MHD FREE CONVECTIVE FLOW PAST A SEMI-INFINITE VERTICAL PERMEABLE MOVING PLATE WITH HEAT ABSORPTION

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Abstract

This paper deals with the influences of heat and mass transfer on two dimentional MHD free convective, laminar and boundary layer flow of a viscous fluid along a semi-infinite vertical permeable moving plate in the presence of uniform transverse magnetic field and heat absorption. The governing equations have been solved by perturbation technique. Numerical evalution of the analytical results has been performed and some graphical results for the velocity, temperature and concentration profiles with in the boundary layer and tabulated results for the local values of the skin-friction coefficient, Nusselt number and Sherwood number are presented and discussed in detail.

Key words: MHD, Free convective, Heat transfer, Mass transfer, Heat absorption

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1. Introduction

Fluid dynamics of various fluids have many engineering and industrial applications. In particular combined heat and mass transfer from different processes with porous media has a wide range of applications in engineering and industry such as enhanced oil recovery, underground energy transport, geothermal reservoirs, cooling of nuclear reactors, drying of porous solids, packed-bed catalytic reactors and thermal insulation. Gribben [1] has considered the MHD boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of a pressure gradient. He has obtained solutions for large and small magnetic Prandtl numbers using the method of matched asymptotic expansion.

Takhar and Ram [2] have studied the MHD free porous convection heat transfer of water at 4^0C through a porous medium. Soundalegkar [3] obtained approximate solutions for the two-dimensional flow of an incompressible viscous fluid past an infinite porous vertical plate with constant suction velocity normal to the plate, the difference between the temperature of the plate and the free stream being moderately large causing the free convection currents. Raptis and Kafousias [4] have studied the influence of a magnetic field upon the steady free convective flow through a porous medium bounded by an infinite vertical plate with constant suction velocity. Raptis [5] has studied mathematically the case of time-varying two-dimensional natural convective heat transfer of an incompressible, electrically-conducting viscous fluid via a highly porous medium bounded by an infinite vertical porous plate. Chamkha [6] has investigated hydromagnetic natural convection from an isothermal inclined surface adjacent to a thermally stratified porous medium. Bian et al. [7] have reported on the effect of an electromagnetic field on natural convection in an inclined porous medium. A great number of Darcian porous MHD studies have been performed examining the effects of magnetic filed on hydrodynamic flow without heat transfer in various configurations,e.g., in channels and past plates, wedgs, etc. [8,9]. Alam et al. [10] studied the problem of free convective heat and mass transfer flow past an inclined semi-infinite heated surface of a steady electrically conducting viscous incompressible fluid in the presence of a magnetic field and heat generation.

Muthucumaraswamy and Senthih [11] considered heat and mass transfer effect on a moving vertical plate in the presence of thermal radiation. Chen [12] studied heat and mass transfer in

MHD flow by natural convection from a permeable inclined surface with variable wall temperature and concentration. Masthanrao et al. [13] investigated chemical reaction and combined buoyancy effects of thermal and mass diffusion on MHD convective flow along an infinite vertical porous plate in the presence of hall current with variable suction and heat generation. Balamurugan et al. [14] studied the problem of unsteady MHD free convective flow past a moving vertical plate with time dependent suction and chemical reaction in a slip flow regime. Ramaprasad et al. [15] have analyzed unsteady MHD free convective heat and mass transfer flow past an inclined moving surface with heat absorption.

In this paper heat and mass transfer effects, on unsteady free convective flow along a vertical permeable plate embedded in a fluid-saturated porous medium in the presence of mass blowing or suction, have been investigated. Effects of magnetic field and absorption are also investigated. In all the earlier investigations it is noticed that the authors have not considered ε^2. In the present work, the set of ordinary differential equations are solved by considering ε^2 in regular perturbation method.

2. Formulation of the problem:

An unsteady two-dimensinal flow of a laminar, incompressible, viscous, electrically conducting and heat-absorbing fluid past a semi vertical permeable moving plate embedded in a uniform porous medium subjected to a uniform transverse magnetic field in the presence of thermal and concentration buoyancy has been considered.

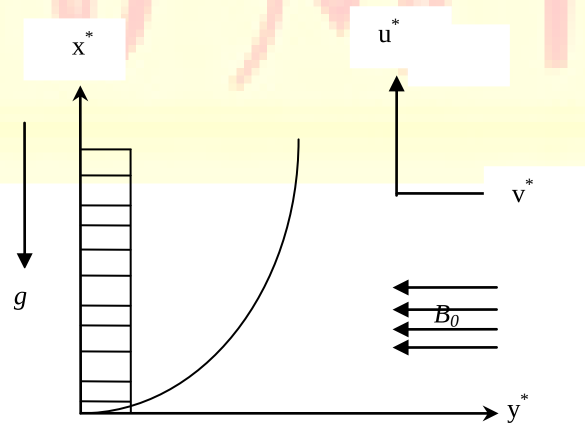

Figure 1. Flow Configuration

It is assumed that there is no applied voltage which implies the absence of an electrical field. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall effect are negligible. A consequence of the small magnetic Reynolds number is the uncoupling of the Navier-Stokes equations from Maxwell's equations. The governing equations for this investigation are based on the balances of mass, linear momentum, energy and concentration species. Taking into consideration the assumptions made above, these equations can be written in Cartesian frame of reference, as follows:

$$\frac{\partial \upsilon^*}{\partial y^*} = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial t^*} + \upsilon^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^2} + g\beta_T (T - T_\infty) + g\beta_c (c - c_\infty) - \nu \frac{u^*}{K^*} - \frac{\sigma}{\rho} B_0^2 u^* \quad (2)$$

$$\frac{\partial T}{\partial t^*} + \upsilon^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} - \frac{Q_0}{\rho c_p}(T - T_\infty) \quad (3)$$

$$\frac{\partial c}{\partial t^*} + \upsilon^* \frac{\partial c}{\partial y^*} = D \frac{\partial^2 c}{\partial y^{*2}} \quad (4)$$

where x^* and y^* are the dimensional distances along and perpendicular to the plate respectively and t^* is the dimentional time. u^* and v^* are the components of dimentional velocities along x^* and y^* directions respectively, ρ is the fluid density, ν is the kinemetic viscosity, c_p is the specific heat at constant pressure, σ is the fluid eletrical conductivity, B_0 is the magnetic induction, K^* is the permeability of the porous medium, T is the dimensional temperature, Q_0 is the dimentional heat absorpction coefficient, c is the dimentional concentration, α is the fluid thermal diffusivity, D is the mass diffusivity, g is the gravitational acceleration, and β_T and β_c are the thermal and concentration expansion coefficients respectively. The magnetic and viscous dissipations are neglected in this study. The third and fourth terms on the RHS of the momentum Eq. (2) denote the thermal and concentration buoyancy respectively. Also, the last term of Eq.(3) denotes the heat absorption' It is assumed that the permeable plate moves with a constant velocity in the direction of fluid flow, and the free stream velocity follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and the concentration at the wall as well as the suction velocity are exponentially varying with time.

Under these assumptions, the apporpriate boundary conditions for the velocity, temperature and concentration fields are

$$u^* = u_p^*, T = T_w + \varepsilon(T_w - T_\infty)e^{n^*t^*}, c = c_w + \varepsilon(c_w - c_\infty)e^{n^*t^*} \quad at \quad y^* = 0 \tag{5}$$

$$u^* \to U_\infty^* = U_0(1 + \varepsilon e^{n^*t^*}), T \to T_\infty, \quad c \to c_\infty \; as \quad y^* \to \infty \tag{6}$$

where u_p^*, c_w and T_w are the wall dimensional velocity, concentration and temperature respectively. U_∞^*, c_∞, and T_∞ are the free stream dimentional velocity, concentration and temperature respectively. U_0 and n^* are constants.

It is clear from Eq.(1) that the suction velocity at the plate surface is a function of time only. It is assumed that it takes the following exponential form:

$$\upsilon^* = -V_0(1 + \varepsilon A e^{n^*t^*}) \tag{7}$$

where A is a real positive constant, ε and εA are small less than unity, and V_0 is a scale suction velocity which has non-zero positive constant. Outside the boundary layer, Eq (2) gives

$$-\frac{1}{\rho} = \frac{\partial U_\infty^*}{\partial t^*} + \frac{\upsilon}{K^*}U_\infty^* + \frac{\sigma}{\rho}B_0^2 U_\infty^* \tag{8}$$

It is convenient to employ the following dimensionless variables:

$$u = \frac{u^*}{U_0}, \quad \upsilon = \frac{\upsilon^*}{V_0}, \quad y = \frac{V_0 y^*}{\upsilon}, \quad U_\infty = \frac{U_\infty^*}{U_0}, \quad U_p = \frac{u_p^*}{U_0}, \quad t = \frac{t^* V_0^2}{\upsilon},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{c - c_\infty}{c_w - c_\infty}, \quad n = \frac{n^*\upsilon}{V_0^2}, \quad K = \frac{K^* V_0^2}{\upsilon^2}, \quad \Pr = \frac{\upsilon\rho c_p}{k} = \frac{\upsilon}{\alpha}, \tag{9}$$

$$Sc = \frac{\upsilon}{D}, M = \frac{\sigma\upsilon B_0^2}{\rho V_0^2}, \quad G_T = \frac{\upsilon\beta_T g(T_w - T_\infty)}{U_0 V_0^2}, \quad G_c = \frac{\upsilon\beta_c g(c_w - c_\infty)}{U_0 V_0^2}, \phi = \frac{\upsilon Q_0}{\rho c_p V_0^2}$$

In view of Eqs. (7)-(9), Eqs.(2)-(4) reduce to the following dimensionless form:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial y} = \frac{\partial U_\infty}{\partial t} + \frac{\partial^2 u}{\partial y^2} + G_T + G_c + N(U_\infty - u) \tag{10}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - \phi\theta \tag{11}$$

$$\frac{\partial C}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad (12)$$

where $N=(M+\frac{1}{K})$ and G_c , G_T, P_r, ϕ and S_c are the solutal Grashof number, thermal Grashof number, Prandtl number, dimensionless heat absorption coefficient, and the Schmidt number respectively. The dimensionless form of the boundary conditions (5) and (6) become

$$u=U_p, \quad \theta=1+\varepsilon e^{nt}, \quad C=1+\varepsilon e^{nt} \quad at \quad y=0 \quad (13)$$

$$u\to U_\infty, \quad \theta\to 0, \quad C\to 0 \quad as \quad y\to\infty \quad (14)$$

3. Solution of the problem:

Eqs. (10) – (12) represent a set of partial differential equations that cannot be solved in closed form. However, it can be reduced to a set of ordinary differential equations in dimensionless form that can be solved analytically. This can be done by representing the velocity, temperature and concentration as

$$u = f_0(y) + \varepsilon e^{nt} f_1(y) + \varepsilon^2 e^{2nt} f_2(y) + O(\varepsilon^3) + \cdots\cdots \quad (15)$$

$$\theta = g_0(y) + \varepsilon e^{nt} g_1(y) + \varepsilon^2 e^{2nt} g_2(y) + O(\varepsilon^3) + \cdots\cdots \quad (16)$$

$$C = h_0(y) + \varepsilon e^{nt} h_1(y) + \varepsilon^2 e^{2nt} h_2(y) + O(\varepsilon^3) + \cdots\cdots \quad (17)$$

Substisuting Eqs. (15) – (17) in to Eqs. (10) – (12), and equating the coefficient of ε^0, ε^1 and ε^2, we get the following pairs of equations for (f_0, g_0, h_0), (f_1, g_1, h_1) and (f_2, g_2, h_2).

$$f_0'' + f_0^1 - Nf_0 = -G_T g_0 - G_c h_0 - N \quad (18)$$

$$f_1'' + f_1' - (N+n) f_1 = -Af_0' - n - N - G_T g_1 - G_c h_1 \quad (19)$$

$$f_2'' + f_2' - (N+2n) f_2 = -Af_1' - G_T g_2 - G_c h_2 \quad (20)$$

$$g_0'' + \Pr g_0^1 - \phi \Pr g_0 = 0 \quad (21)$$

$$g_1'' + \Pr g_1' - (n+\phi)\Pr g_1 = -\Pr Ag_0' \quad (22)$$

$$g_2'' + \Pr g_2' - \Pr(2n+\phi) g_2 = -\Pr Ag_1' \quad (23)$$

$$h_0'' + Sch_0' = 0 \tag{24}$$

$$h_1'' + Sch_1' - Scnh_1 = -ScAh_0^1 \tag{25}$$

$$h_2'' + Sch_2' - 2nSch_2 = -ScAh_1' \tag{26}$$

where a prime refers to ordinary differentiation with respect to y. The corresponding boundary conditions can be written as

$$f_0 = U_p,\ f_1 = 0,\ f_2 = 0,\quad g_0 = 1, g_1 = 1, g_2 = 0, h_0 = 1, h_1 = 1, h_2 = 0 \quad \text{at} \quad y = 0$$

$$f_0 = 1,\ f_1 = 1,\ f_2 = 0, g_0 \to 0, g_1 \to 0, g_2 \to 0, h_0 \to 0, h_1 \to 0, h_2 \to 0 \quad \text{at } y \to \infty \tag{27}$$

The solutions of equations (18) – (26) subject to the boundary conditions (27) are

$$f_0 = B_{10}e^{-\lambda_1 y} + B_8 e^{-m_3 y} + B_9 e^{-Scy} + 1 \tag{28}$$

$$f_1 = B_{18}e^{-\lambda_2 y} + B_{11}e^{-\lambda_1 y} + B_{12}e^{-m_3 y} - B_{14}e^{-m_4 y} - B_{15}e^{-m_3 y} - B_{16}e^{-m_1 y} + B_{17}e^{-Scy} + 1 \tag{29}$$

$$\begin{aligned} f_2 = {} & B_{33}e^{-\lambda_3 y} + B_{19}e^{-\lambda_2 y} + B_{20}e^{-\lambda_1 y} + B_{21}e^{-m_3 y} + B_{22}e^{-Scy} - B_{23}e^{-m_4 y} - B_{24}e^{-m_3 y} - B_{25}e^{-m_1 y} \\ & + B_{26}e^{-Scy} - B_{27}e^{-m_5 y} - B_{28}e^{-m_4 y} - B_{29}e^{-m_3 y} + B_{30}e^{-m_2 y} - B_{31}e^{-m_1 y} - B_{32}e^{-Scy} \end{aligned} \tag{30}$$

$$g_0 = e^{-m_3 y} \tag{31}$$

$$g_1 = (1 - B_4)e^{-m_4 y} + B_4 e^{-m_3 y} \tag{32}$$

$$g_2 = B_7 e^{-m_5 y} + B_5 e^{-m_4 y} + B_6 e^{-m_3 y} \tag{33}$$

$$h_0 = e^{-Scy} \tag{34}$$

$$h_1 = k_1 e^{-m_1 y} - k_2 e^{-Scy} \tag{35}$$

$$h_2 = -B_1 e^{-m_1 y} + B_2 e^{-m_1 y} + B_3 e^{-Scy} \tag{36}$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(y,t) = (1 + B_8 e^{-m_3 y} + B_9 e^{-Scy} + B_{10} e^{-\lambda_1 y}) + \varepsilon e^{nt}(1 + B_{11} e^{-\lambda_1 y} + B_{12} e^{-m_3 y} + B_{17} e^{-Scy} + B_{18} e^{-\lambda_2 y} - B_{14} e^{-m_4 y} - B_{15} e^{-m_3 y} - B_{16} e^{-m_1 y}) + \varepsilon^2 e^{2nt}(B_{19} e^{-\lambda_2 y} + B_{20} e^{-\lambda_1 y} + B_{21} e^{-m_3 y} + B_{22} e^{-Scy} + B_{26} e^{-Scy} + B_{30} e^{-m_3 y} + B_{33} e^{-\lambda_3 y} - B_{23} e^{-m_4 y} - B_{24} e^{-m_3 y} - B_{25} e^{-m_1 y} - B_{27} e^{-m_5 y} - B_{28} e^{-m_4 y} - B_{29} e^{-m_3 y} - B_{31} e^{-m_1 y} - B_{32} e^{-Scy}) \quad (37)$$

$$\theta(y,t) = e^{-m_3 y} + \varepsilon e^{nt}(e^{-m_4 y}(1 - B_4) + B_4 e^{-m_3 y}) + \varepsilon^2 e^{2nt}(B_5 e^{-m_4 y} + B_6 e^{-m_3 y} + B_7 e^{-m_5 y}) \quad (38)$$

$$C(y,t) = e^{-Scy} + \varepsilon e^{nt}(k_1 e^{-m_1 y} - k_2 e^{-Scy}) + \varepsilon^2 e^{2nt}(-B_1 e^{-m_2 y} + B_2 e^{-m_1 y} + B_3 e^{-Scy}) \quad (39)$$

The skin-friction coefficient, the Nusselt number and the sherwood number are important physical parameters for this type of boundary-layer flow. These parameters can determined as follows:

Skinfriction coefficient or shearing stress:

$$\mathrm{Cf} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = (-\lambda_1 B_{10} - m_3 B_8 - ScB_9) + \varepsilon e^{nt}(-\lambda_2 B_{18} - \lambda_1 B_{11} - m_3 B_{12} - ScB_{13} + m_4 B_{14} + m_3 B_{15} + m_1 B_{16} - ScB_{17}) + \varepsilon^2 e^{2nt}(-\lambda_3 B_{33} - \lambda_2 B_{19} - \lambda_1 B_{20} - m_3 B_{21} - ScB_{22} + m_4 B_{23} + m_3 B_{24} + m_1 B_{25} - ScB_{26} + m_5 B_{27} + m_4 B_{28} + m_3 B_{29} - m_3 B_{30} + m_1 B_{31} + ScB_{22}) \quad (40)$$

Nusslet number:

$$\mathrm{Nu} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -m_3 + \varepsilon e^{nt}(-m_4(1 - B_4) - m_3 B_4) + \varepsilon^2 e^{2nt}(-m_5 B_7 - m_4 B_5 - m_3 B_6) \quad (41)$$

Sherwood number:

$$\mathrm{Sh} = \left(\frac{\partial c}{\partial y}\right)_{y=0} = -Sc + \varepsilon e^{nt}(-m_1 k_1 + Sck_2) + \varepsilon^2 e^{2nt}(m_2 B_1 - m_1 B_2 - ScB_3) \quad (42)$$

4. Results and Discussion:

Numerical evaluation of the analytical results reported in the previous section was performed and a representative set of results is reported graphically in Figs. 2–4. These results are obtained to illustrate the influence of the solutal Grashof number Gc, the Prandtl number Pr and the Schmidt number Sc on the velocity, temperature and the concentration profiles.

Fig. 2 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number. The velocity distribution attains a distinctive maximum value in the vicinity of the plate surface and then decreases properly to approach the free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increases in the concentration buoyancy effects represented by Gc. This is evident in the increases of U as Gc increases in Fig. 2.

Fig.3. reveals the temperature profiles for different values of Prandtl number Pr. It is observed that the temperature decreases for an increase in the value of Prandtl number Pr. The reason is that smaller values of Prandtl number are responsible for increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface extra rapidly for higher values of Pr. Hence, in the case of larger Prandtl number the thermal boundary layer is thinner and the rate of heat transfer is reduced.

The concentration profiles are plotted in Fig. 4 for various values of Schmidt number Sc. From this figure, it is noticed that the concentration decreases with an increase in the values of the Schmidt number Sc. A comparison of curves in the figure shows a decrease in concentration with an increase in Schmidt number Sc. Actually it is true, since the increase of Sc means decrease of molecular diffusivity and therefore decreases in concentration boundary layer.

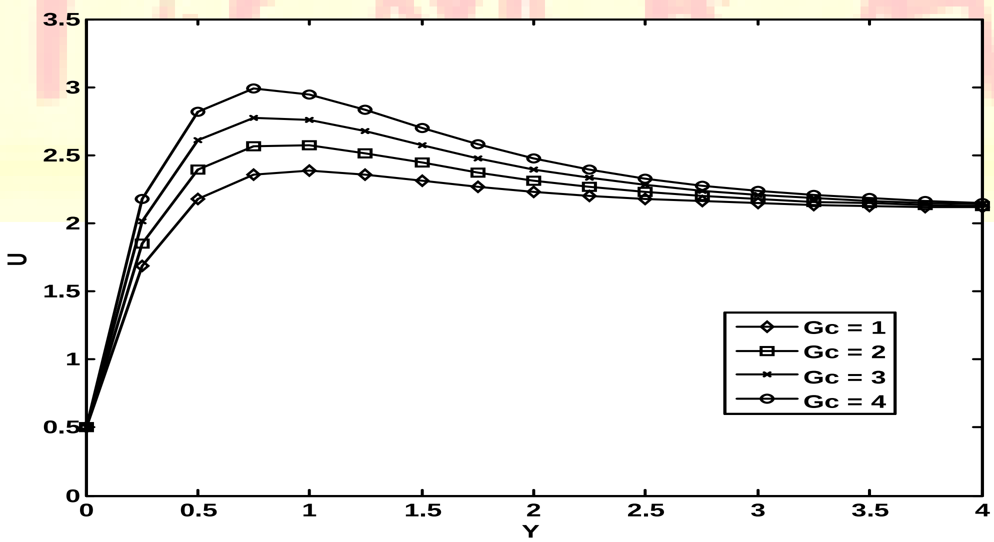

Figure 2. Effects of Gc on velocity profiles when A = 0.5, Pr = 0.7, G_T = 2.0, U_P = 0.5, k = 0.5, ε = 0.2, M = 1.0, t = 1, n = 0.1, Sc = 0.6, ϕ = 1.0.

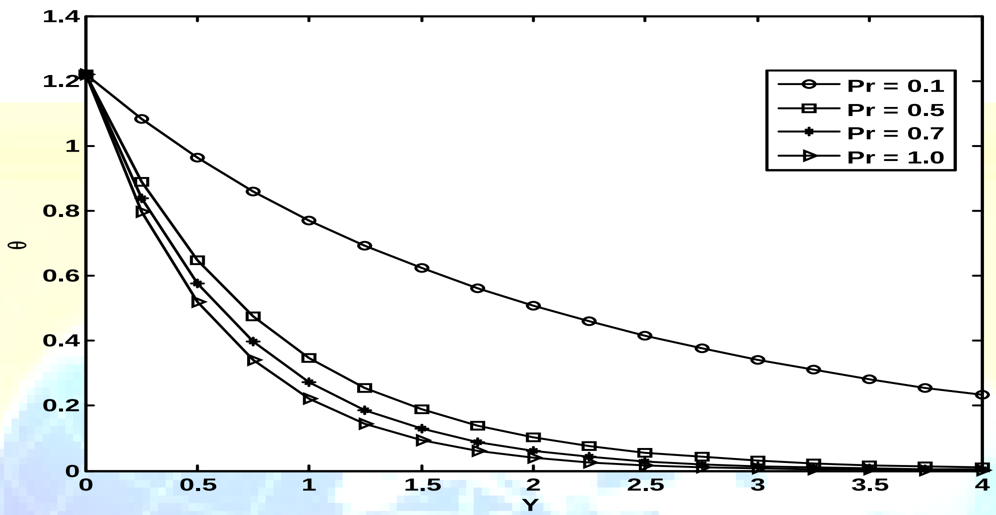

Figure 3. Effects of Pr on temperature profiles when A = 0.5, G_T = 2.0, G_C = 1.0, U_P = 0.5, k = 0.5, ε = 0.2, M = 1.0, t = 1, n = 0.1, Sc = 0.6, ϕ = 1.0.

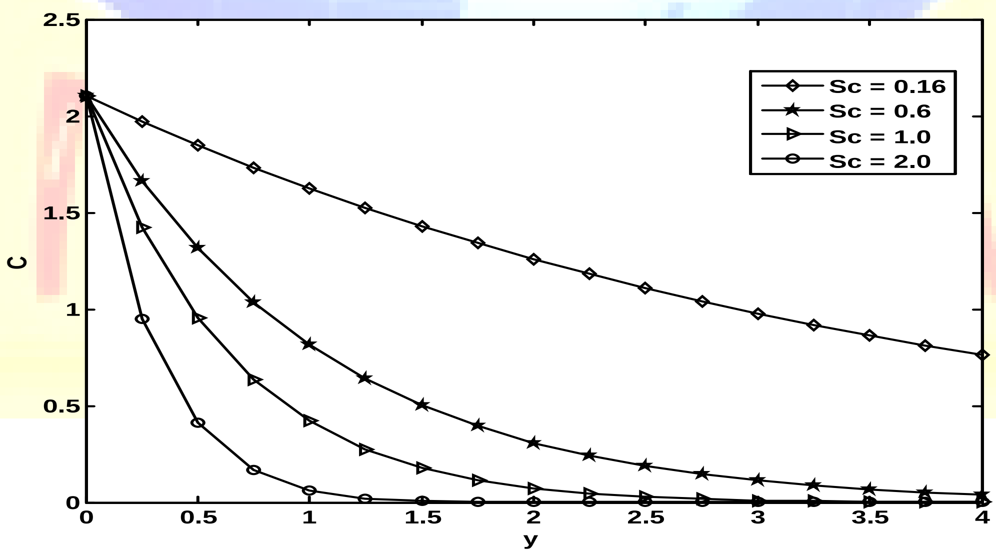

Figure 4. Effects of Sc on concentration profiles when A = 0.5, Pr = 0.7, G_T = 2.0, G_C = 1.0, U_P = 0.5, k = 0.5, ε = 0.2, M = 1.0, t = 1, n = 0.1, ϕ = 1.0.

Table: 1

ϕ	Pr	Nu
0	0.7	- 1.3649
1	0.7	- 2.0821
1	0.5	- 1.5522
1	1.0	- 1.8340

Table: 2

Sc	Sh
0.16	- 0.5470
0.6	- 1.9705
1.0	- 3.2730
2.0	- 6.5368

Table: 3

Gc	ϕ	K	M	Sc	G_T	Pr	Cf
1	1.0	0.5	1.0	0.6	2.0	0.7	3.5140
2	1.0	0.5	1.0	0.6	2.0	0.7	4.1443
3	1.0	0.5	1.0	0.6	2.0	0.7	4.7745
4	1.0	0.5	1.0	0.6	2.0	0.7	5.4047
1	0	0.5	1.0	0.6	2.0	0.7	3.7610
1.0	2	0.5	1.0	0.6	2.0	0.7	3.4092
1.0	3	0.5	1.0	0.6	2.0	0.7	3.3415
1.0	1.0	0.1	1.0	0.6	2.0	0.7	3.9917
1.0	1.0	0.2	1.0	0.6	2.0	0.7	3.6462
1.0	1.0	0.4	1.0	0.6	2.0	0.7	3.5187
1.0	1.0	0.5	2	0.6	2.0	0.7	3.5393
1.0	1.0	0.5	3	0.6	2.0	0.7	3.5871
1.0	1.0	0.5	4	0.6	2.0	0.7	3.6462

1.0	1.0	0.5	1.0	0.2	2.0	0.7	3.7000
1.0	1.0	0.5	1.0	0.78	2.0	0.7	3.4553
1.0	1.0	0.5	1.0	1	2.0	0.7	3.3968
1.0	1.0	0.5	1.0	0.6	1	0.7	3.0413
1.0	1.0	0.5	1.0	0.6	3	0.7	3.9867
1.0	1.0	0.5	1.0	0.6	4	0.7	4.4595
1.0	1.0	0.5	1.0	0.6	2.0	0.1	4.0510
1.0	1.0	0.5	1.0	0.6	2.0	0.5	3.6212
1.0	1.0	0.5	1.0	0.6	2.0	1	3.3906

Table 1 shows the effect of ϕ and Pr on Nusselt number. The Nusselt number decreases with an increase in ϕ and Pr. From table 2 it is noted that Sherwood number decreases due to an increase in Schmidt number. Table 3 shows the effect of Gc, ϕ, K, M, Sc, G_T, Pr on skin friction Cf. It can be observed that the skin friction coefficient increases with an increase in Gc, M, and G_T whereas it decreases with an increase of ϕ, K, Sc, and Pr.

5. Conclusions:

The present study is carried out to investigate the influences of heat and mass transfer on two dimentional MHD free convective, laminar and boundary layer flow of a viscous fluid along a semi-infinite vertical permeable moving plate in the presence of uniform transverse magnetic field and heat absorption. The dimensionless governing equations are solved by using the perturbation technique. The results for velocity, temperature and concentration are obtained and plotted graphically. The numerical results for skin friction, Nussle number and Sherwood number are computed in tables. It should be mentioned here that in the absence of the concentration buoyancy and heat asorption and $\varepsilon^2 = 0$, all of the flow and heat transfer solutions reported above are consistent with those reported earlier by Kim [16]. The main conclusions of this study are as follows.

1. Velocity of the fluid increases with an increasing value of Gc.

2. Temperature of the fluid decreases with an increasing values of Pr.

3. The fluid concentration decreases with increasing values of Sc.

4. Coefficient of skin friction received positive impact in case of Gc, M, and G_T, while negative effect in the case of ϕ, K, Sc, and Pr.

5. Nusselt number decreases with an increase in ϕ and Pr.

6. Sherwood number decreases for increasing values of Sc.

6. Appexdix:

$$\lambda_1 = \frac{1+\sqrt{1+4N}}{2}, \lambda_2 = \frac{1+\sqrt{1+4(N+n)}}{2}, \lambda_3 = \frac{1+\sqrt{1+4(N+2n)}}{2}, k_1 = 1+\frac{ASc}{n},$$

$$k_2 = \frac{ASc}{n}, m_1 = \frac{Sc+\sqrt{Sc^2+4nSc}}{2}, m_2 = \frac{Sc+\sqrt{Sc^2+8nSc}}{2}, m_3 = \frac{\Pr+\sqrt{\Pr^2+4\phi\Pr}}{2},$$

$$m_4 = \frac{\Pr+\sqrt{\Pr^2+4(n+\phi)\Pr}}{2}, m_5 = \frac{\Pr+\sqrt{\Pr^2+4(2n+\phi)\Pr}}{2}, \quad B_3 = \frac{Ak_2Sc}{2n}$$

$$B_1 = \frac{ScAk_1m_1}{m_1^2 - Scm_1 - 2nSc} + \frac{AK_2Sc}{2n}, B_2 = \frac{ScAk_1m_1}{m_1^2 - Scm_1 - 2nSc}, B_3 = \frac{Ak_2Sc}{2n}$$

$$B_4 = \frac{\Pr Am_3}{m_3^2 - m_3\Pr - (n+\phi)\Pr}, B_5 = \frac{\Pr Am_4(1-B_4)}{m_4^2 - m_4\Pr - (2n+\phi)\Pr}, B_6 = \frac{\Pr Am_3B_4}{m_3^2 - m_3\Pr - (2n+\phi)\Pr},$$

$$B_7 = -B_5 - B_6, B_8 = -\frac{G_T}{m_3^2 - m_3 - N}, B_9 = -\frac{G_c}{Sc^2 - Sc - N}, B_{10} = U_P - B_8 - B_9 - 1,$$

$$B_{11} = \frac{A\lambda_1 B_{10}}{\lambda_1^2 - \lambda_1 - (N+n)}, B_{12} = \frac{Am_3B_8}{m_3^2 - m_3 - (N+n)}, B_{13} = \frac{AScB_9}{Sc^2 - Sc - (N+n)}$$

$$B_{14} = \frac{G_T(1-B_4)}{m_4^2 - m_4 - (N+n)}, B_{15} = \frac{G_T B_4}{m_3^2 - m_3 - (N+n)}, B_{16} = \frac{Gck_1}{m_1^2 - m_1 - (N+n)},$$

$$B_{17} = \frac{Gck_2}{Sc^2 - Sc - (N+n)} \quad B_{18} = B_{14} + B_{15} + B_{16} - B_{11} - B_{12} - B_{13} - B_{17} - 1$$

$$B_{19} = \frac{A\lambda_2 B_{18}}{\lambda_2^2 - \lambda_2 - (N+2n)}, B_{20} = \frac{A\lambda_1 B_{11}}{\lambda_1^2 - \lambda_1 - (N+2n)}, B_{21} = \frac{Am_3B_{12}}{m_3^2 - m_3 - (N+2n)}$$

$$B_{22} = \frac{AScB_{13}}{Sc^2 - Sc - (N+2n)}, B_{23} = \frac{Am_4B_{14}}{m_4^2 - m_4 - (N+2n)}, B_{24} = \frac{Am_3B_{15}}{m_3^2 - m_3 - (N+2n)}$$

$$B_{25} = \frac{Am_1 B_{16}}{m_1^2 - m_1 - (N+2n)}, B_{26} = \frac{AScB_{17}}{Sc^2 - Sc - (N+2n)}, B_{27} = \frac{G_T B_7}{m_5^2 - m_5 - (N+2n)}$$

$$B_{28} = \frac{G_T B_7}{m_4^2 - m_4 - (N+2n)}, B_{29} = \frac{G_T B_6}{m_3^2 - m_3 - (N+2n)}, B_{31} = \frac{GcB_2}{m_1^2 - m_1 - (N+2n)}$$

$$B_{32} = \frac{G_c B_3}{Sc^2 - Sc - (N+2n)},$$

$$B_{33} = B_{23} + B_{24} + B_{25} + B_{27} + B_{28} + B_{29} + B_{31} + B_{32} - B_{19} - B_{20} - B_{21} - B_{22} - B_{26} - B_{30}$$

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