

## OPTIMIZATION OF VENDOR SELECTION PROBLEM USING LINEAR INTUITIONISTIC FUZZY MODEL

Prabjot Kaur\*

Rahul Kumar\*

### ABSTRACT-

*In this paper we propose a linear intuitionistic fuzzy linear model to select the best vendors and determine the order quantity for allocation. The model is a reformulation of Ghodspour and Brien (1998) model where the objective is maximization of the objective function value representing the total value of purchasing. The selection of vendor is based on criteria like cost, quality, delivery, service and capacity constraints. Often data related to these criteria are uncertain or incomplete, so data can be best represented by triangular intuitionistic fuzzy number's. The model considers the objective function represented by triangular intuitionistic fuzzy number and the constraints are deterministic. The solution of the linear model is based on value and ambiguity indexes. A ranking function is used to convert the intuitionistic fuzzy linear model to crisp linear model and solution is obtained by using optimization software. A numerical example illustrates our methodology to compare a previous model in literature.*

**Keyword-** Vendor selection, triangular intuitionistic fuzzy number (TIFN), Optimization, Ranking Function

\* Department of Mathematics, Birla Institute of Technology, Mesra, Ranchi, Jharkhand, India

## 1.0 Introduction

Due to changing business conditions like competition, new information technologies and fragmentation of markets the vendor selection problem is becoming complex and vital activity for organizations. The supplier selection decisions determine how many and which suppliers should be selected as supply sources and how order quantities should be allocated among the selected suppliers [3]. It can be argued that it is extremely difficult for any one supplier/vendor to excel in all dimensions of performance due to multicriteria nature of the problem involving both qualitative and quantitative criteria. At the time of making decisions, the value of many criterion and constraints are expressed in vague terms such as “high in quality” or “cheap in price”. Deterministic models cannot easily take this vagueness into account. The vagueness can be handled by intuitionistic fuzzy sets **Atanassov(1986)** an extension of fuzzy sets. The advantage of IFS is that they have an additional degree of freedom over fuzzy sets. Literature review reveals that mathematical programming techniques have been applied to issues related to allocation of orders to vendors. **Moore and Fearon (1973)**, **Gaballa (1974)** discussed the use of linear programming and applied mathematical programming in a real life application case. **Anthony and Buffa (1977)** developed a single objective linear programming model to support strategic purchasing scheduling. **Bender et al. (1985)** applied single objective programming to develop a commercial computerized model for vendor selection at IBM. **Kingsman (1988)** discussed conceptually linear programming problem(LPP) and dynamic programming tools for purchasing raw materials with fluctuating prices. **Turner (1988)** proposed the use of mixed integer optimisation model. **Chaudhry et al. (1993)** used linear and mixed binary integer programming models to solve cost minimizing problems of vendor selection with price breaks. **Rosenthal et al. (1995)**, **Jayaraman et al. (1999)**, **Ghodsypour and O'Brien (2001)** developed a mixed integer linear/non linear program that would find the purchasing strategy for the buyer to minimize the total purchase cost. **Ghodsypour, and O'Brien (2006)** developed an asymmetric fuzzy multi-objective linear model that enables the decision-makers to assign different weights to various criteria in the problem. **Amid, Ghodsypour, and O'Brien (2009)** presented a fuzzy weighted additive and mixed integer linear programming method that includes minimizing the net cost, minimizing the net rejected items and minimizing the net late deliveries objective functions with capacity and demand requirement constraints under price breaks in a supply chain. **Shirkouhi, Shakouri and Keramati(2013)** used a two phase fuzzy multiobjective linear

programming model for allocation under multi-price level and multiproduct. Though fuzzy sets allows the degree of satisfiability of each alternative with respect to a set of criteria with respect to incomplete and vague information in decision making problems. But intuitionistic sets are one degree better than fuzzy sets because of an additional degree of possibility to represent imperfect information in real world problem in a better way. A very few applications of intuitionistic fuzzy in optimization of vendor selection problem has been used. **Shahrokhi, Bernard and shidpour (2011)** used IFS and LP to select suppliers for manufacturing firms. Other general applications of IFS in optimization being **Dubey and Mehra (2011)** solved LPP with data as triangular intuitionistic fuzzy numbers, **Li(2008)** extended the LPP for solving MADM problems under Intuitionistic Fuzzy environment and **Nagoorgani (2012)** solved the intuitionistic fuzzy linear programming.

The summary of the paper is as follows: Section 1 briefly discusses the definition and literature review of vendor selection problem. Section 2 discusses the basics of intuitionistic fuzzy sets and definitions used in the problem. Section 3 the methodology to be used in the problem for selection of vendor. Section 4 gives an illustration of a numerical example used as case study. Section 5 results and discussions of our numerical example and finally conclusions in section 6.

## 2.0 Preliminaries

**Definition 1 :** Given a fixed set  $X = \{x_1, x_2, \dots, x_n\}$ , an intuitionistic fuzzy set (IFS) is defined as  $\bar{A} = (\langle x_i, t_A(x_i), f_A(x_i) \rangle / x_i \in X)$  which assigns to each element  $x_i$ , a membership degree  $t_A(x_i)$  and a non-membership degree  $f_A(x_i)$  under the condition  $0 \leq t_A(x_i) + f_A(x_i) \leq 1$ , for all  $x_i \in X$ .

**Definition 2:** A (TIFN)  $\bar{A} = (a_1, a_2, a_3; w_a)(a_1', a_2, a_3'; u_a)$  is an IFS in  $R$  with the following membership function  $\mu_{\bar{A}}(x)$  and non-membership  $\vartheta_{\bar{A}}(x)$ .

$$\mu_{\bar{A}}(x) = \begin{cases} \frac{(x-a_1)w_a}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ w_a & x = a_2 \\ \frac{(a_3-x)w_a}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \vartheta_{\bar{A}}(x) = \begin{cases} \frac{a_2 - x + u_a(x-a_1')}{a_2 - a_1'}, & a_1' \leq x \leq a_2 \\ u_a, & x = a_2 \\ \frac{x-a_2+u_a(a_3'-x)}{a_3'-a_2'}, & a_2 \leq x \leq a_3' \\ 1, & \text{otherwise} \end{cases}$$

The values  $w_a$  and  $u_a$  respectively represent the maximum degree of membership and the non-membership such that  $0 \leq w_a \leq 1, 0 \leq u_a \leq 1$ .

**Definition 3:** Arithmetic Operations of TIFN is given by:

If  $\bar{A} = (a_1, a_2, a_3; w_a)(a_1', a_2, a_3'; u_a)$  and  $\bar{B} = (b_1, b_2, b_3; w_b)(b_1', b_2, b_3'; u_b)$  are two TIFNs, and  $k$  be a real number then

$$\bar{A} + \bar{B} = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3; \min\{w_a, w_b\})(a_1' + b_1', a_2 + b_2, a_3' + b_3'; \max\{u_a, u_b\})\}$$

is also a TIFN. ...(1)

$$k\bar{A} = \text{is a TIFN } \{(ka_1, ka_2, ka_3; w_a)(ka_1', ka_2, ka_3'; u_a)\}. k > 0.$$

**Definition 4:** Let  $\bar{A} = (a_1, a_2, a_3; w_a)(a_1', a_2, a_3'; u_a)$  be a TIFN. Then value and the ambiguity of  $\bar{A}$  is given as follows:

1. The value of the membership function of  $\bar{A}$  is

$$V_\mu(\bar{A}) = \frac{(a_1 + 4a_2 + a_3)w_a}{6} \quad \dots(2)$$

2. The value of the non-membership function is

$$V_\nu(\bar{A}) = \frac{(a_1' + 4a_2 + a_3')(1 - u_a)}{6} \quad \dots(3)$$

3. The ambiguity of the membership function of  $\bar{A}$  is

$$A_\mu(\bar{A}) = \frac{(a_3 - a_1)w_a}{6} \quad \dots(4)$$

4. The ambiguity of non-membership function  $\bar{A}$  is

$$A_\nu(\bar{A}) = \frac{(a_3' - a_1')(1 - u_a)}{6} \quad \dots(5)$$

$$\text{Also } A_\mu(\bar{A}) \leq A_\nu(\bar{A}).$$

**Definition 5:** Let  $\bar{A} = (a_1, a_2, a_3; w_a)(a_1', a_2, a_3'; u_a)$  be a TIFN. Then the value index and ambiguity index of  $\bar{A}$  is defined as follows:

$$V(\bar{A}, \lambda) = V_\mu(\bar{A}) + \lambda(V_\nu(\bar{A}) - V_\mu(\bar{A})) \quad \dots(6)$$

$$\text{and } A(\bar{A}, \lambda) = A_\nu(\bar{A}) - \lambda(A_\nu(\bar{A}) - A_\mu(\bar{A})) \quad \dots(7)$$

where  $\lambda \in [0, 1]$  is a weight which represents the decision maker's preference information.

**Definition 6:** Ranking relation  $F(\bar{A}, \lambda) = V(\bar{A}, \lambda) - A(\bar{A}, \lambda) \quad \dots(8)$

### 3.0 Methodology

#### FORMULATIONS

#### 3.1 THE LINEAR CRISP MODEL FOR VENDOR SELECTION

Model parameters:

$R_i$  Final ratings of  $i^{\text{th}}$  supplier

$X_i$  Order quantity for  $i^{\text{th}}$  supplier

$V_i$  Capacity of  $i^{\text{th}}$  supplier

$D'$  Demand for the period

$q_i$  Defect percent of  $i^{\text{th}}$  supplier

$Q$  Buyer's maximum acceptable defect rate

The integrated linear programming model (**Ghodsypour and Brien (1998)**) is formulated as follows:

$$\text{Max (TVP)} = \sum_{i=1}^n R_i X_i \quad \dots(9)$$

Subject to:

$$\sum_{i=1}^n X_i = D' \quad (\text{Demand constraint}),$$

$$\sum_{i=1}^n X_i q_i \leq QD' \quad (\text{Aggregate quality constraint}) \quad \dots(10)$$

$$X_i \leq V_i, i = 1, 2, \dots, n \quad (\text{Vendor's capacity constraints})$$

$$X_i \geq 0, i = 1, 2, \dots, n \quad (\text{Non-negativity constraint})$$

where the objective of the model is to maximize the total value of purchasing and the constraints represented by equation (10).

### 3.2 PROPOSED INTUITIONISTIC FUZZY MODEL

In problem formulation we define the objective function by TIFN's, i.e. by pairs of membership and non-membership functions  $[(a_1, a_2, a_3)(a'_1, a_2, a'_3)]$ . The final model is as follows:

$$\text{Max (TVP)} = \sum_{i=1}^n R_i [(a_1, a_2, a_3)(a'_1, a_2, a'_3)] X_i$$

Subject to :

$$\sum_{i=1}^n X_i = D$$

$$\sum_{i=1}^n X_i q_i = QD \quad \dots (11)$$

$$X_i \leq V_i, i=1,2,\dots,n$$

$$X_i \geq 0, i=1,2,\dots,n.$$

### 3.4 CRISP FORMULATIONS OF THE MODEL

The elements in the objective function are intuitionistic fuzzy numbers and the constraints are in the crisp form. To convert the intuitionistic fuzzy objection function to crisp form [6] using the equation (7), for predefined  $\lambda \in [0,1]$ , IFLP model(refer equation(11)) is equivalent to the following crisp optimization problem.

$$\begin{aligned} & \max(1 - \lambda) \min_j \{w_{\tilde{c}_j}\} \sum_{j=1}^n \frac{V_{\mu}(\tilde{c}_j)}{w_{\tilde{c}_j}} x_j - (1 - \lambda) \min_j \{1 - u_{\tilde{c}_j}\} \sum_{j=1}^n \frac{A_{\nu}(\tilde{c}_j)x_j}{1-u_{\tilde{c}_j}} + \\ & \lambda \min_j \{1 - u_{\tilde{c}_j}\} \sum_{j=1}^n \frac{V_{\nu}(\tilde{c}_j)}{1-u_{\tilde{c}_j}} x_j - \lambda \min_j \{w_{\tilde{c}_j}\} \sum_{j=1}^n \frac{A_{\mu}(\tilde{c}_j)x_j}{w_{\tilde{c}_j}} \end{aligned} \quad \dots(12)$$

Subject to:

$$\begin{aligned} \sum_{i=1}^n X_i &= D \\ \sum_{i=1}^n X_i q_i &= QD \end{aligned} \quad \dots (13)$$

$$X_i \leq V_i, i=1,2,\dots,n$$

$$X_i \geq 0, i=1,2,\dots,n.$$

For optimistic attitude  $\lambda=1$ , equation (\*) reduces to

$$\begin{aligned} & \max \min_j \{1 - u_{\tilde{c}_j}\} \sum_{j=1}^n \frac{V_{\nu}(\tilde{c}_j)x_j}{1-u_{\tilde{c}_j}} - \min_j \{w_{\tilde{c}_j}\} \sum_{j=1}^n \frac{A_{\mu}(\tilde{c}_j)x_j}{w_{\tilde{c}_j}} \\ & \text{Subject to:} \end{aligned} \quad \dots(14)$$

$$\begin{aligned} \sum_{i=1}^n X_i &= D \\ \sum_{i=1}^n X_i q_i &= QD \end{aligned} \quad \dots (15)$$

$$X_i \leq V_i, i=1,2,\dots,n$$

$$X_i \geq 0, i=1,2,\dots,n.$$

For pessimistic  $\lambda=0$  (\*) reduces to

$$\begin{aligned} & \max \min_j \{w_{\tilde{c}_j}\} \sum_{j=1}^n \frac{V_{\mu}(\tilde{c}_j)}{w_{\tilde{c}_j}} x_j - (1 - \lambda) \min_j \{1 - u_{\tilde{c}_j}\} \sum_{j=1}^n \frac{A_{\nu}(\tilde{c}_j)x_j}{1-u_{\tilde{c}_j}} + \\ & \min_j \{w_{\tilde{c}_j}\} \sum_{j=1}^n \frac{V_{\mu}(\tilde{c}_j)}{w_{\tilde{c}_j}} x_j - \min_j \{1 - u_{\tilde{c}_j}\} \sum_{j=1}^n \frac{A_{\nu}(\tilde{c}_j)x_j}{1-u_{\tilde{c}_j}} \end{aligned}$$

$$\text{Subject to:} \quad \dots(16)$$

$$\begin{aligned} \sum_{i=1}^n X_i &= D \\ \sum_{i=1}^n X_i q_i &= QD \quad \dots (17) \\ X_i &\leq V_i, i=1,2,\dots,n \\ X_i &\geq 0, i=1,2,\dots,n. \end{aligned}$$

The above crisp LPP is solved by optimization software **Tora 2.0**. We obtain solution of the model for the various decision makers.

#### 4.0 Numerical Example

Assume that the management of a JIT manufacturer decides to choose their best suppliers and assign their optimum order quantities to maximize the TVP. The main criteria for supplier selection are cost, quality and service. According to the corporate strategies the quality includes defects and process capability while service involves on-time delivery, response to changes and process flexibility. Four suppliers are included in the evaluation process and their cost, quality, On time delivery and capacities are presented in Table 1. The demand is 1000 units and the maximum acceptable defect rate is 0.02.

**Table 1: The crisp data for the various suppliers**

Supplier	Cost	Quality	one time delivery	capacity
A <sub>1</sub>	30	.03	.95	400
A <sub>2</sub>	40	.05	.98	700
A <sub>3</sub>	50	.01	.85	600
A <sub>4</sub>	45	.06	.92	500

**Step-1 :** The crisp data ( Table 1) is converted to intuitionistic TFNs and intuitionistic weights for various suppliers (Table 2). The data of capacity is kept in crisp form. In Table 3 the data has been normalized and aggregated using equation (1) of definition 3.

**Table 2: Intuitionistic Fuzzy Data for the various supplier**

Supplier	Weight (w <sub>i</sub> , u <sub>i</sub> )	Cost	Quality	On time delivery	Capacity
A <sub>1</sub>	(1,0)	(25,30,45) (20,30,50)	(0.02,0.03,0.05) (0.01,0.03,0.06)	(0.80,0.95,1.00) ) (0.75,0.95,1.50) )	400

A <sub>2</sub>	(3/4,1/4)	(35,40,50) (30,40,60)	(0.03,0.05,0.06) (0.02,0.05,0.07)	(0.90,0.98,0.99) ) (0.88,0.98,1.01) )	700
A <sub>3</sub>	(1/2,1/4)	(40,50,60) (35,50,70)	(0.009,0.01,0.03) (0.008,0.01,0.04)	(0.81,0.85,0.87) ) (0.80,0.92,1.05) )	600
A <sub>4</sub>	(3/4,1/5)	(40,45,55) (30,45,60)	(0.03,0.06,0.07) (0.02,0.06,0.08)	(0.85,0.92,1.00) ) (0.80,0.92,1.05) )	500

**Table 3: Normalization and aggregation of Intuitionistic Fuzzy Data for the various supplier**

Supplier	Cost	Quality	OTD	Capacity	Aggregation of Cost, Quality & OTD
A <sub>1</sub>	(.36,.43,.69) (.29,.43,.72)	(.25,.38,.63) (.13,.38,.75)	(.53,.63,.67) (.5,.63,1)	400	(1.14,1.44,1.94) (.92,1.44,2.47)
A <sub>2</sub>	(.50,.57,.71) (.43,.57,.86)	(.38,.63,.75) (.25,.63,.88)	(.6,.65,.66) (.59,.65,.68)	700	(1.48,1.85,2.12) (1.27,1.85,2.42)
A <sub>3</sub>	(.57,.71,.86) (.5,.71,1)	(.11,.13,.38) (.1,.13,.5)	(.54,.57,.58) (.53,.57,.59)	600	(1.22,1.41,1.82) (1.13,1.41,2.09)
A <sub>4</sub>	(.57,.64,.79) (.42,.64,.86)	(.38,.75,.88) (.25,.75,1)	(.57,.61,.67) (.53,.61,.70)	500	(1.52,2.00,2.34) (1.20,2.00,2.56)

**Step 2:** From equation(11) our intuitionistic model becomes

$$\text{Max}Z=(1.14,1.44,1.94)(.92,1.44,2.47)x_1+(1.48,1.85,2.12)(1.27,1.85,2.42)x_2+(1.22,1.41,1.82)(1.13,1.41,2.09)x_3+(1.52,2.00,2.34)(1.20,2.00,2.56)x_4$$

Subject to:

$$\begin{aligned} x_1+x_2+x_3+x_4 &= 1000 \\ 0.03x_1+0.05x_2+0.01x_3+0.06x_4 &\leq 20 \quad \dots(18) \\ x_1 &\leq 400 \\ x_2 &\leq 700 \\ x_3 &\leq 600 \\ x_4 &\leq 500 \\ x_i &\geq 0, i=1,2,3,4. \end{aligned}$$

Step 3: Using equations (12)-(17) for defuzzification: in equation (18) we get

For  $\lambda=0$ , our model becomes

$$\text{Max } Z = .3307x_1 + .6292x_2 + .483x_3 + .648x_4$$

Subject to:

$$\begin{aligned} x_1+x_2+x_3+x_4 &= 1000 \\ 0.03x_1+0.05x_2+0.01x_3+0.06x_4 &\leq 20 \quad \dots(19) \\ x_1 &\leq 400 \\ x_2 &\leq 700 \\ x_3 &\leq 600 \\ x_4 &\leq 500 \\ x_i &\geq 0, i=1,2,3,4. \end{aligned}$$

For  $\lambda=0.5$ , our model becomes

$$\text{Max } Z = .6705x_1 + .95425x_2 + .74375x_3 + .9905x_4$$

Subject to:

$$\begin{aligned} x_1+x_2+x_3+x_4 &= 1000 \\ 0.03x_1+0.05x_2+0.01x_3+0.06x_4 &\leq 20 \\ x_1 &\leq 400 \\ x_2 &\leq 700 \quad \dots(20) \\ x_3 &\leq 600 \\ x_4 &\leq 500 \\ x_i &\geq 0, i=1,2,3,4. \end{aligned}$$

For  $\lambda=1$ , our model becomes

$$\text{Max } Z = 1.0103x_1 + 1.2793x_2 + 1.0075x_3 + 1.333x_4$$

Subject to:

$$\begin{aligned}
 &x_1+x_2+x_3+x_4=1000 \\
 &0.03x_1+0.05x_2+0.01x_3+0.06x_4\leq 20 \\
 &x_1\leq 400 \qquad \dots(21) \\
 &x_2\leq 700 \\
 &x_3\leq 600 \\
 &x_4\leq 500 \\
 &x_i\geq 0, i=1,2,3,4.
 \end{aligned}$$

Step 4: The deterministic converted model (Equations 19,20,21) has been solved using Tora 2.0. The results are shown in Tables 4 and 5.

### 5.0 Results and Discussion

For various decision makers, pessimistic, moderate and optimistic the value of purchasing and allocation of order is as follows:

**Table 4: allocation of order to various suppliers under various decision makers.**

Supplier	Allocation of order for $\lambda=0$	Allocation of order for $\lambda=0.5$	Allocation of order for $\lambda=1$
A <sub>1</sub>	300	300	300
A <sub>2</sub>	100	100	100
A <sub>3</sub>	600	600	600
A <sub>4</sub>	0	0	0

**Table 5: The value of objective function for various decision makers**

Decision Makers	$\lambda=0$	$\lambda=0.5$	$\lambda=1$
Z	452	743	1036

We observe that the maximum value of the objective function (Table 5) is for an optimistic decision maker and then a moderate and then a pessimistic decision maker. The allocation of order is same for all decision makers (Table 4). Looking at the market scenario it is better to allocate allocations to maximum vendors as possible for supply in time and need. The best scenario being for a optimistic decision maker and worst case other decision makers decisions can be considered after all allocation is same for all.

## 6.0 Conclusions

An intuitionistic fuzzy model has been proposed and solved in this paper for allocation of order quantity to vendors. The objective value function was treated as intuitionistic fuzzy subject to crisp constraints. The optimization model was solved by a different approach. The advantage of IFSs is that it provides a way for explaining lack of information in the human decision making process .

The future scope of this work being working with real life data and implementing the above model for various applications.

## REFERENCES

1. **Ghodsypour, S. H., & O'Brien, C. (1998).** A decision support system for supplier selection using an integrated analytical hierarchy process and linear programming. *International Journal of Production Economics*, 199–212.
2. **Shahrokhi, Bernard and Shidpour,** An integrated method using intuitionistic fuzzy set and linear programming for supplier selection problem, 18th IFAC World Congress Milano (Italy) August 28 - September 2, 2011.
3. **Atanassov, K. (1986).** Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 79, 403-405.
4. **Ghodsypour, S.H. and Brien, C.O. (2001)** The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint, *International J. of Production Economics*, vol 73, 15-27.
5. **Jayaraman, V. and Srivastava, R. and Benton, W.C. (1999)** Supplier Selection and order quantity allocation: A comprehensive Model, *The J. of Supply chain Management: A global review of Purchasing and Supply*, Spring, 50-57.
6. **Dipti Dubey and Aparna Mehra (2011).** Linear Programming with triangular intuitionistic fuzzy numbers, *EUSFLAT-LFA 2011*, 563-569.
7. **A. Nagoorgani ,** A New Approach on Solving Intuitionistic Fuzzy Linear Programming Problem, *Applied Mathematical Sciences*, Vol. 6, 2012, no. 70, 3467 – 3474.

8. **Deng-Feng Li**, Extension of the LINMAP for multiattribute decision making under Atanassov's intuitionistic fuzzy environment, *Fuzzy Optim Decis Making* (2008) 7:17–34.
9. **Shirkouhi, Shakouri and Keramati (2013)**, Supplier selection and order allocation problem using a two phase fuzzy multiobjective linear programming . *Applied Mathematical Modelling* 37, 9308-9323.
10. **Anthony, T.F. and Buffa, F.P.** (1977) Strategic purchase scheduling, *J. of Purchasing and Materials Management*, 27-31.
11. **Gaballa, A.A.** (1974) Minimum cost allocation of tenders, *Operations Research Quartely*, vol 25(3), 389-398.
12. **.Bender, P.S., Brown, R.W., Isaac, M.H. and Shapiro, J.F.** (1985) Improving purchasing productivity at IBM with a normative decision support system, *Interfaces*, vol 15(3), 106-115.
13. **Kingsman, B.G.** (1988) Purchasing raw materials with uncertain fluctuating prices, *European J. of Operational Research*, vol 25, 358-372.
14. **Chaudhry S.S., Forst F.G. and Zydiak J.L.** (1993), 'Vendor selection with price breaks', *European Journal of Operational Research*, Vol. 70, pp. 52-66.
15. **Amid, A., Ghodsypour, S.H., O'Brien, C.,** 2006. Fuzzy multiobjective linear model for supplier selection in a supply chain. *International Journal of Production Economics* 104 (2), 394–407.
16. **Turner, I** (1988), An independent system for the evaluation of contract tenders, *Journal of operational Research Society*, 39/6, 551-561.
17. **Moore and Fearon** (1972), computer assisted decision making in purchasing, *Journal of purchasing*, 9/4, 5-25.

18. **Rosenthal, E.C., Zydiak, J.L., Chaudhry, S.S.**, 1995. Vendor selection with bundling. Decision Sciences 26 (1), 35-48.
  
19. **Amid ,Ghodsypur ,O'Brien (2009)** A weighted additive fuzzy multiobjective model for the supplier selection problem under price break in supply chain,Internation Journal of Production Economics,121(2),323-332.

