ON SB$\hat{g}$ – CONNECTED AND SB$\hat{g}$–COMPACT SPACES

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Abstract

The purpose of this paper is to introduce a new type of connected spaces called sb$\hat{g}$-connected space in topological spaces. The notion of sb$\hat{g}$-compact spaces and sb$\hat{g}$-Lindelof spaces are also introduced and their properties are studied. We discuss their relationship with already existing concepts. We also introduce sb$\hat{g}$-closure and discuss their properties.

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1. Introduction

In 1974, Das defined the concepts of semi-connectedness in topology and investigated its properties. Compactness is one of the most important, useful and fundamental concepts in topology. In 1981, Dorsett introduced and studied the concept of semi-compact spaces. Since then, Hanna and Dorsett, Ganster and Mohammad S.Sarask investigated the properties of semi-compact spaces. In 1990, Ganster defined and investigated semi-lindelof spaces. The notion of connectedness and compactness are useful of not only general topology but also of other advanced branches of Mathematics.

In 2015, K.BalaDeepaArasi and S.Navaneetha Krishnan introduced and studied the properties of sbĝ-closed sets in topological spaces. In this paper, we introduce the concepts of sbĝ-connected spaces, sbĝ-compact spaces and sbĝ-lindelof spaces. Also, we investigate their basic properties.

2. Preliminaries

Throughout this paper (X, τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), Cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1:[1] A subset A of a topological space (X,τ) is called a sbĝ-closed set if sCl(A) ⊆ U whenever A ⊆ U and U is bĝ-open in X. The family of all sbĝ-closed sets of X are denoted by sbĝ-C(X).

Definition 2.2:[1] The complement of a sbĝ-closed set is called sbĝ-open set. The family of all sbĝ-open sets of X are denoted by sbĝ-O(X).

Definition 2.3:[13] A topological space X is said to be connected if X cannot be expressed as the union of two disjoint non-empty open sets in X.

Definition 2.4:[9] A collection B of open sets in X is called an open cover of A ⊆ X if A ⊆ $\cup\{U_\alpha : U_\alpha \in B\}$ holds.

Definition 2.5:[10] A topological space X is said to be compact if every open cover of X has a finite subcover.

Definition 2.6:[9] A topological space X is said to be Lindelof if every cover of X by open sets contains a countable subcover.

Definition 2.7: A function f: (X,τ) → (Y,σ) is called a

1) sbĝ-continuous[2] if f^{-1}(V) is sbĝ-closed in X for every closed set V in Y.
2) sbĝ-irresolute[2] if f^{-1}(V) is sbĝ-closed in X for every sbĝ-closed set V in Y.
3) stronglysbĝ-continuous[3] if f^{-1}(V) is closed in X for every sbĝ-closed set V in Y.
4) sbĝ - open map[2] if f(V) is sbĝ-open in Y for every open set V in X.
5) contrasbĝ-continuous map[3] if f^{-1}(V) is sbĝ-closed in (X,τ) for every open set V in (Y,σ).

Definition 2.8:[12] A space (X, τ) is said to be locally indiscrete if every open subset of X is closed in X.

Definition 2.9:[1] A Space (X,τ) is called a $T_{sb\hat{g}}$-space if every sbĝ-closed set in X is closed.

Theorem 2.10:[12] A topological space X is connected if and only if the only clopen subsets of X are ϕ and X.

3. sbĝ–Connectedness

We introduce the following definitions.

Definition 3.1: Atopological space (X,τ) is called a sbĝ–connected space, if (X,τ) cannot be written as a disjoint union of two non–empty sbĝ–open sets. A subset of (X,τ) is sbĝ–connected if it is sbĝ–connected as a subspace of (X,τ).

Definition 3.2: A subset A of a topological space (X,τ) is called sbĝ-regular if it is both sbĝ-open and sbĝ-closed.

Theorem 3.3:A topological space X is sbĝ-connected if only if the only sbĝ-regular subsets of X are ϕ and X itself.

Proof:

Necessity:

Suppose X is a sbĝ-connected space. Let A be non-empty proper subset of X that is, sbĝ-regular. Then A and X\A are non-empty sbĝ-regular set. This is contradiction to our assumption.

Sufficiency:

Suppose X = A ∪ B where A and B are disjoint non-empty sbĝ-open sets. Then A = X\B is sbĝ-closed. Thus A is a non-empty proper subset that is, sbĝ-regular. This is contradiction to our assumption. Therefore, X is sbĝ-connected.

Theorem 3.4: A topological space X is sbĝ-connected if and if every sbĝ-continuous function of X into a discrete space Y with atleast two points is a constant function.

Proof:

Necessity:

Let f be a sbĝ-continuous function of the sbĝ-connected space into the discrete space Y. Then for each y ∈ Y, $f^{-1}(\{y\})$ is a sbĝ-regular set of X. Since X is sbĝ-connected, $f^{-1}(\{y\})$ = ϕ or X. If $f^{-1}(\{y\})$ = ϕ for all y ∈ Y, then f ceases to be a function. Therefore, f^{-1} $(\{y_0\})$ = X for a unique y_0∈ Y. This implies f(X) = { y_0 } and hence f is a constant function.

Sufficiency:

Let U be a sbĝ-regular set in X. Suppose U = ϕ. We claim that U = X. Otherwise, choose two fixed points y_1 and y_2 in Y. Define f: X → Y by $f(x) = \begin{cases} y_1 & \text{if } x \in U \\ y_2 & \text{otherwise} \end{cases}$

Then for an open set V in Y, $f^{-1}(V) = \begin{cases} U & \text{if V contains } y_1 \text{ only} \\ X\backslash U & \text{if V contains } y_2 \text{ only} \\ X & \text{if V contains both } y_1 \text{ and } y_2 \\ \Phi & \text{otherwise.} \end{cases}$

In all the cases $f^{-1}(V)$ is sbĝ-open in X. Hence f is non-constant sbĝ-continuous function of X into Y. This is a contradiction to our assumption. This proves that the only sbĝ-regular subsets of X are ϕ and X. Hence, X is sbĝ-connected.

Theorem 3.5: Every sbĝ–connected space is connected.

Proof: Let (X,τ) be a sbĝ–connected space. Suppose that (X,τ) is not connected. Then X=A⋃B where A and B are disjoint non–empty open sets in (X,τ). By proposition 3.4 in [1], A and B are sbĝ–open sets. Therefore, X=A⋃B, where A and B are disjoint non–empty sbĝ–open sets in (X,τ). This contradicts the fact that (X,τ) is sbĝ–connected and so (X,τ) is connected.

The converse of the above theorem need not be true as shown in the following example.

Example 3.6: Let X={a,b,c} and τ = {X, Φ}. Then (X,τ) is a connected space but not a sbĝ–connected space, because X = {a}∪{b,c}, where {a} and {b,c} are sbĝ–open sets in (X,τ).

Theorem 3.7: If (X,τ) is a $T_{sb\hat{g}}$– space and connected, then (X,τ) is sbĝ–connected.

Proof: Suppose X is not sbĝ–connected. Let A and B are two non–empty disjoint sbĝ–open subsets of X such that X =A∪B. Since X is a $T_{sb\hat{g}}$–space, A and B are open which is a contradiction to our assumption that X is connected. Hence X is sbĝ–connected.

Theorem 3.8: If f: (X,τ) → (Y, σ) is a sbĝ–continuous surjection and (X,τ) issbĝ–connected, then (Y,σ) is connected.

Proof: Suppose (Y,σ) is not connected, then Y=A∪B, where A and B are non–empty disjoint open sets of (Y,σ). Since f is a sbĝ–continuous onto map, X=$f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)\ and\ f^{-1}(B)$ are disjoint non–empty sbĝ–open sets in (X,τ). This contradicts the fact that (X,τ) is sbĝ–connected and so (Y,σ) is connected.

Theorem 3.9: If f: (X,τ) → (Y,σ) is a sbĝ–irresolute surjection and (X,τ) is sbĝ–connected, then so is Y.

Proof: Suppose (Y,σ) is not sbĝ–connected, then Y = A∪B, where A and B are disjoint non–empty sbĝ–open sets of (Y,σ). Since f is sbĝ–irresolute and onto, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)\ and\ f^{-1}(B)$ are disjoint non–empty sbĝ–open sets in (X,τ). This contradicts the fact that (X,τ) is sbĝ–connected and so (Y,σ) is connected.

Theorem 3.10: If f: (X,τ) → (Y,σ) is strongly sbĝ–continuous onto map, where (X,τ) is a connected space, then (Y,σ) is sbĝ–connected.

Proof: Suppose (Y,σ) is not sbĝ–connected, then Y = A∪B where A and B are disjoint non–empty sbĝ–open sets of (Y,σ). Since f is strongly sbĝ–continuous and onto, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)\ and\ f^{-1}(B)$ are disjoint non–empty open sets in (X,τ). This contradicts the fact that (X,τ) is connected and so (Y,σ) is sbĝ–connected.

Theorem 3.11: If f: (X,τ) → (Y,σ) is sbĝ-open and sbĝ-closed injection and Y is sbĝ-connected, then X is connected.

Proof: Let A be a clopen subset of X. Then, f(A) is sbĝ-regular in Y. Since Y is sbĝ-connected, f(A) = ϕ or Y. Hence A = ϕ or X. By theorem 2.10, X is connected.

Theorem 3.12: A contra sbĝ-continuous image of a sbĝ-connected space is connected.

Proof: Let (X,τ) → (Y,σ) be a contra sbĝ-continuous function from sbĝ-connected space X onto a space Y. Assume that Y is disconnected. Then, Y = A ∪ B where A and B are non-empty clopen sets in Y with A∩B = ϕ. Since f is contra sbĝ-continuous, we have $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty sbĝ-open sets in X with $f^{-1}(A) \cup f^{-1}(B)$= $f^{-1}(A \cup B)$=$f^{-1}(Y)$ = X and $f^{-1}(A) \cap f^{-1}(B)$ = $f^{-1}(A \cap B)$ = $f^{-1}(\phi)$ = ϕ. This shows that X is not sbĝ-connected which is a contradiction. Thus, Y is connected.

Theorem 3.13: Let X be a locally indiscrete space. Then the following are equivalent.

a) X is connected

b) X is sbĝ-connected.

Proof:

Follows from the definitions 2.3, 2.8 and 3.1.

4. sbĝ–Compact Spaces

We introduce the following definitions.

Definition 4.1: A collection $\{A_i$, i ∈ I} of sbĝ–open sets in topological spaces (X,τ) is called a sbĝ–open cover of a subset B is B ⊑∪$\{A_i$, i ∈ I}.

Definition 4.2: A topological space (X,τ) is said to be sbĝ–compact, if every sbĝ–open cover of X has a finite sbĝ–subcover.

Definition 4.3: A subset B of a topological space (X,τ) is said to be sbĝ–compact relative to X, if for every collection $\{A_i$, i ∈ I} of sbĝ–open subsets of X such that B ⊑∪$\{A_i$, i ∈ I}, there exists a finite subset I_0 of I such that B ⊑∪$\{A_i$, i ∈$I_0\}$.

Definition 4.4: A subset B of a topological space (X,τ) is said to be sbĝ–compact if B is sbĝ–compact as a subspace of (X,τ).

Theorem 4.5: A sbĝ–closed subset of sbĝ–compact space is sbĝ–compact relative to (X,τ).

Proof: Let A be a sbĝ–closed subset of a sbĝ–compact space X. Then A^C is sbĝ–open in X. Let S be a cover of A in X by sbĝ–open sets in X. Then {S,A^C} is a sbĝ–open cover of X. Since X is sbĝ–compact, it has a finite subcover say $\{C_1,C_2,\ldots\ldots\ldots C_n\}$. If thissubcovercontains A^C, we discard it. Otherwise we leave the subcover as it is. Hence we obtain a finite sbĝ–open subcover of A and so A is sbĝ–compact relative to X.

Theorem 4.6: A space X is sbĝ-compact if and only if every family of sbĝ-closed sets in X with empty intersection has a finite subfamily with empty intersection.

Proof: Suppose X is sbĝ-compact and $\{F_\alpha : \alpha \epsilon \Delta\}$ is a family of sbĝ-closed sets in X such that $\cap\{F_\alpha : \alpha \epsilon \Delta\}$ = ϕ. Then, $\cup\{X\backslash F_\alpha : \alpha \epsilon \Delta\}$ is a sbĝ-open cover for X. Since X is sbĝ-compact, this cover has finite subcover, say {X\\$F_{\alpha_1}$, X\\$F_{\alpha_2}$, ……X\\$F_{\alpha_n}$} for X. That is, X = $\cup\{X\backslash F_{\alpha_i} : i = 1,2,\ldots.n\}$. This implies that $\cap_{i=1}^{n} F_{\alpha_i}$ = ϕ.

Conversely, Suppose that every family of sbĝ-closed sets in X which has empty intersection. Let $\{U_\alpha : \alpha \epsilon \Delta\}$ be a sbĝ-open cover for X. Then $\cup\{U_\alpha : \alpha \epsilon \Delta\}$ = X. Taking the complements, we get$\cap\{X\backslash U_\alpha : \alpha \epsilon \Delta\}$ = ϕ. Since $X\backslash U_\alpha$ is sbĝ-closed for each $\alpha \epsilon \Delta$. By the assumption, there is a finite subfamily, {X\\$U_{\alpha_1}$, X\\$U_{\alpha_2}$, ……X\\$U_{\alpha_n}$} with empty intersection. That is, $\cap_{i=1}^{n} U_{\alpha_i}$ = ϕ. Taking the complements on both sides, we get $\cup_{i=1}^{n} U_{\alpha_i}$ = X. Hence, X is sbĝ-compact.

Theorem 4.7: A sbĝ–continuous image of a sbĝ–compact space is compact.

Proof: Let f: (X,τ) → (Y,σ) be a sbĝ–continuous onto map, where (X,τ) is a sbĝ–compact space. Let $\{A_i$, i ∈ I} be an open cover of (Y, σ). Then $\{f^{-1}(A_i)$, i ∈ I} is a sbĝ–open cover of (X,τ).

Since (X,τ) is sbĝ–compact, it has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2) \dots \dots f^{-1}(A_n)\}$. Since f is onto, $\{A_1, A_2 \dots \dots A_n\}$ is a finite open cover of (Y,σ) and so (Y,σ) is compact.

Theorem4.8: If a map f: (X,τ) → (Y,σ) is sbĝ–irresolute and a subset B is sbĝ–compact relative to (X,τ), then the image f(B) is sbĝ–compact relative to (Y,σ).

Proof: Let $\{A_i$, i ∈ I} be any collection of sbĝ–open subsets in (Y, σ). Since f is sbĝ–irresolute, $\{f^{-1}(A_i)$, i ∈ I} is also a collection of sbĝ–open sets in (X,τ). Now, since B is sbĝ–compact relative to (X,τ), for every collection $\{f^{-1}(A_i)$, i ∈ I} of sbĝ–open sets in (X,τ) such that B $\sqsubseteq \bigcup_{i \in I} f^{-1}(A_i)$, there exists a finite subsets I_0 of I such that B $\sqsubseteq \bigcup_{i \in I_0} f^{-1}(A_i)$. Therefore, f(B) $\sqsubseteq \bigcup_{i \in I_0} A_i$ and so f(B) is sbĝ–compact relative to (Y,σ).

Theorem 4.9: If f: (X,τ) → (Y,σ) is a strongly sbĝ–continuous onto map where (X,τ) is a compact space, then (Y,σ) is sbĝ–compact.

Proof: Let $\{A_i$, i ∈ I} be a sbĝ–open cover of (Y, σ). Then $\{f^{-1}(A_i)$, i ∈ I} is an open cover of (X,τ), since f is strongly sbĝ–continuous. Since (X,τ) is compact, it has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2) \dots \dots f^{-1}(A_n)\}$ and since f is onto, $\{A_1, A_2 \dots \dots A_n\}$ is a finitesubcover of (Y,σ) and hence (Y,σ) is sbĝ–compact.

Theorem 4.10: If f: (X,τ) → (Y,σ) is sbĝ-open function and Y is sbĝ-compact, then X is compact.

Proof: Let $\{V_\alpha\}$ be an open cover for X. Then, {f(V_α)} is a cover of Y by sbĝ-open set. Since Y is sbĝ-compact,

{f(V_α)} contains a finite subcover, namely { f($V_{\alpha 1}$), f($V_{\alpha 2}$),…… f($V_{\alpha n}$)}. Then { $V_{\alpha 1}$,$V_{\alpha 2}$, ……. $V_{\alpha n}$} is a finite subcover for X. Thus X is compact.

Definition 4.11: A space X is said to be sbĝ-Lindelof if every cover of X by sbĝ-open sets contain a countable subcover.

Remark 4.12: Every finite space is sbĝ-compact and every countable space is sbĝ-Lindelof.

Theorem 4.13: A space X is sbĝ-Lindelof if and only if every family of sbĝ-closed sets in X with empty intersection has a countable subfamily with empty intersection.

Proof: Suppose X is sbĝ-Lindelof and $\{F_\alpha : \alpha \epsilon \Delta\}$ is a family of sbĝ-closed sets in X such that $\cap\{F_\alpha : \alpha \epsilon \Delta\}$ = ϕ. Then, $\cup\{X \backslash F_\alpha : \alpha \epsilon \Delta\}$ is a sbĝ-open cover for X. Since X is sbĝ-Lindelof, this cover has countable subcover, say {X\\$F_{\alpha_i}$: i = 1,2,3,….} for X. That is, X = $\cup\{X \backslash F_{\alpha_i} : i = 1,2, \ldots\ldots\}$. This implies that $\cap_i (X \backslash F_{\alpha_i})$ = ϕ.

Conversely, Suppose that every family of sbĝ-closed sets in X which has empty intersection. Let $\{U_\alpha : \alpha \epsilon \Delta\}$ be a sbĝ-open cover for X. Then $\cup\{U_\alpha : \alpha \epsilon \Delta\}$ = X. Taking the complements, we get$\cap \{X \backslash U_\alpha : \alpha \epsilon \Delta\}$ = ϕ. Since $X \backslash U_\alpha$ is sbĝ-closed for each $\alpha \epsilon \Delta$. By the assumption, there is a countable subfamily, {X\\$U_{\alpha_i}$: i = 1,2,3,…..} with empty intersection. That is, $\cap_i (X \backslash U_{\alpha_i})$= ϕ. Taking the complements on both sides, we get $\cup_i U_{\alpha_i}$= X. Hence, X is sbĝ-Lindelof.

Theorem 4.14: Let f: (X,τ) → (Y,σ) be a sbĝ-continuous surjection and X be sbĝ-Lindelof. Then Y is Lindelof.

Proof: Let $\{V_\alpha\}$ be an open cover for Y. Since f is sbĝ-continuous function, $\{f^{-1}(V_\alpha)\}$ is a cover of X by sbĝ-open sets. Since X is sbĝ-Lindelof, $\{f^{-1}(V_\alpha)\}$ contains a countable subcover, namely $\{f^{-1}(V_{\alpha_n})\}$. Then $\{V_{\alpha_n}\}$ is a countable subcover for Y. Thus, Y is Lindelof.

Theorem 4.15: Let f: (X,τ) → (Y,σ) be a sbĝ-irresolute surjection and X be sbĝ-Lindelof. Then Y is sbĝ-Lindelof.

Proof: Let $\{V_\alpha\}$ be sbĝ-open cover for Y. Since f is sbĝ-irresolute function, $\{f^{-1}(V_\alpha)\}$ is a cover of X by sbĝ-open sets. Since X is sbĝ-Lindelof, $\{f^{-1}(V_\alpha)\}$ contains a countable subcover, namely $\{f^{-1}(V_{\alpha_n})\}$. Then $\{V_{\alpha_n}\}$ is a countable subcover for Y. Thus, Y is sbĝ-Lindelof.

Theorem 4.16: Let f: (X,τ) → (Y,σ) be a sbĝ-open function and Y be sbĝ-Lindelof. Then Y is Lindelof.

Proof: Let $\{V_\alpha\}$ be an open cover for X. Since f is sbĝ-open function, $\{f(V_\alpha)\}$ is a cover of Y by sbĝ-open sets. Since Y is sbĝ-Lindelof, {f(V_α)} contains a countable subcover, namely $\{f(V_{\alpha_n})\}$. Then $\{V_{\alpha_n}\}$ is a countable subcover for X. Thus, Y is Lindelof.

5. sbĝ-Closure

We introduce the following definition

Definition 5.1: Let A be a subset of a topological space (X,τ). Then the sbĝ–closure of A is defined to be the intersection of all sbĝ–closed sets containing A and is denoted by sbĝ–cl(A). That is, sbĝ–cl(A) = ∩{F: A⊑F and F ∈ sbĝ–C(X)}

Always A ⊑sbĝ–cl(A).

Remark 5.2: sbĝ–cl(A) is the smallest sbĝ–closed set containing A.

Theorem 5.3: Let A and B be subsets of a topological space (X,τ). Then

(i) sbĝ–cl(Φ) = Φ and sbĝ–cl(X) = X.

(ii) If A⊑ B, then sbĝ–cl(A) ⊑sbĝ–cl(B).

(iii) sbĝ–cl(A∩B) ⊑sbĝ–cl(A) ∩sbĝ–cl(B).

(iv) sbĝ–cl(A∪B) =sbĝ–cl(A) ∪ sbĝ–cl(B).

(v) A is a sbĝ–closed set in (X,τ) if and only if A = sbĝ–cl(A).

(vi) sbĝ–cl(sbĝ–cl(A)) = sbĝ–cl(A).

Proof:

(i) Obvious.

(ii) We have A ⊑ B ⊑sbĝ–cl(B). But sbĝ–cl(A) is the smallest sbĝ–closed set containing A. Hence,sbĝ–cl(A) ⊑sbĝ–cl(B).

(iii) We have A∩B ⊑ A and A∩B ⊑ B. From theorem 5.3(ii), sbĝ–cl(A∩B) ⊑sbĝ–cl(A) and sbĝ–cl(A∩B) ⊑sbĝ–cl(B). Hence,sbĝ–cl (A∩B) ⊑sbĝ–cl (A) ∩ sbĝ–cl (B).

(iv) Since A⊑A∪B and B⊑A∪B. From the above subdivision (ii), we have,sbĝ–cl(A) ⊑sbĝ–cl(A∪B) and sbĝ–cl(B) ⊑sbĝ–cl(A∪B). Hence, sbĝ–cl (A)∪ sbĝ–cl (B) ⊑ sbĝ–cl (A∪B). On the other hand, A ⊑sbĝ–cl(A) and B ⊑sbĝ–cl(B) implies that A∪B ⊑sbĝ–cl(A) ∪sbĝ–cl(B). But, sbĝ–cl(A∪B) is the smallest sbĝ–closed set containing A∪B. Hence, sbĝ–cl(A∪B) ⊑ sbĝ–cl(A) ∪ sbĝ–cl(B). Thus, sbĝ–cl (A∪B) =sbĝ–cl (A) ∪sbĝ–cl (B).

(v) **Necessity:** Suppose that A is a sbĝ–closed set in X. By remark 5.2, A ⊑sbĝ–cl(A). From definition 5.1 and hypothesis, we have sbĝ–cl(A)⊑A. Therefore, A= sbĝ–cl(A).

Sufficiency: Suppose that A = sbĝ–cl(A). From definition 5.1, sbĝ–cl(A) is a sbĝ–closed set in X.

(vi) From definition 5.1, sbĝ–cl(A) is a sbĝ–closed set in X. By (v), sbĝ–cl(sbĝ–cl(A)) = sbĝ–cl(A).

Remark 5.4: The reversible inclusion of theorem 5.3 (iii) is not true in general from the following example.

Example 5.5: Let X={a,b,c,d} with a topology τ = {X, Φ, {a},{b,c,d}}.

sbĝ–C(X) = {X, Φ, {a}, {b,c,d}}

If A = {b} and B = {c}, then sbĝ–cl(A) = {b,c,d} and sbĝ–cl(B) = {b,c,d}. Here, A∩B = Φ, sbĝ–cl(A∩B)=Φ.

But,sbĝ–cl (A)∩sbĝ–cl (B)={b,c,d}.

Hence, sbĝ–cl (A) ∩ sbĝ–cl (B)⋢sbĝ–cl (A∩B).

Remark 5.6: From theorem 5.3, subdivision (i), (iv) and (vi), we can say that sbĝ–closure is the kuratowski closure operator on (X, τ).

Theorem 5.7: In a topological space (X,τ), for every x∈X, x∈sbĝ–cl(A) if and only if U∩A≠ Φ for every sbĝ–open set U containing x.

Proof:

Necessity: Let x ∈sbĝ–cl(A) and suppose that there exists a sbĝ–open set U containing x such that U∩A=Φ. Then A ⊑U^c and U^c is a sbĝ–closed set. By remark 5.2, sbĝ–cl(A) ⊑U^c ⇒ x ∈ U^c ⇒ x ∉ U, a contradiction. Hence, U∩A≠ Φ.

Sufficiency: Let x ∉ sbĝ–cl(A). Then there exists a sbĝ–closed set F containing A such that x∉ F. Hence F^c is a sbĝ–open set containing x such that F^c⊑A^c. Therefore,F^c∩A= Φ which contradicts the hypothesis. Hence, x ∈ sbĝ–cl(A).

Definition 5.8: A point x in a topological space (X,τ) is called a sbĝ–interior point of a subset A of X if there exists some sbĝ–open set U containing x such that U⊑ A. The set of all sbĝ–interior points of A is called the sbĝ–interior of A and is denoted by sbĝ–int(A).

Remark 5.9:sbĝ–int(A) is the union of all sbĝ–open sets contained in A, hence sbĝ–int(A) is the largest sbĝ–open set contained in A.

Theorem 5.10: If A is a subset of a topological space (X, τ) then

(i) sbĝ–int(X \ A) = X \ sbĝ–cl(A).

(ii) sbĝ–cl(X \ A) = X \ sbĝ–int(A).

Proof:

(i) We have,sbĝ–int(A)⊑A⊑sbĝ–cl(A). Hence, X \ sbĝ–cl(A)⊑X \ A⊑X \ sbĝ–int(A). Then, X \ sbĝ–cl(A) is the sbĝ–open set contained in X \ A. But,sbĝ–int(X \ A) is the largest sbĝ–open set contained in X \ A. Therefore, X \ sbĝ–cl(A) ⊑sbĝ–int (X \ A). On the other hand, if x ∈sbĝ–int (X \ A), there exists a sbĝ–open set U containing x such that U⊑X\A. Hence, U∩A=Φ. Therefore,x∉ sbĝ–cl(A) and hence x∈ X \ sbĝ–cl(A). Thus sbĝ–int(X \ A) ⊑X \sbĝ–cl(A).

(ii) We have,sbĝ–int(A)⊑A⊑sbĝ–cl(A). Hence, X \ sbĝ–cl(A)⊑X \ A⊑X \ sbĝ–int(A). Then X \ sbĝ–int(A) is the sbĝ–closed set containing X \ A. But sbĝ–cl(X \ A) is the smallest sbĝ–closed set containing X \ A. Therefore, sbĝ–cl(X \ A) ⊑ X \ sbĝ–int(A). On the other hand, if x ∈ X \ sbĝ–int(A) ⇒ x∉ sbĝ–int(A)

⇒x∉ sbĝ–int(X \ A^c)

⇒x∉ X \ sbĝ–cl(A^c) [From Subdivision (i)]

⇒ x ∈ sbĝ–cl(X \ A)

Hence,X \ sbĝ–int(A) ⊑sbĝ–cl(X \ A). Thus,sbĝ–cl(X \ A) = X \ sbĝ–int(A).

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