	0	1	[0]
C	[0]	0	1	1
D	1	5	[0]	1

N=n, the assignment can be done for the above table

A------2,B------4,C-----1,D------3

Example 4.3:

An air-line operates seven days a week has time-table shown below. Crews must have a Minimum layover (rest) time of 5 hrs,between flights. Obtain the pair of flights that minimimizes layover time away from home. For any given pair the crews will e based at the city that result in the smaller layover.

Delhi-Jaipur			Jaipur- Delhi		
Flight No	Depart	Arrive	Flight No	Depart	Arrive
1	7.00Am	8.00AM	101	8.00AM	9.15AM
2	8.00AM	9.00AM	102	8.30AM	9.45AM
3	1.30PM	2.30PM	103	12.00Noon	1.15PM
4	6.30PM	7.30PM	104	5.30PM	6.45PM

For each pair, mention the town where the crews should be based.

Solution:

Step1: construct the table for layour times between flights when crew is based at Delhi, for simplicity , consider 15 minutes = 1unit.

Table 1: layover times when crew based at Delhi

Flight No	101	102	103	104
1	96	98	112	38
2	92	94	108	34
3	70	72	86	108
4	50	52	66	88

Since, the crew must have a minimum layover of 5 hrs between flights

The layover time between flights 1 and 101 will be 24 hrs (96 units)from 8.00 AM to 8.00 AM next day i.e flight 1 arrives jaipur at 8.00 am and leaves the jaipur 8.00 am next day because of minimum layover is 5 hrs between flights and other flights is there in between so flight will be there next day only.

Flight 1 to 102 will be (98units) 8.00 am arrives jaipur leaves jaipur 8.30 am next day= 24 hrs+30 minutes

Flight 1 to 103 will be (112 units) 8.00 am arrives jaipur leaves jaipur 12.00 noon next day= 24 hrs +4 hrs =112 units

Flight 1 to 104 will be (38 units)8.00 am arrives jaipur leaves jaipur 5.30 pm on the same day = 9 hrs +30 min = 38 mins

The layover time between Flight 2 to 101 will be (9.00 am arrival and depart from jaipur 8.00 am next day) =23 hrs = 92 units

Flight 2 to 102 will be (9.00 am arrives jaipur and depart from jaipur 8.30 am next day) = 23 hrs +30 minutes = 94 units

Flight 2 to 103 will be (9.00 am arrives jaipur and depart from jaipur 12.00 noon next day) = 24 hrs +3 hrs = 108 units

Flight 2 to 104 will be (9.00 am arrives jaipur and depart from jaipur 5.30 pm same day) = 8 hrs +30 minutes= 34 units

The layover time between Flight 3 to 101 will be (2.30 pm arrival and depart from jaipur 8.00 am next day) =17 hrs + 30 minutes = 70 units

Flight 3 to 102 will be (2.30 pm arrives jaipur and depart from jaipur 8.30 am next day) = 18hrs = 72 units

Flight 3 to 103 will be (2.30 pm arrives jaipur and depart from jaipur 12.00 noon next day) = 21hrs + 30 minutes = 86 units

Flight 3 to 104 will be (2.30 pm arrives jaipur and depart from jaipur 5.30 pm next day) = 24hrs+3hrs= 108 units

The layover time between Flight 4 to 101 will be (7.30 pm arrival and depart from jaipur 8.00 am next day) =12 hrs +30 minutes = 50 units

Flight 4 to 102 will be (7.30 pm arrives jaipur and depart from jaipur 8.30 am next day) = 13hrs = 52 units

Flight 4 to 103 will be (7.30 pm arrives jaipur and depart from jaipur 12.00 noon next day) = 16hrs + 30 minutes = 66 units

Flight 4 to 104 will be (7.30 pm arrives jaipur and depart from jaipur 5.30 pm next day) = 22hrs= 88 units

Step2: Table 2: layover times when crew based at jaipur

Flight No	101	102	103	104
1	87	85	71	49
2	91	89	75	53
3	113	111	97	75
4	37	35	21	95

Since, the crew must have a minimum layover of 5 hrs between flights

The layover time between flights 101 and 1 will be 21 hrs+ 45 minutes (87 units) from 9.15 AM to 7.00 AM next day by flight no 1 i.e flight 101 arrives Delhi at 9.15 am and leaves the Delhi 7.00 am next day by flight no 1 because of minimum layover is 5 hrs between flights and no other flights is there in between so flight will there next day only.

Flight 101 to 2 will be (91 units) 9.15 am arrives Delhi leaves Delhi 8.00 am next day= 22 hrs+45minutes

Flight 101 to 3 will be (113 units) 9.15 am arrives Delhi leaves Delhi 1.30 pm next day= 28 hrs +15 minutes = 113 units

Flight 101 to 4 will be (38 units) 9.15 am arrives Delhi leaves Delhi 6.30 pm on the same day = 9 hrs +15 min = 37 mins

The layover time between Flight 102 to 1 will be (9.45 am arrival and depart from Delhi 7.00 am next day) =21 hrs+15 minutes = 85 units

Flight 102 to 2 will be (9.45 am arrives Delhi and depart from Delhi 8.00 am next day) = 22hrs + 15 minutes = 89 units

Flight 102 to 3 will be (9.45 am arrives Delhi and depart from Delhi 1.30 pm next day) = 27 hrs +45 minutes = 111 units

Flight 102 to 4 will be (9.45 am arrives Delhi and depart from Delhi 6.30 pm same day) = 8 hrs +45 minutes= 35 units

The layover time between Flight 103 to 1 will be (1.15 pm arrival and depart from Delhi 7.00 am next day) =17hrs + 45 minutes = 71 units

Flight 103 to 2 will be (1.15 pm arrives Delhi and depart from Delhi 8.00 am next day) = 18hrs + 45 minutes = 75 units

Flight 103 to 3 will be (1.15 pm arrives Delhi and depart from Delhi 1.30 pm next day) = 24 hrs +15 minutes = 97 units

Flight 103 to 4 will be (1.15 pm arrives Delhi and depart from Delhi 6.30 pm same day) = 5 hrs +15 minutes= 21 units

The layover time between Flight 104 to 1 will be (6.45 pm arrival and depart from Delhi 7.00 am next day) =12hrs + 15 minutes = 49 units

Flight 104 to 2 will be (6.45 pm arrives Delhi and depart from Delhi 8.00 am next day) = 13hrs + 15 minutes = 53 units

Flight 104 to 3 will be (6.45 pm arrives Delhi and depart from Delhi 1.30 pm next day) =18 hrs +45 minutes = 75 units

Flight 104 to 4 will be (6.45 pm arrives Delhi and depart from Delhi 6.30 pm same day) = 23 hrs + 45 minutes= 95 units

Step 3: construct the table for minimum layover times between flights with the help of Table 1 and Table 2 layover times denote that the crew is based at jaipur.

Table 3

Flight No	101	102	103	104
1	87	85	71	38
2	91	89	75	34
3	70	72	86	75
4	37	35	21	88

Using Hungarian method we solve the above table and the assignments are as shown in the table

Flight No	101	102	103	104
1	37	33	33	**0**
2	45	41	**0**	0
3	**0**	0	28	17
4	4	**0**	0	67

The optimal assignments are

Flight 1-104

Flight 2-103

Flight 3-101

Flight 4-102

Conclusions:

In this paper, a new and simple modal was introduced for solving assignment problems . As considerable number of problems has been so for presented for Assignment problem in which the Hungarian method is more convenient method therefore this paper present a three different models for solving assignment problems

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